

TREE STATION SIERRA LEONE TRIANGULATION

TEXT BOOK

OF

TOPOGRAPHICAL AND GEOGRAPHICAL SURVEYING

BY

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PREFACE.

THE methods described in this book are chiefly based on the actual experience of Royal Engineer Officers who are, or have been engaged in carrying out geographical and topographical surveys

Chapter IV, on 'Traverses,' is mainly written by Captain W J Johnston Captain H D Pearson and Captain C J Heath

Chapter VII, 'Surveys under Active Service Conditions,' is from the writings of Colonel Sir Thomas Holdich, K C M G, K C I E, Colonel Gore, C S I, late Surveyor General of India, Colonel the Hon M G Talbot Director of Surveys in the Sudan, and Colonel Wahab, C I E

Chapter VIII, 'Levelling,' is due to Captain H L Crosthwait and Captain C Russell Brown

A portion of Chapter X, the use of the theodolite for contouring is taken from a report by Major P H du P Casgrain

Chapter XII, 'The Reproduction of Maps in the Field,' has been written by Captain B A G Shelley with some additions by Lieutenant F V Thompson

Chapter XIV, 'Field Astronomy,' is based on the 'Notes on Astronomy,' written for the School of Military Engineering by Colonel A C MacDonnell, re-arranged by Captain C J Heath

In Chapter XVI the description of 'Photo-longitudes' is taken from the account published by Major E H Hills, C M G, in the 'Memoirs of the Royal Astronomical Society,' Vol LIII 1897

The section on 'Occultations' has been written by Captain C H Foulkes

The 'Graphic Method of predicting the circumstances of an Occultation' is due to Colonel S C N Grant C M G, and permission to use the diagrams explanatory of this method has been kindly granted by the Royal Geographical Society Appendix VIII is also due to Colonel Grant

The extension of the 'Graphic Method' is from a paper by Mr A C D Crommelin

Tables I, II, IX, XI and XII were computed by Major C H Enthoven

Tables XVII and XVIII were computed by Lieutenant G F Evans

The diagrams for rapidly determining the Time and Altitude of a star on the Prime Vertical were devised by Captain C J Heath

The Star Charts were drawn by Captain H F Winsloe

Mr A R Hinks, M A, of Cambridge Observatory, has very kindly read and criticised the chapters which deal with field astronomical methods, these chapters have benefited greatly in consequence

The reproductions of British and Foreign Topographical Maps were executed under the direction of Colonel D A Johnston, C B, Director General of the Ordnance Survey Most of

the other plates were drawn in the Topographical Section of the War Office under the direction of Major E. H. Hills, C.M.G., a few, however, have been produced by H.M. Stationery Office.

Plate V. is from a plane table sheet of Kroonstad by Lieut Colonel H. M. Jackson, Surveyor General of the Transvaal.

Plate VI. is from a plane table survey executed in Somaliland under Captain G. A. Beazeley.

Plate VII. is from an original field sheet lent by Lieut Colonel F. B. Longe, Surveyor General of India.

The field methods described are, for the most part, those in use by the Survey of India, but advantage has been taken of recent experience in mapping and exploring various territories in Africa and elsewhere to include useful methods which are not commonly employed in India.

CONTENTS.

LIST OF PLATES.

TEXT BOOK OF TOPOGRAPHICAL SURVEYING.

CHAPTER I.

INTRODUCTION. GENERAL PRINCIPLES.

Surveying is the process of accurately determining the relative positions of the features of a portion of the Earth's surface.

Topography (τόπος, γράφω, a description of a place) is the delineation of the natural and artificial features of a locality.

Topographical Surveying is thus the process of accurately determining the relative positions of the surface features of the earth, and of delineating them on a map. The surface features are of two kinds: artificial, that is to say those produced by the agency of man, such as towns, roads or railways; and natural features, such as rivers, lakes, forests, hill features and the undulations of the ground.

A Topographical Map should depict all the surface features which can be legibly drawn; all the accidents of the surface should be shown which distinguish it from a level, featureless plain. The amount of information presented on a topographical map is thus conditioned by two principal factors, the complexity of the surface features, and the scale of the map.

Since it is not customary to construct maps in the field on scales smaller than about $\frac{1}{500,000}$, this scale will be taken as the lower limit for topographical maps. Maps on scales smaller than this are compilations or atlas maps.

It is not the primary intention of a topographical map to show property boundaries; this is the function of a large-scale *Cadastral Plan*. Nor, on the other hand, has a topographical survey for its object the investigation of any scientific questions connected with the shape and dimensions of the earth; this is the domain of a *Geodetic Survey*.

The production of a map, in the ordinary sense of the word, is, in fact, the aim of a topographical survey.

The scales of topographical maps may vary from 6 inches to 1 mile to $\frac{1}{8}$ inch to 1 mile, or from about $\frac{1}{10,000}$ to $\frac{1}{500,000}$, but for military and administrative purposes, the most useful scales are 2 inches, 1 inch, $\frac{1}{2}$ inch, and $\frac{1}{4}$ inch to 1 mile, or say from $\frac{1}{30,000}$ to $\frac{1}{250,000}$, and this book is to be considered as dealing mainly with maps on such scales.

Topographical maps naturally vary in accuracy, according to the conditions under which they have been produced. Thus a plane table survey made rapidly on active service will be wanting in that refinement of accuracy which is to be expected of a map deliberately executed in peace time. In a broad sense, even a military sketch may be considered a rough

topographical map, but military sketches will not be considered in this book, which will deal with —

(1) Deliberate topographical surveys,
(2) Topographical surveys executed on service,
(3) Geographical surveys, such as are required for the delimitation of an international boundary
(4) Exploratory surveys

Topographical geographical and exploratory surveys merge into one another, and there is no very definite dividing line between them. Generally speaking, there should be no detectable error in a topographical map, whilst geographical surveys are somewhat rough and ready. The value of a survey on active service depends on the extent to which it assists the operations in progress and minute accuracy must often, in such a case, be sacrificed.

The extent of the field before the topographer may be realised from the fact that, excluding Canada, Australia, New Zealand and India, the total unmapped area of the British Empire amounts to about 3,700 000 square miles.

Main Principle to Observe in Carrying Out a Topographical Survey

The main principle to observe in all map making is to work from the whole to the part, and not from the part to the whole. It would be entirely wrong, for instance, to map a block of four English counties by making independent surveys of each county and then joining the surveys together. It is still more incorrect to attempt to compile a map of a large country by piecing together farm surveys. Such a proceeding, which has several times been attempted, results in a thoroughly untrustworthy map. The technical reasons for this will be better understood later on; meanwhile it must be accepted as an axiom that however large or however small may be the area to be surveyed, it must be treated as a whole, and that all over the area a number of carefully determined points must be fixed, forming, as it were, an accurate framework or skeleton on which the detail is to depend. It is clear that such a method prevents the accumulation of any system of errors except that of the accurate framework.

The accuracy of a map will thus ultimately depend on the accuracy of the framework, which will consist of points fixed by one of the following methods —

(1) Triangulation,
(2) Traverses,
(3) Astronomical determinations, or
(4) Combinations of the above

Of these triangulation is always to be preferred, and should be adopted whenever possible.

Triangulation.—It is an elementary proposition in trigonometry that, given the angles of a triangle and the length of one side, the lengths of the other sides can be calculated.

If then, we have a chain of triangles as below, and know all the angles and the length of one side, we can compute the lengths of all the other sides.

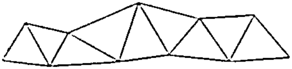


Fig. 1

The same clearly applies to a network or to any figure built up of triangles

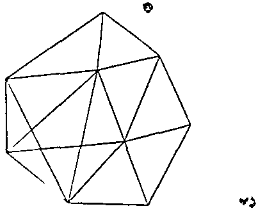


Fig 2

Now imagine that these points are objects on the earth's surface and that at each object horizontal angles have been observed to those surrounding it, and suppose that the horizontal distance between any two of these points has been measured, it is clear that the distance between any two connected points is easily computed and plotted on paper.

The measured side is called the *base*.

CLASSES OF TRIANGULATION

(1) **Principal Triangulation** or **Geodetic Triangulation** is the main triangulation of a country carried out with every refinement of care, of which the sides vary in length from about 10 to 50 miles (though rays well over 100 miles have been used), in which no precaution is neglected, and of which the computations are most exact and laborious. The triangular error of a principal triangle, *i.e.* the algebraic sum of the errors of the three angles of the triangle, would not be expected to exceed 1 second.

(2) **Secondary Triangulation**.—In a regularly surveyed country this would break up the principal triangles into sides of 5 to 10 miles in length, or else be carried out in the form of secondary chains connecting the principal chains. Triangular error, say, 2 to 5 seconds.

The observations are not usually reduced so rigorously as in principal work.

(3) **Tertiary Triangulation**.—The minor triangulation of a regularly surveyed country covering the whole surface of the land with triangles the sides of which are from 1 to 5 miles in length. Triangular error, say, 5 to 15 seconds.

In the Ordnance Survey of the United Kingdom—

The sides of the Principal Triangles average	35½ miles	
Secondary	5	,,
Tertiary ,,	1¼	

In the Survey of India the average length of the sides of the principal triangles is 32 miles.

(4) **Topographical Triangulation**.—Though it may be considered certain that in course of time the whole land surface of the world will be systematically surveyed and covered with principal, secondary, and tertiary triangles, still for some time to come there will be, in many cases, the need for topographical maps produced rapidly and with a minimum of expense without the aid of rigorous triangulation. As an example of a geographical survey carried out rapidly without waiting for a rigid framework, the survey of Burma may be

mentioned This country, which was annexed at the end of 1885 was rapidly surveyed on the scale of 4 miles to 1 inch The triangulation for this survey was carried out during the execution of the topography the sides were of very varying lengths the points observed were mainly pagodas or well defined hill tops, the sole object was to provide points for the plane tablers to work on, and provided there was no plotable error on the particular scale used, no further accuracy was demanded This sort of triangulation, of which the sole intention is to provide a framework sufficiently accurate for the scale in use, and which will never be used for geodetic or cadastral work or for larger scale topographical maps, may be called a *topographical triangulation*

(5) **Rapid Triangulation** —A topographical triangulation carried out during an expedition, or when marching with a column on active service, in this many devices, which would be objectionable in more deliberate triangulation are used to expedite the work

Other Methods of Forming the Framework

As has been mentioned on p 2 the other methods of forming a framework of accurately fixed points on which the detail of the map is to depend are *Traverse Astronomical Determinations* or combinations of these methods with each other or with triangulation

Traverses are described in Chapter IV

Astronomical Determinations are dealt with in Chapters XIV, XV, and XVI

Note

The framework should always be trigonometrical where the circumstances permit The two chief hindrances to triangulation are the absence of hills or the existence of forest, and the most difficult country to triangulate is a flat forest clad region In such a region the expense of triangulation may render it necessary to execute a framework of traverses

Generally speaking, astronomical determinations are to be avoided a map of which the framework is entirely astronomical falls far short of the standard of a good topographical map In exploratory and geographical surveys however it may be necessary to make a free use of astronomical methods

BASES

If the angles of a triangulation are correctly observed, any error in the measurement of the base will be reproduced in the triangulation. Thus if the measurement makes the base 1/[illegible] longer than it really is, and the furthest point of a triangulation is really 500 miles distant from the base, the triangulation will place that point at a distance of 500·1 miles, or ·1 mile too far.

Bases are of various degrees of accuracy according to the class of triangulation which is to depend on them, viz.:—

(1) Base lines of Geodetic Triangulation with a probable error of 1/[illegible] to 1/[illegible].
(2) Base lines for accurate Secondary work with a probable error of 1/[illegible] to 1/[illegible].
(3) Base lines for Topographical work with a probable error of 1/[illegible] to 1/[illegible].
(4) Rough base lines.

The measurement of a Geodetic Base line is a very special operation which occupies some weeks and costs much money. There is a large literature on the subject which will not be further referred to in this book.

Topographical Bases.—The *length* of a topographical base will depend on the stretch of level or evenly sloping ground available. Such bases have varied in length from ½ mile to 3 or 4 miles. *It is [illegible] important to be able to extend [illegible] by [illegible] well-conditioned triangles [illegible] from the ends of the base.*

The base at Mandalay measured in 1886 for the Topographical Survey of Upper Burma was 1 mile long.

The base for the Bloemfontein 2-inch Survey in 1900 was 5300 feet.

Base for Bermuda was 2890 feet.

Base for Guernsey was 3165 feet.

Base for the Sierra Leone Liberia Boundary was 3073 feet.

Site for a Base.—The ideal site is one on level even ground from which good views of the surrounding country can be obtained.

Measurement of Base.—A topographical base should always be measured with a steel tape. This should be carefully compared with a standard steel tape before and after using. The error of the standard at a given temperature should be known. This can be ascertained at home at the Ordnance Survey Office, Southampton; in India at the offices of the Survey of India; in South Africa at the Cape Observatory.

The standard tape itself should not be used.

Steel tapes can be bought of the following lengths: 100, 300 or 400 feet or 50 or 100 metres.

The line of the base should first be cleared of trees, bushes or high grass; ant heaps and such small mounds should be removed.

Each terminal point of the base should, if time allows, be marked by burying a marked stone 2 feet under ground; accurately centred over this should be built a small masonry pillar, surface flush with the ground. A fine mark should be made in a metal plug sunk into the surface of the pillar.

Having marked the terminal points of the base, set a theodolite over one of them and direct it on the other end, over which a pole should have been fixed; with the aid of the theodolite align bamboo poles at intervals of about 300 yards. By means of this alignment

hammer into the ground square-headed pickets accurately in the line of the base, and at tape lengths from each other, starting from one end of the base. The tops of the pickets should be just flush with the ground; on these pickets nail strips of wood or zinc.

In measuring the base, stretch the tape from terminal point to first picket, and from picket to picket; the tension should be taken on a spring balance and should be about 15 lb. The end of the tape should be marked by a fine pencil line on the wood strip, or by a cut with a knife. Take the temperature of the tape at frequent intervals by letting the bulb of the thermometer come in contact with the under side of the tape.

At the other terminal the small space between the last graduation and the end of the base should be measured with a pair of dividers.

Now measure the inclinations to horizontal from peg to peg with level or theodolite.

If the section consists of one or two fairly uniform slopes it is sufficient to take the inclinations of these only, noting where the changes of slope occur.

Minor undulations crossed by the tape are to be disregarded.

The base should be thus measured *at least once in each direction*, on a dull day, if possible, or in early morning or late afternoon.

Base Computations.—The measured length of the base is subject to five corrections*:—

i. For *standard*,
ii. " *temperature*,
iii. " *inclination*,
iv. " *sag*,
v. " *height above sea*.

The following case may be taken as an example:—

One tape was kept as a reference tape and not used for measuring. This, when tested at Southampton, had been found to be 0.05 feet short at 62° F.

The base tape (supposed to be 100 feet long) used was 0.02 feet long on reference tape at 71° F.

The temperature of base tape during measurement was 82° F.

Length as measured 5301.24 feet.

Height of base line above sea 4,520 feet.

For 2,300 feet, base had a slope of 1 in 15; remainder level.

i. *Standard.*—

Reference tape was 0.05 short at 62° F.

Now steel expands .00000625 of its length for 1° F.

Therefore, at 71° it was $\frac{9 \times 100 \times 6.25}{10\,000\,000}$ longer, or .0056 feet longer.

Reference tape was therefore 0.0444 short at 71° F.
Base " " " 0.0200 long on reference tape at 71°.

" 0.0244 short at 71°. (*a*)

ii. *Temperature.*—

Temperature of base tape during measurement was 82°, when computed was 71°.

$$\text{Increase of length } \frac{11 \times 100 \times 6.25}{10\,000{,}000}, \text{ or } 0.0069 \qquad (b)$$

Subtracting (*b*) from (*a*) we find that the base tape during measurement was 0.0175 short; correction for standard and temperature is therefore

$$-.0175 \times \frac{5301}{100} = -0.928 \text{ feet}.$$

* There is a sixth correction for *elastic extension*, which may be disregarded if the tape is always stretched at the same tension.

iii *Inclination* —

For 2 300 feet the base had a slope of 1° 15′

Correction = − 2 300 (1 − cos 1° 15′) = 0 550

iv *Sag* —

In the measurement of a base it is desirable that the whole length of the tape should as far as possible be supported by the ground, so that no correction is necessary for the sag of the tape Where the ground is uneven, it is customary to support the tape at intervals by pickets, but it may happen in the measurement of a base that a nullah or ravine has to be crossed In such a case the sag s the difference between the length when suspended and the length when laid on a plane surface, must be determined and corrected for

If s = the correction for sag

l = the length of the tape suspended between two supports (in feet)

w = the weight of the tape in lbs

t = the tension applied in lbs

$s = l \frac{w^2}{24t^2}$ approximately

See Appendix V

Height above sea —

All measurements are supposed to be made at sea level If the base is measured at a mean height h above sea level it will require a correction of $\frac{h}{R}$ × length of base where R the radius of the earth may be taken as 20 900 000 feet

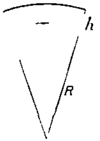


Fig 3

In this case h = 4 520

Then correction = $\frac{4,520 \times 5\ 301}{20\ 900\ 000}$ = 1 146

Collecting these corrections

	Feet
i and ii Standard and temperature	0 928
iii Inclination	0 550
iv Sag	− 0 000
v Height above sea	− 1 146
Total correction	2 624
Measured length	5301 240
Reduced length of base	5298 62

It is clear that corrections iii and iv must always be negative, i and ii may be either positive or negative

hammer into the ground square headed pickets accurately in the line of the base and at tape lengths from each other, starting from one end of the base. The tops of the pickets should be just flush with the ground; on these pickets nail strips of wood or zinc.

In measuring the base stretch the tape from terminal point to first picket and from picket to picket; the tension should be taken on a spring balance and should be about 15 lb. The end of the tape should be marked by a fine pencil line on the wood strip, or by a cut with a knife. Take the temperature of the tape at frequent intervals by letting the bulb of the thermometer come in contact with the under side of the tape.

At the other terminal, the small space between the last graduation and the end of the base should be measured with a pair of dividers.

Now measure the inclinations to horizontal from peg to peg with level or theodolite.

If the section consists of one or two fairly uniform slopes it is sufficient to take the inclinations of these only, noting where the changes of slope occur.

Minor undulations crossed by the tape are to be disregarded.

The base should be thus measured *at least once in each direction* on a dull day, if possible, or in early morning or late afternoon.

Base Computations.—The measured length of the base is subject to five corrections*:—

i. For *standard*.
ii. " *temperature*.
iii. " *inclination*.
iv. " *sag*.
v. " *height above sea*.

The following case may be taken as an example:—

One tape was kept as a reference tape and not used for measuring. This when tested at Southampton had been found to be 0·05 feet short at 62° F.

The base tape (supposed to be 100 feet long) used was 0·02 feet long on reference tape at 71° F.

The temperature of base tape during measurement was 82° F.

Length as measured 5301·24 feet.

Height of base line above sea 4520 feet.

For 2300 feet base had a slope of 1 in 15, remainder under level.

i. *Standard*.—

Reference tape was 0·05 short at 62° F.

Now steel expands ·00000625 of its length for 1° F.

Therefore at 71° it was $\frac{9 \times 100 \times 6 \cdot 25}{10,000,000}$ longer, or ·0056 feet longer.

Reference tape was therefore 0·0444 short at 71° F.

Base " 0·0200 long on reference tape at 71°.

0·0244 short at 71°. (*a*)

ii. *Temperature*.—

Temperature of base tape during measurement was 82°; when compared was 71°.

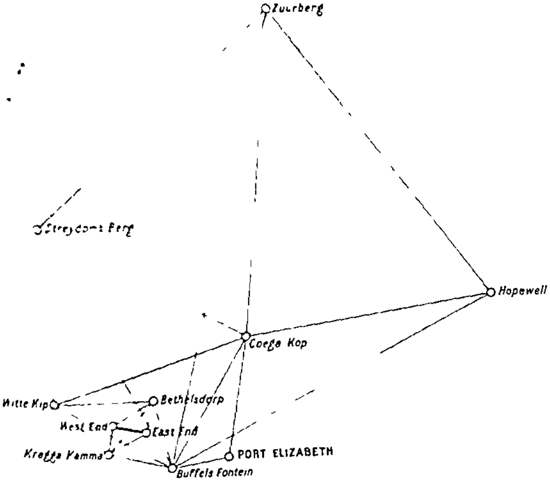
$$\text{Increase of length } \frac{11 \times 100 \times 6 \cdot 25}{10,000,000} \text{ or } 0 \cdot 0069 \qquad (b)$$

Subtracting (*b*) from (*a*) we find that the base tape during measurement was 0·0175 short; correction for standard and temperature is therefore

$$- \cdot 0175 \times \frac{5301}{100} = -0 \cdot 928 \text{ feet}$$

* There is also a correction for *elastic extension*, which may be disregarded if the tape is always stretched to the same tension.

PORT ELIZABETH BASE EXTENSION



Scale 11·7 Miles = 1 Inch

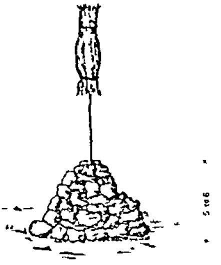
expansion of any metal or alloy, *i.e.* $\frac{1}{10}$ that of platinum or $\frac{1}{20}$ that of brass (Less than $\frac{1}{1,000,000}$ for 1° Centigrade)

Tapes of this metal are now available, 25 metres long, or of special lengths if required

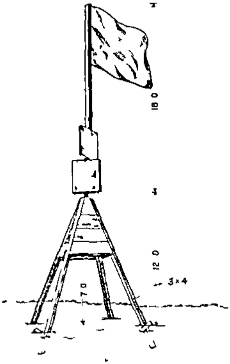
The agent for the sale of Invar in England is Mr T H A Baugh, 92, Hatton Garden, London The metal requires special treatment in manufacture and is liable to slow residual variations of length, it is therefore necessary that tapes used for the measurement of accurate bases should be tested by exact standards before and after measurement

The use of Invar for the measurement of good topographical bases renders negligible the errors caused by uncertainty of tape temperatures

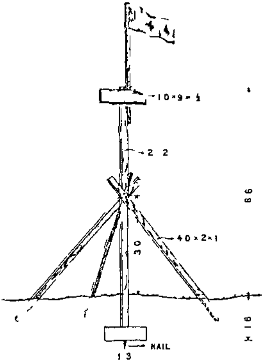
TRIGONOMETRICAL SIGNALS FOR SHORT DISTANCES



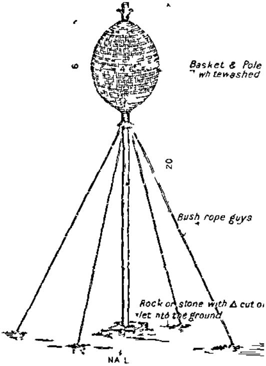
INDIAN POLE AND BRUSH SIGNAL



QUADRIPOD SIGNAL



TRIGONOMETRICAL POLE



W T AFRICAN BASKET SIGNAL

should be some 4 or 5 inches apart on paper. Thus for triangulation for a 1 inch map the sides would be 4 or 5 miles long; for a ¼ inch map some 16 to 20 miles long. But so much will depend on the nature of the country that this rule must only be treated as a rough guide.

Chain or Net.—When the extension of the base is completed in a large country, such as India or South Africa, the geodetic triangulation is carried out in the form of chains of triangles running for preference N. and S., and E. and W. thus:—

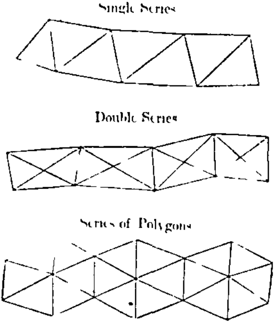


Fig. 4.

Secondary chains are run to fill up the spaces between the principal chains.

In small countries, such as the United Kingdom, the country is covered with a network of principal triangles.

Triangulation executed rapidly during a march or along a frontier during a boundary survey will usually take the form of a chain, but of course less symmetrical and less well conditioned than a principal or secondary chain.

A topographical triangulation, which is essentially required to provide points for the detail surveyors, will necessarily cover with a net the whole country to be surveyed.

Intersected Points.—In tertiary, topographical and rapid triangulation, angles are often observed to objects which are not visited; such objects are called intersected points, and should always be fixed by at least three rays. The extension of the triangulation should not, except in the case of rapid triangulation, depend on these points, which are, however, very often most useful to the detail surveyors. Such objects will be spires, tall chimneys, pagodas, or inaccessible rocks or trees.

The Theodolite.—The angles of a triangulation are invariably observed with a theodolite, which is an instrument devised for the measurement of horizontal and vertical angles.

Theodolites are designated by the diameter in inches of their engraved horizontal circles.

In times past in geodetic triangulation theodolites have been used with circles as much as 36 inches in diameter; but improvements in graduation of the circles have made it possible to obtain even better results with far smaller instruments. In Burma, for instance, the geodetic triangulation was executed with a 12 inch, and in South Africa with a 10 inch theodolite.

For ordinary topographical work it may be assumed that the triangulation will be carried out with one of the following instruments —

(1) 5 inch micrometer transit theodolite
(2) 6 inch vernier „ „
(3) 5 inch „ „ „

A transit theodolite is one in which the telescope can be rotated vertically end for end and in which the vertical circle is graduated throughout its circumference

Verniers and micrometers are different means of reading graduated circles

Verniers—

Principle of the Vernier—If the length of six, three divisions of a primary scale be taken and divided into four (fig 5) it is evident that the length of each of the latter is ¾ that of a primary division and that the difference between the length of one of the four divisions and

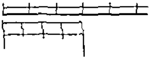


Fig 5

the length of a primary division is ¼ of a primary division and generally if $(n-1)$ primary divisions be divided into n divisions to form a vernier, then the difference in length between a primary and a vernier division is $\frac{1}{n}$th of a primary division

The same result obtains if we take six, five divisions of a primary scale and divide it into four for a vernier (fig 6) each of the vernier divisions is then ⁵⁄₄ of a primary division,

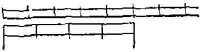


Fig 6

and the difference between their lengths is as before, ¼ of a primary division and generally if $n+1$ primary divisions be divided into n vernier divisions the difference between the length of a primary and a vernier division is $\frac{1}{n}$th of a primary division

For the application of this principle see fig 7 below which represents a portion of the graduated arc of a theodolite

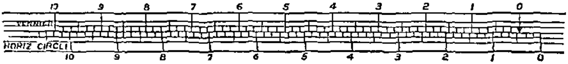
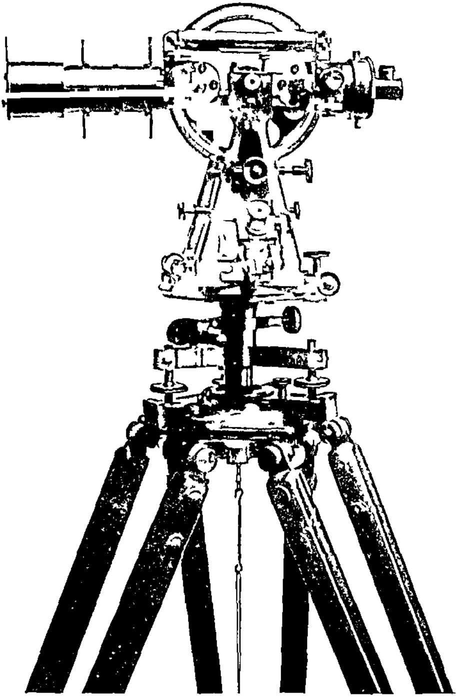


Fig 7

In this example the lower set of numbers represents degrees, these in turn are divided into six divisions each representing 10 minutes

The upper set of figures represents minutes, these are also divided into six divisions each being equal to 10 seconds To read the present example it will *be seen that the arrow of the* vernier plate comes between the second and third divisions of the lower plate, which indicates the reading to be between 20 and 30

PLATE III



5 Micrometer Transit Theodolite

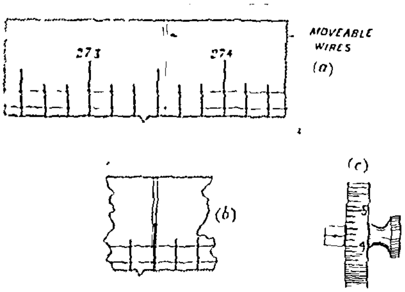


Fig. 8

lines are the divisions of the limb, the numbers are degrees, and the limb in this example is divided into 10 minute spaces. At the *bottom* of the field will be seen a V shaped notch in the frame of the diaphragm. *It is the position* of this notch which it is required to measure.

It will be at once seen that the notch is somewhere between *273° 20′* and *273° 30′*. Two fine vertical lines will be noticed; these can be moved laterally by turning the graduated micrometer head, of which a separate sketch is shown in fig. 8 (*c*). These should be moved laterally until they occupy a position one on each side of a graduation on the limb, as in (*b*), and the graduation selected should be that which is nearest to the notch; it is *immaterial* which side. The minutes and seconds can now be read off the graduated head of *the micrometer* (*c*); in this example the graduated head reads 4′ + between 10″ and 20″, by estimation 4′ 17″. The complete reading is therefore 273° 24′ 17″.

Taking Runs —It is part of the principle of this apparatus that a lateral movement of the parallel wires from one division of the limb to the next should be accomplished in a complete revolution of the micrometer head This should be tested occasionally before beginning work, the test is called "taking runs" If the mean of two or three runs does not differ from the proper amount by more than 2 or 3 seconds, the error for topographical work may be neglected Should there be a large error, it means that the microscope is not properly adjusted

Adjustment for "Run"—The adjustment will be described as applied to a 5 Micro microscope theodolite by Messrs Troughton & Simms (See Plate III)

Take runs four times each on each microscope Determine the error of the mean of the two microscopes, and correct this by altering one microscope twice the amount Thus supposing that the mean of two microscopes, instead of reading 10 in traversing the wires from one division of the limb to the next, reads 10 5, correct one of the microscopes so that it shall read 10 less than it did in a run *i e* the image of the limb in the field of the microscope must be diminished in size

Now if it is wished to increase or diminish the power of the microscope it will be necessary to increase or decrease the distance between the object glass and the wires

The microscope has at the end a screw in the body, *b*, a clamping ring embracing the

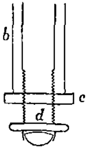


Fig 9

object glass cell *c* and a cell *d* carrying the object-glass the cell having a long screw cut on it for the purpose of adjustment

First release the ring *c* then move *d* towards the limb if it is desired to increase the readings then take care to make fast again

A little adjustment for focus will be necessary after every alteration, this is done by releasing the screw in the aluminium socket and sliding the whole microscope towards or from the graduations as may be necessary

In practice it is found that the microscopes very rarely need adjustment Instruments of this class have recently been much used on boundary commissions and even on active service Their advantages over vernier instruments are, that they are more accurate and more quickly read no one accustomed to micrometers would willingly go back to verniers

Examination of Theodolite—On receiving a theodolite examine it to see if it is in good order

Inspect the stand and see that there is no shakiness in any part of it When the feet are shod with metal it sometimes happens that shrinkage of the wood allows a little play in one of the feet which is quite enough to vitiate the observations

The theodolite box should be opened, the positions of the object and eye ends of the telescope marked on the box, and the places of the tangent screws of the upper and lower plates carefully noted

Setting up at a Station—Supposing now, that the observer has reached the station of observation with his instrument.* The first thing to do is to place the stand with its legs wide apart, so as to give a firm base and with its centre exactly over the station dot, its head being as nearly level as the eye can judge. Dust the grooves of the tribrach, and having taken out the lower portion of the theodolite—remembering always to hold it by the strongest parts—place it gently on the stand and clamp it to the tribrach.

It is well at this stage to level the upper plate approximately, as it is quite possible that it may be necessary to alter the position of the legs in order to avoid bringing undue strain on the foot screws. If the legs are moved, care must be taken to keep the instrument centred over the station dot.

Having centred and approximately levelled the lower portion of the theodolite, open the cleats and dust the Y supports of the transit axis of the telescope. Take out the telescope, wipe the transit axis and lower it gently into the Y's; close the cleats and screw them down. Take the cap off the object glass and focus the telescope. Dust with a soft camel's hair brush the horizontal and vertical circles. Set the vernier or micrometer lenses. Make quite sure that there is no shake in any part of the theodolite or stand, and that the feet of the latter rest upon firm ground into which there is no fear of their sinking during the observations. In loose or marshy soil hammer in pickets on which the feet of the theodolite may rest.

See that the tangent screws are in about the centre of their run.

THE ADJUSTMENTS OF THE THEODOLITE

Adjustments of the Telescope, Focus and Parallax.—To focus the telescope first remove the cap from the object glass. If this moves on a hinge no remark is necessary; but if it pulls off care must be taken to draw it with a screwing up motion to avoid any chance of the object glass being partially unscrewed.

The telescope should now be pointed to the sky and the eye-piece pushed in or pulled out, also with a screwing up motion, until the wires are seen with perfect distinctness.

The telescope is now directed on to some distant and well defined object and the object glass moved in or out until distinct vision is obtained. When this has been done the foci of the object glass and the eye-piece should coincide on the plane of the wires.

To test the focus, move the eye slightly to one side and note if the object appears to move relatively to the wires. If it does not move the focus is perfect, but if there is movement, it is due to imperfect focus or *parallax*, which must be eliminated before observing is commenced. To effect this note whether the object moves with the eye or against it—*i.e.* if when the eye is moved to the left the object moves to the left or to the right of the wires. If the former, it shows that the focus of the object glass is beyond the wires and the object glass must therefore be brought nearer to the observer. If the latter the focus is too near, and the object glass must be moved further out.

From what has been said above, it will be seen that the proper position of the eye-piece varies with the eyesight of the individual observer, but not with the distance of the object; while the position of the object glass is unaffected by the eyesight of the observer but varies with the distance of the object.

The object glass, however, will be set at the same position, called *Solar or Stellar focus*, for all objects so far distant that the rays falling on it from them may be taken as practically parallel to one another.

In the case of a theodolite of the class described this applies to all objects not less than half a mile from the observer, but for nearer objects at any distance over 250 yards the change required is very slight.

* For very accurate work with large instruments isolated pillars are built. In this case it is assumed that the legs of the stand rest upon the natural soil or rock.

Taking Runs.—It is part of the principle of this apparatus that a lateral movement of the parallel wires from one division of the limb to the next should be accomplished in a complete revolution of the micrometer head. This should be tested occasionally before beginning work; the test is called "taking runs." If the mean of two or three runs does not differ from the proper amount by more than 2 or 3 seconds, the error, for topographical work may be neglected. Should there be a large error it means that the microscope is not properly adjusted.

Adjustment for "Run".—The adjustment will be described as applied to a 5" Micro-microscope theodolite by Messrs. Troughton & Simms. (See Plate III.)

Take runs four times each on each microscope. Determine the error of the mean of the two microscopes, and correct this by altering one microscope twice the amount. Thus supposing that the mean of two microscopes, instead of reading 10' in traversing the wires from one division of the limb to the next reads 10' 5", correct one of the microscopes so that it shall read 10" less than it did in a run, *i.e.* the image of the limb in the field of the microscope must be diminished in size.

Now if it is wished to increase or diminish the power of the microscope, it will be necessary to increase or decrease the distance between the object glass and the wires.

The microscope has at the end a screw in the body, *b* a clamping ring embracing the

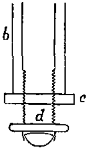


Fig. 9

object glass cell *c* and a cell *d* carrying the object glass, the cell having a long screw cut on it for the purpose of adjustment.

First release the ring *c* then move *d* towards the limb if it is desired to increase the reading, then take care to make fast again.

A little adjustment for focus will be necessary after every alteration; this is done by releasing the screw in the aluminium socket and sliding the whole microscope towards or from the graduations as may be necessary.

In practice it is found that the microscopes very rarely need adjustment. Instruments of this class have recently been much used on boundary commissions and even on active service. Their advantages over vernier instruments are, that they are more accurate and more quickly read. No one accustomed to micrometers would willingly go back to verniers.

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It is well at this stage to level the upper plate approximately, as it is quite possible that it may be necessary to alter the position of the legs in order to avoid bringing undue strain on the foot-screws. If the legs are moved, care must be taken to keep the instrument centred over the station dot.

Having centred and approximately levelled the lower portion of the theodolite, open the clamps and dust the Y supports of the transit axis of the telescope. Take out the telescope, wipe the transit axis and lower it gently into the Y's; close the cleats and screw them down. Take the cap off the object glass and focus the telescope. Dust with a soft camel's hair brush the horizontal and vertical circles. Set the vernier or microscope lenses. Make quite sure that there is no shake in any part of the theodolite or stand, and that the feet of the latter rest upon firm ground, into which there is no fear of their sinking during the observations. In loose or marshy soil hammer in pickets on which the feet of the theodolite may rest.

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The telescope is now directed on to some distant and well-defined object, and the object glass moved in or out until distinct vision is obtained. When this has been done the foci of the object glass and the eye-piece should coincide on the plane of the wires.

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From what has been said above, it will be seen that the proper position of the eye-piece varies with the eyesight of the individual observer, but not with the distance of the object; while the position of the object glass is unaffected by the eyesight of the observer, but varies with the distance of the object.

The object glass, however, will be set at the same position, called *stellar* or *solar focus*, for all objects so far distant that the rays falling on it from them may be taken as practically parallel to one another.

In the case of a theodolite of the class described, this applies to all objects not less than half a mile from the observer; but for nearer objects at any distance over 250 yards the change required is very slight.

* For very accurate work with large instruments isolated pillars are built. In this case it is assumed that the legs of the stand rest upon the natural soil or rock.

When the proper adjustment of the eye-piece and of the object glass for the solar focus have been found it is convenient to mark the tubes, so that on subsequent occasions focus may be found at once

Adjustment for Collimation in Azimuth—A theodolite is said to be collimated in azimuth when the *line of collimation*, i.e., the imaginary straight line joining the intersection of the wires and the optical centre of the object glass cuts the transit axis or axis of *vertical* motion at right angles

In fig 10 let tt_1 denote the transit axis, then when the line of collimation, which for shortness call c coincides with the perpendicular to tt_1, or the line pp_1, the instrument is in collimation and if the telescope be made to describe a semicircle in its Y s, its line of collimation produced will pass through the zenith, describing a great circle But if c be represented by ow, then the semicircle traced out will be represented by oo_1 the false zenith being z_1 while the arc described will be that of a small circle In other words, by changing face, the object end or o, will arrive at o_1, while the wires, w, will describe a similar small circle and reach w_1 Whence if any object on the horizon give a horizontal reading at face left of o and then the pivots of the instrument be changed and the telescope be turned sufficiently in azimuth to intersect the object the two readings thus obtained will instead of exactly coinciding, disagree by oo_1

The error in collimation will, therefore, be $\frac{oo_1}{2} = oz$ *

Description of the Diaphragm—Before entering into the methods of collimating, a short description of the diaphragm is necessary

The wires the arrangement of which varies in different theodolites, are attached to a metal ring or diaphragm, whose position in the tube of the telescope can be regulated by

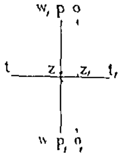


Fig 10

means of four capstan headed screws, a, b, c, d (see fig 11), which pass loosely through slots, S, in the tube so as to allow the diaphragm to be moved as required Each of these slots is covered by a small plate to exclude dirt

If it is necessary to move the diaphragm towards b, loosen the screw a and slacken the screws c and d then by screwing b into the tube, the diaphragm is drawn in its direction When it has been moved sufficiently, tighten up the three screws a c, d

If an ordinary eye piece is used with the usual inverting telescope with the result that the image is seen upside down, it must be remembered that, if the object that has to be intersected appears to the left of the wire the wire must be shifted to the left and the diaphragm moved accordingly

* A theodolite is said to be face left or face right when the vertical circle is on the observer's left or right Face is changed by turning the telescope through 180 degrees vertically so as to bring the object glass where the eye piece was before If now the instrument is turned in azimuth so as to bring the eye piece to the observer's eye the vertical circle will be found to be on the opposite side to that it occupied previously with reference to the observer

Methods of Collimating in Azimuth.—There are three methods of collimating in azimuth:

(1) By changing pivots. This process would be used for all small theodolites the telescope of which cannot revolve through a complete vertical circle.

(2) By changing face. This is applicable to those theodolites only whose telescope can revolve through a complete vertical circle (*i.e.* transit theodolites).

(3) By Gauss's method. This is applicable to all theodolites, but as it requires two other telescopes fitted with cross wires it is comparatively seldom available, and will, therefore, not be treated of further.

Collimating by Changing Pivots.—Throw open the clips and see that the clip screws of the index arm are barely biting their shoulder and no more. Intersect the object azimuthally, and note the reading, taking care that the clamps are holding firmly. Next, release the clip screws, raise the telescope gently and revolve the transit axis through 180 degrees, so that the level comes underneath, leaving the index arm and clip screws upwards. Replace the telescope gently with the pivots changed and with the object glass still pointing towards the object. If the wires in this second position of the telescope still intersect the object, the instrument is in collimation; if not, complete the intersection by the tangent screw of the upper plate and note the second reading. Set the instrument to the mean of the two readings, and then, looking into the telescope, move the diaphragm by its collimating screws *a*, *b*, *c*, *d*, fig. 11, until the

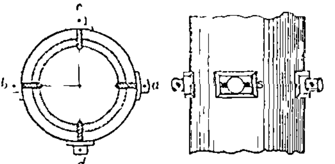


Fig. 11.

wires again intersect the object. Repeat the process until the intersection remains constant on changing pivots, remembering that half the apparent is the real error.*

Collimating by Changing Face.—Intersect and note the reading of any fixed object; change face, intersect and read again. The two readings should differ by exactly 180 degrees. If they do not, the error is corrected as above.

In collimating by changing face, any errors that may exist in the graduation of the limb are involved in the process.

In collimating an instrument it is desirable that the object selected for intersection should be at such a distance that the object glass be set to the focus which will be employed during the subsequent observations, as the line of collimation is liable to be distorted when the focus is altered.

To make the Vertical and Horizontal Wires respectively Vertical and Horizontal.—As the correct positions for these wires are cut on the diaphragm by the maker so that the lines cut each other at right angles, it follows that to adjust one wire is to adjust both, and this may be done by the following method:—

Level the instrument with care, intersect any small well defined object with the vertical wire, and see if it continues bisected along the wire when the telescope is moved in a vertical plane. If this is not the case, the collimating screws must be slackened sufficiently to allow the diaphragm to be revolved until this condition is secured, when they must be again

* The same result can be obtained by estimating half the error as it appears in the telescope and dispensing with the readings.

tightened. It will now be found that the horizontal wire, if properly placed by the maker, will continue to bisect an object on which it has been placed when the instrument is turned in azimuth.

Levelling the Theodolite.—This consists in making the vertical axis truly perpendicular to the horizon.

Place the longer of the levels, *o*, of the horizontal plate parallel to any two foot screws, *e*, *e*, and by suitably turning these latter bring the bubble into the centre of its tube. In doing this the observer must remember always to turn the foot screws in opposite directions, that is to say, to work both hands either towards or away from himself. Then turn the upper plate 180 degrees in azimuth. Supposing the bubble to occupy a different place from what it did before, move it by means of the same two foot screws into the mean of these two places, and it is evident that if the upper plate be turned another 180 degrees until it occupies its original position, the bubble will continue to occupy that mean place, called the *zero of the level*. Now turn the upper plate 90 degrees in azimuth, and by means only of the third and hitherto unused foot screws bring the bubble into its mean place.

The instrument is now approximately level, and the above described process must be continued until the level remains steady in whatever direction the telescope may be pointed. The mean place is not necessarily in the centre of the tube, but if the deviation is considerable the bubble can be brought into the centre by means of the two drawing screws which secure the level to the horizontal plate. The length of the bubble varies inversely as the temperature, and consequently the positions of the ends of the bubble alter with a change of temperature. It is convenient to have one end of the level marked and to watch that end only while levelling.

The levelling is now as perfect as it can be made by the body level. As the level attached to the microscope arms is usually more delicate the final levelling can be completed with this by bringing the bubble approximately into the centre of its run by the clip screws, and then proceeding exactly as above described.

Level on Microscope Arms.—If the level is on the vernier or microscope arms, it is necessary to have it well adjusted.

To do this level the instrument, bring the bubble into the centre by means of the clip screws *H*, intersect the object and note the reading; change face and set the bubble by the clips as before, and intersect again. The mean of the two readings will be the true elevation. Set the vertical circle to this reading and intersect the object by the clip screws, and, keeping the instrument clamped, bring the bubble of the level into the centre by means of its drawing screws. It is clear that where the error is small the discrepancy between the two faces may be eliminated in a similar way to that used for collimation in azimuth. But the method just described is better.

When a level is carried on the vernier arms of the vertical limb it is desirable to know the value of the level scale for the purpose of correcting the observed vertical angles. See p. 19.

To level the Transit Axis. Level the upper plate. Place the striding level on the pivots and by tapping it gently sideways bring the bubble of the cross level into the centre of its run.

When the bubble of the striding level is steady, read and register the number of the level scale at which one end of it stands, say the end nearer to the cross level.

Now reverse the level end for end, cross level, read and register the same end of the bubble as before. If the transit axis is level the same end of the bubble will read the same on both occasions. If it does not read the same, half the apparent is the real error, which can be corrected by the capstan headed screw, *k*, which raises or lowers the Y above it.

Unless the error of collimation and the dislevelment of the transit axis are very large, it is better not to try to correct them, as their effect can be eliminated by a suitable method of observation.

Determination of Value of One Division of Level Scale

The level is usually graduated, but, if not, make a paper scale of equal parts, reading outwards from the centre towards both ends, and paste it on to the side of the level, so that the centre of the scale corresponds with the centre of the run of the bubble, assuming that the level is on the vernier arms (as it is only in that position that it affords a means of correcting the vertical angles), by means of the clip screws move the bubble up to one end of its run, say towards the object end, so that the object end of the bubble corresponds approximately with the extreme reading of the scale. Intersect with the horizontal wire some convenient object for observing. Read and record one end of the bubble, say the object end, and the vertical angle. Now by means of the clip screws bring the bubble back towards the eye as far as you can, taking care that it is really floating and within the graduations of the scale. Intersect the same object as before, and record the vertical angle and the reading of the object end of the bubble in its new position. The difference between the two readings of the object end of the bubble gives the dislevelment in terms of divisions of the scale, and the difference between the two vertical angles gives the same dislevelment in minutes and seconds of arc. Dividing this angular measurement by the number of divisions of dislevelment you obtain the value in arc of one division of the scale.

Thus:—

	Elevation	Object end of bubble
1st observation	7° 3 28	18
2nd "	7 0 0	5
Difference	0 3 28	13

Value of one division = $\frac{208}{13}$ = 16.

This operation must be repeated several times in order to get a good mean value. The bubble of a level is very susceptible to changes of temperature, so care must be taken that it is not exposed to such changes while this operation is being performed. Should there be any chance of the bubble altering its length while you are determining the value of the divisions of the scale, it will be necessary to read and record both ends of the bubble. In observing, as described previously, for each vertical angle taken, the readings of both ends of the bubble must be recorded. *To apply the correction* the rule is as follows:—

Divide the difference between the sums of the readings of the object end and eye end by the total number of readings and the result will be the dislevelment in terms of divisions of the scale. Multiply this result by the angular value of one division of the scale and you obtain the angular correction for dislevelment to be applied to the mean vertical angle. Supposing two observations are taken to a point one face left and one face right and the readings are as follows:—

	O	I
F L	5	8
F R	7	6
	12	14

tightened It will now be found that the horizontal wire if properly placed by the maker, will continue to bisect an object on which it has been placed when the instrument is turned in azimuth

Levelling the Theodolite —This consists in making the vertical axis truly perpendicular to the horizon

Place the longer of the levels, *o*, of the horizontal plate parallel to any two foot screws, *c*, *c*, and by suitably turning these latter bring the bubble into the centre of its tube In doing this the observer must remember always to turn the foot screws in opposite directions, that is to say, to work both hands either towards or away from himself Then turn the upper plate 180 degrees in azimuth Supposing the bubble to occupy a different place from what it did before, move it by means of the same two foot screws into the mean of these two places, and it is evident that if the upper plate be turned another 180 degrees until it occupies its original position, the bubble will continue to occupy that mean place, called the *zero of the level* Now turn the upper plate 90 degrees in azimuth, and by means only of the third and hitherto unused foot screws bring the bubble into its mean place

The instrument is now approximately level, and the above described process must be continued until the level remains steady in whatever direction the telescope may be pointed The mean place is not necessarily in the centre of the tube, but if the deviation is considerable the bubble can be brought into the centre by means of the two drawing screws which secure the level to the horizontal plate The length of the bubble varies inversely as the temperature and consequently the positions of the ends of the bubble alter with a change of temperature It is convenient to have one end of the level marked, and to watch that end only while levelling

The levelling is now as perfect as it can be made by the body level As the level attached to the microscope arms is usually more delicate, the final levelling can be completed with this by bringing the bubble approximately into the centre of its run by the clip screws, and then proceeding exactly as above described

Level on Microscope Arms —If the level is on the vernier or microscope arms, it is necessary to have it well adjusted

To do this level the instrument bring the bubble into the centre by means of the clip screws *H* intersect the object and note the reading change face and set the bubble by the clips as before and intersect again The mean of the two readings will be the true elevation Set the vertical circle to this reading and intersect the object by the clip screws and, keeping the instrument clamped bring the bubble of the level into the centre by means of its drawing screws It is clear that where the error is small the discrepancy between the two faces may be eliminated in a similar way to that used for collimation in azimuth But the method just described is better

When a level is carried on the vernier arms of the vertical limb it is desirable to know the value of the level scale, for the purpose of correcting the observed vertical angles See p 19

To Level Transit Axis Level the upper plate Place the striding level on the pivots, and by tapping it gently sideways bring the bubble of the cross level into the centre of its run

When the bubble of the striding level is steady, read and register the number of the level scale at which one end of it stands say the end nearer to the cross level

Now reverse the level end for end, cross level read and register the same end of the bubble as before If the transit axis is level, the same end of the bubble will read the same on both occasions If it does not read the same, half the apparent is the real error, which can be corrected by the capstan headed screw, *k*, which raises or lowers the Y above it

Unless the error of collimation and the dislevelment of the transit axis are very large, it is better not to try to correct them as their effect can be eliminated by a suitable method of observation

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Thus —

	Elevation			Object end of bubble
1st observation	7	3	28	18
2nd ,,	7	0	0	5
Difference	0	3	28	13

Value of one division $= \frac{208}{13} = 16$

This operation must be repeated several times in order to get a good mean value The bubble of a level is very susceptible to changes of temperature, so care must be taken that it is not exposed to such changes while this operation is being performed Should there be any chance of the bubble altering its length while you are determining the value of the divisions of the scale, it will be necessary to read and record both ends of the bubble In observing, as described previously for each vertical angle taken, the readings of both ends of the bubble must be recorded *To apply the correction* the rule is as follows —

Divide the difference between the sums of the readings of the object end and eye end by the total number of readings and the result will be the dislevelment in terms of divisions of the scale Multiply this result by the angular value of one division of the scale, and you obtain the angular correction for dislevelment to be applied to the mean vertical angle Supposing two observations are taken to a point one face left and one face right, and the readings are as follows —

	O	E
F L	5	8
F R	7	6
	12	14

In this case the sum of the readings of the eye ends exceeds that of the object end by 2. The number of readings of ends of the bubble is 4. So to get the dislevelment in terms of division of the scale we must divide 2 by $4 = \frac{1}{2}$. Suppose the value of one division of the level scale is 16 seconds, then to get the correction we must multiply $\frac{1}{2}$ by 16 seconds $= 8$ seconds. The 8 seconds of arc must be applied to the mean of the two observed angles. With regard to its sign, the eye end being in excess, the correction must be subtracted from an elevation and added to a depression. If the object end were in excess, the process would, of course, be reversed.

$$\text{Generally correction to altitude} = + \frac{O - E}{\text{number of readings}} \times \text{value of 1 division}$$

If the level graduations read from one end and not outwards from the centre, book the readings of the zero end as negative, and then apply the above rule.

Method of Observing Horizontal Angles.—Observations taken to elevated objects are subject to two great sources of error, viz., dislevelment of the transit axis and want of adjustment of the line of collimation. These causes of error are generally large in small instruments, and although capable of rectification, adjustments do not long remain perfect, hence the system of observation should be such as to cancel these errors. This is attained by changing face, *i.e.*, by observing the same angle with the vertical circle or face alternately to the left and to the right. This system should be adopted in all theodolite observations, for it is as effectual as it is simple.

The angles at a station are taken thus: supposing the observer to be at B, and that it is required to observe the angles to the signals at C, D, E.

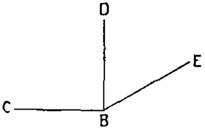


Fig 12

Clamp the upper plate to the lower, having set A vernier or microscope to about 0° on the horizontal limb. Loosen the lower clamp and turn the whole instrument till station C comes into the field of the telescope, now tighten the lower clamp, and intersect C with the lower tangent screw; see that the vernier still reads about 0. Now loosen the upper clamp and turn the upper plate and telescope completely round till C comes into the field of view, taking care not to overshoot it, clamp the upper plate and intersect.

Read off both verniers or microscopes and enter in angle book. The reading of A vernier should be identical with the setting. If the difference is greater than can be accounted for by a slight difference in the intersection, there must be a defect in the instrument, which should be examined.

Now loose the upper clamp, and turn the telescope on to D, clamp, intersect with the upper tangent screw without overshooting. Read the verniers or microscopes, and book the readings. Repeat the same process with E, and so on, with a continuous movement from left to right: this is termed **"swinging right."**

It is generally desirable to intersect the zero station again at the end of the swing, so as to detect any movement of the instrument which may have occurred. If such has occurred, the observations must be taken again. In some surveys the starting point is called the R.O., or Referring Object; there is no necessity to use this term in topographical work.

Suppose that during these observations the vertical circle had been to the right of the telescope, this set would be described as one round of angles, **face right** swing right, on zero 0° The **zero or arc** is the setting of the first station or signal read during a round of angles In the geodetic triangulation of Burma, for instance, the successive rounds of angles were taken on the following zeros 0° 0, 20° 1, 40° 2, 60 3, and so on, 9 zeros in all The object of changing zero is to minimize the effects of errors of graduation and to prevent any bias due to reading the same angle constantly on the same portion of the arc

The number of changes of face, of swings, and the number of zeros will depend on the class of the work

In geodetic triangulation the number of zeros will probably be at least 6, with 2 faces on *each zero, 2 swings on each face*

In secondary work probably 2 or 3 zeros each of 2 faces and 2 swings

In tertiary, 2 zeros, 2 faces, 1 swing on each face

In topographical work the numbers will naturally vary with the time available The following system is generally suitable —

Zero 0°, F R, swing R,
 „ 0°, F L, „ L,
 „ 35°, F L, „ L,
 „ 35°, F R „ R

or 4 rounds of angles at each station

But if time does not allow of so many then

Zero 0°, F R, swing R,
 „ 35°, F L „ L

The reason for taking 35 as the second zero is that, whether there are 2 or 3 microscopes or verniers, they will not fall on changing zero over the same part of the arc But any arbitrary change—not 60 nor 180 —will do

Method of Observing Vertical Angles —To observe vertical angles see that the instrument is perfectly levelled Direct the telescope on the object leaving the horizontal clamp free, clamp the vertical circle when the object comes into the field and intersect with the tangent screw It is generally convenient to intersect with a portion of the horizontal wire at one side of the intersection of the cross wires If this is done care must be taken to intersect with exactly the same spot on the wire when the telescope is reversed on changing face, remembering that if the spot appeared to be on the left of the centre F L it will be on the right when the object is observed F R

Read and record both the verniers, noting if the angle is an elevation or a depression If there is a level on the vernier arms, enter the readings of both ends of the bubble under the headings of O or object end, and E or eye end

If the object was intersected from *above* the first time it should be intersected from *below* when a second observation is taken

Now change face and observe the object F R in exactly the same manner The mean of the F L and F R observations should give the true elevation or depression If however, there is a level on the vernier arms and the sum of the readings of the two ends do not exactly agree, a correction will have to be applied See p 20

The height above their station dots of the instrument and of the objects observed should be entered

A vertical angle is of very small value if it has not been observed on both faces

Generally, *when taking horizontal angles leave the vertical arc unclamped, and when* taking vertical angles leave the horizontal arc unclamped This implies that the *horizontal* and vertical angles should not be taken simultaneously, and that the object should *not be*

observed on intersection of the cross wires but separately on vertical and horizontal wires. If time presses, however, this restriction may be ignored, and vertical and horizontal angles may be taken together.

General Remarks on Observations.—It is a general rule of all regular surveys that all the observations should be entered in ink before the observer leaves his station. A description of each station should also be entered in the angle book, in order to enable any other observer or the detail surveyor to recognise it without difficulty.

This description should contain not only such information as would enable anyone to identify both the general locality and the exact spot used as a station, but should also state the most convenient camping ground in the neighbourhood, and the best direction from which to ascend the hill in the case of a hill station. The nature of the mark left should also be given, and its height above the station dot.

Satellite Stations.—It sometimes happens that angles are required from some point such as a tree, steeple, temple, or pagoda, where it is impossible to erect the theodolite. This case never occurs in geodetic or good secondary triangulation, but it is not very uncommon in rapid work, where the stations have not been laid out beforehand, and the observer is dependent on natural objects for his marks. In such cases the theodolite is set up at the nearest convenient point, called a *satellite station*, at a short distance from the object, and

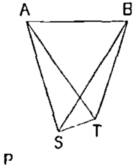


Fig 13

the angles taken from this station can afterwards be reduced to the true values at the object itself.

Supposing now that the triangulator has observed the trunk of a tree on the summit of a hill, and that on his arrival he finds that it is impossible for him to observe with his theodolite nearer to the tree than several feet, he would put up his theodolite in the most convenient place as near to the tree as possible, and observe his round of angles as usual, taking care to include the centre of the trunk of the tree. He must also measure accurately the distance from his station to the trunk of the tree. He can now correct his angles in either of two ways:—

(1) If a number of angles have been taken from the station and only two or three have been taken to the tree, and if it is possible to raise upon the station a mark sufficiently conspicuous to be visible from future stations, it is preferable to correct the two or three angles already taken to the tree.

(2) If on the other hand only a few angles have been taken at the station, and if it is impossible to erect on it as useful a mark as the tree, then it is advisable to reduce the angles taken at the station to what they would have been had they been taken on the spot occupied by the tree. This is called reduction to the centre.

Let AB be two stations from which the tree, T, has been observed, and let S be the station or spot where the theodolite was put up.

(1) In the first case the angles SAB and ABS are required, the angles TAB and A being known. Evidently SAB = TAB + SAT and $\sin SAT = \sin TSA \frac{ST}{AT}$. AT can computed approximately or measured with sufficient accuracy from the working chart of triangulation.

(2) In the second case, which is the usual one, ATB is required. Produce TS to P.

Then
$$BSP = BTS + SBT$$
$$ASP = ATS + SAT$$

Subtracting (2) from (1)

$$ASB = ATB + SBT - SAT, \text{ whence } ATB = ASB + SAT - SBT$$

Now $\sin SAT = \sin TSA \frac{ST}{AT}$, AT being found in any approximate way, such as measurement from the chart of triangulation, and $\sin SBT = \sin TSB \frac{ST}{BT}$, BT being found in a approximate way.

COMPUTATIONS

Lengths of the Sides. Suppose AB to be the measured base, C a point in triangulation, the angles A, B, C having been observed.

Now since the surface of the earth is a curved surface the horizontal angles at A, B, and have been observed in planes which are very slightly inclined to each other, and, if perfec observed, the sum of A, B, and C will exceed 180° by a small quantity known as the **spheri excess**. *In geodetic and secondary triangulation* this quantity is always computed subtracted from the sum of the three angles and any remaining excess or defect on 180° is error to be adjusted by an elaborate process which will not here be described.

But in tertiary and topographical triangulation the procedure is very simple, viz divide the excess or defect on 180° equally between the three angles. The angles th corrected are the *corrected angles*.

By elementary plane trigonometry, we have

$$b = c \times \frac{\sin B}{\sin C} \quad \text{and} \quad a = c \times \frac{\sin A}{\sin C}$$

c being the known base.

The most convenient arrangement for logarithmic computation is shown on p. 24, t ends of the known side being written first and the stations entered in order in the direction motion of the hands of a watch.

The other triangles are solved in succession in the same way.

It is sometimes, though not often necessary to compute the third side of a triangle which two sides a, b, and the included angle C are known. The following formula is used:

$$\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2},$$

whence $\frac{A - B}{2}$ is found, and

$$\frac{A + B}{2} = 90 - \frac{C}{2}$$

From these A and B are found and the triangle can be solved in the ordinary way.

It is desirable, whenever possible, to obtain more than one value for the length of a sid the mean being taken in subsequent computations.

FORM A.—(S.M.L. Topographical Survey, Form I.)

Computation of Sides of Triangles

$$\text{Formula}\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Triangle	Letter of station	Name of station	Mean of observed angles —or satellite angles reduced to centre	Correction	Corrected angles	Logarithmic computation	Sides (log feet)
						log AB 4 3937724	
14	A	Dernet Fort	96 50 26	+ 3	96 50 29	L sin 9 9968067	4 4853938 (BC)
	B	Callan Hill	29 38 26	+ 3	29 38 29	L sin = 9 6942276	4 1827247 (AC)
	C	Stoke Ch.	53 30 59	+ 3	53 31 2	L cosec = 10 0917247	
			179 59 51	+ 9	180 0 0		
						log DB = 4 5323728	
15	D	Naval Hospital	56 28 36	−10	56 28 26	L sin = 9 9209741	4 4854049 (BC)
	B	Callan Hill	55 16 26	− 9	55 16 17	L sin = 9 9147976	4 4792277 (CD)
	C	Stoke Ch.	68 15 26	− 9	68 15 17	L cosec = 10 0320580	
			180 0 28	−28	180 0 0		

Co-ordinates or Latitudes and Longitudes—The next step in the computations is to put the results in a form which permits of accurate plotting, and which enables a record to be kept of the position of each trigonometrical point

This can be done in two ways —

I By rectangular co-ordinates,

II By latitudes and longitudes

I Rectangular Co ordinates—If the distance east or west of the starting point is not likely to exceed 100 miles and there is no probability of geographical positions being required accurately, nor of there being any further extension of the triangulation and if the azimuths of the sides are not likely to be wanted, proceed as follows —

(a) Calculate by plane trigonometry the lengths of all the sides of the triangles

(b) Select one station as the initial station and the meridian through that station as the initial meridian, then, treating the earth's surface as a plane calculate the *co-ordinates* of all the stations, the co-ordinates being measured along and at right angles to the initial meridian thus—

If α is the azimuth, or true bearing of the first side AB A the initial point, then x_1 (the co-ordinate at right angles of the meridian) = AB sin α

$$y_1 = AB \cos\alpha$$

$$\left.\begin{aligned} x &= BC \sin\beta \\ y &= BC \cos\beta \end{aligned}\right\} \text{where } \beta = ABC - \alpha$$

Then co-ordinates of A are O, O

" " B " x_1, y_1

" " C " $x_1 + x$, $y_1 - y$, &c

All these points can now be plotted with reference to the initial point, A and its meridian

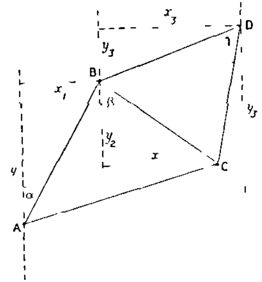


Fig 14

It is important to note that once the initial meridian has been left the angles β, γ, &c, formed by the sides with lines parallel to the meridian, are no longer the true bearings of these lines, and should never be used as such

It is clear that if a chain of triangles runs in an oblique direction it may be convenient to

plot along, and at right angles to, a line of known bearing, running generally in the same direction

II Latitudes and Longitudes *—If the distance east or west of the starting point is likely to exceed 100 miles, or if the geographical positions of the stations are of importance, or if the azimuths of the sides are likely to be wanted, or if the triangulation is likely to be extended in the future then calculate the latitude and longitude of each trigonometrical station having regard to the true shape of the earth, as follows —

It is first necessary to know the latitude and longitude of some trigonometrical station and the azimuth (i e, true bearing) of some ray from that station The latitude, longitude and azimuth are either determined astronomically (see Chapter XIV) or have been brought up by former triangulation In the case of astronomical determination there is no difficulty in obtaining the required latitude and azimuth, but the longitude may present some difficulty, and an approximate value may have to be assumed This will not, in any way, affect the accuracy of the work in itself

Suppose now that A and B are two stations and that we know —

λ – latitude of A,
L – longitude of A,
A – azimuth of B from A,
c – distance in feet at mean sea level between A and B,

then we require λ_b – latitude of B,
L_b – longitude of B,
B – azimuth of A from B

If we put $\lambda_b = \lambda + \Delta\lambda$
$L_b = L + \Delta L$
$B = \pi + A + \Delta A,$

and if $\Delta\lambda = \delta_1\lambda + \delta_2\lambda,$
$\Delta L = \delta_1 L + \delta_2 L$
$\Delta A = \delta_1 A + \delta_2 A,$

then it can be shown that

$\delta_1\lambda = P \cos A \; c,$	$\delta_2\lambda = \delta_1 A \; R \; \sin A \; c,$
$\delta_1 L = \delta_1\lambda \; Q \; \sec\lambda \tan A,$	$\delta_2 L = \delta_2\lambda \; S \; \cot A,$
$\delta_1 A = \delta_1 L \sin\lambda,$	$\delta_2 A = \delta_2 L \; T,$

which are the expressions used in the form for computation

It will be noticed that each term is found consecutively from the preceding one

The quantities P Q, R, S, T will be found by interpolation from Table I The proper signs to be given to $\delta_1\lambda$, $\delta_1 L$, &c, will be found in Tables II (*a*) and (*b*)

Examples of the Computation of Latitude, Longitude, and Reverse Azimuth— This is shown on Form B In this example Sugar Loaf is the station of which the latitude and longitude are known Blorenge that of which they are required The distance Sugar Loaf Blorenge is known and also the azimuth of the ray Sugar Loaf—Blorenge *Azimuths for this computation are always to be reckoned from south by west*

In the example given, the longitude of the known station was *west* of Greenwich, and was therefore booked as a negative quantity, east longitude is booked as a positive quantity The signs of the quantities $\delta_1\lambda$, $\delta_2\lambda$, $\delta_1 L$, $\delta_2 L$, $\delta_1 A$, $\delta_2 A$ are to be booked exactly as found in Tables II (*a*) and (*b*)

Suppose that we start with a triangle ABC, of which the latitude and longitude of A are known, the azimuth of AB, and the lengths of all the sides —

First compute the latitude and longitude of B and reverse azimuth BA Then, by adding

* See also Appendix III

Form B.—(S.M.E. Topographical Survey Form III.)

Latitudes, Longitudes, and Reverse Azimuths

Station A	Sugar Loaf	
Lat. of A (λ) Long. of A (L) Station B	51° 51′ 44·00″ W 3 3 23·00 Blorenge	° ′ ″
Az. B from A (A) log AB (c) in feet	1° 23′ 58″ 4·3556249	° ′ ″
P log cos A log c	3·9939183 $\bar{1}$·9998704 4·3556249	
log $\delta_1\lambda$ Q log sec λ log tan A	2·3494127 $\bar{1}$·9988671 0·2093246 $\bar{2}$·3879194	
log $\delta_1 L$ log sin λ	0·945 38 $\bar{1}$·8957142	
log $\delta_1 A$ R log sin A log c	0·841208 8·37848 $\bar{2}$·38779 4·35562	
log $\delta\lambda$ S log cot A	$\bar{7}$·06317 0·59922 1·61208	
log δL T	2·08447 0·01267	
log $\delta_2 A$	2·00714	
λ $\delta_1\lambda$ $\delta\lambda$	51 51 44·00 − 0 3 43·58 − 0 0 0·00	° ′ ″
Lat. of B	51 48 0·42	° ′ ″
* L $\delta_1 L$ δL	W − 3° 3 23·00 − 0 0 8·82 + 0 0 0·01	° ′ ″
Long. of B	− 3 3 31·81	° ′ ″
π + A δ_1 A δ A	181° 23 58 − 0 0 6·9 + 0 0 0·01	° ′ ″
Az. of A from B	181° 23 51	° ′ ″

* When A is W. of Greenwich give L the negative sign.

the angle at B to this last obtain the azimuth of BC, and, knowing now the latitude and longitude of B and azimuth BC, calculate the latitude and longitude of C

Now, as a check, calculate the latitude and longitude of C direct from A

And generally, to avoid arithmetical errors, compute the latitude and longitude of each point from at least two rays

Latitudes and longitudes should be worked out to two places of decimals of a second

The latitudes and longitudes of all the stations of a triangulation are thus computed successively from those of previously computed stations These latitudes and longitudes are definite and sufficient records of the positions of the stations, and the positions can be plotted on any scale required

Trigonometrical Heights—Having computed the lengths of the sides and the co-ordinates or latitudes and longitudes, the next step is to calculate the heights of the stations above mean sea level For this purpose some initial height is necessary, which may have been determined by former triangulation or may be determined by direct observation of the mean sea level or by careful barometer readings at sea level and at the point in question (see p 87)

We will assume that the height above mean sea level of one station of the triangulation is known, and that vertical angles have been taken at each station to all stations visible therefrom and that these vertical angles have been observed with a theodolite on both faces, and that the means have been corrected for level error

All vertical angles require a correction for refraction The ray of light from a distant object lies, as a rule, in such a curve as to make the object appear higher than it really is The amount of this deflection varies with the place, season and time of day Sometimes, but rarely it makes an object appear lower than it really is

In fig 15 A and B are the summits of two hills, AR, BR the apparent directions of the distant stations (due to refraction) As AR BR are portions of an arc of small curvature, BAR and ABR are sensibly equal Make OC = OA Then it is required to find BC the difference of height of A and B

Let the observed depressions at A and B be D_a and D_b,

$$BAC = OCA - ABO \quad \text{and}$$

$$BAC = BAO - CAO, \quad \text{and since} \quad OCA = CAO,$$

$$BAC = \tfrac{1}{2}(BAO - ABO) = \tfrac{1}{2}(RAO - RBO) \text{ (since } BAR = ABR) = \tfrac{1}{2}(D_b - D_a)$$

Now as AC is a very small part of the earth's surface,

$$BC = AC \tan BAC \text{ (sensibly)}, \quad \text{or}$$

$$h = C \tan \frac{D_b - D_a}{2}, \text{ or if one angle is an elevation}$$

$$h = C \tan \frac{E_a + D_b}{2}, \quad \text{or} \quad C \tan \frac{E_b + D_a}{2}, *$$

where

h is the required difference in height

C the distance between the stations

D_a D_b the angles of depression at A and B

It has been assumed above that the reciprocal observations were taken simultaneously This is usually impracticable, so in order to attain the same result as far as possible, vertical angles should be taken (when time allows) about the same time of day, that time being between noon and 3 p.m., when refraction is fairly constant

* See Form C p 31

Where reciprocal angles have not been observed, it is necessary to adopt some approximate value for the refraction (BAR)

From fig 15 it is clear that, calling the refraction r,

$$r = \frac{c - D_a - D_b}{2},$$

where c is the arc of the earth's surface between A and B

So that whenever reciprocal angles have been observed the amount of the refraction can be found

The result of a very large number of observations all over the world is to show that the

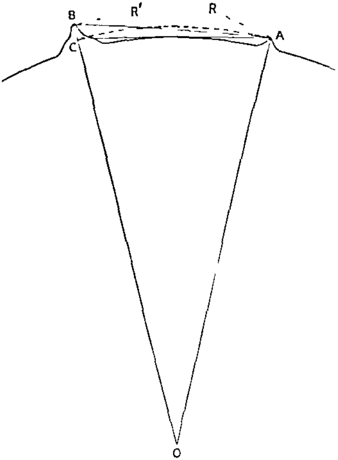


Fig 15

quantity $\frac{r}{c}$, called the **coefficient of refraction** is fairly constant for a given country and climate

The mean coefficient of refraction for South Africa is	064
" " " " Burma	067
" " " " the United Kingdom (for rays not crossing the sea)	075
" " " " the United Kingdom (for rays crossing the sea)	081

Generally the coefficient of refraction may be taken as 070

Having adopted a suitable coefficient of refraction, it is clearly easy to compute the effect

produced by curvature and refraction combined The labour saving table on p 31 has been computed for a coefficient of 070 Its use is as follows —

If a vertical angle has been observed at only one of the stations, if this is a depression call it D, if an elevation E

Then look out the value of K in the Table, corresponding to the distance between the stations

Let

$$S = K - D \quad \text{or} \quad K + E,$$

then the difference in height between the stations

$$= \tan S \times \text{distance between the stations}$$

To get the exact height, a correction must be applied for the heights in feet above their station dots of the instruments at A and B i_a and i_b and of the signals at A and B g_a and g_b

This correction, to be applied to the already obtained difference of height,

$$= + \frac{g_a - g_b + i_a - i_b}{2}$$

Two examples of the computation of heights are given on Form C, one where reciprocal angles have been observed and the other where only one angle has been observed

Datum Level—All heights on a map should be referred to mean sea level The mean sea level for the Ordnance Survey of the United Kingdom was determined at Liverpool The datum level of the Survey of India is the mean sea level at Karachi

Mean Sea Level—This is the mean height of the surface of the sea during an indefinitely long period As determined by the Survey of India the mean sea level is the mean of the hourly heights extending over a period of 5 years Five years is chosen so as to cover the cycle of recurrence of astronomical phenomena producing tidal effect This is the mean sea level which should be used in all civilized countries having an organized survey

It will often however, be impossible on a topographical survey to determine mean sea level so accurately In such a case, if there are any tidal records at the port chosen, the mean of high and low water heights throughout a year will give a good value

If there are no tidal records fix a graduated pole in the sea and observe the heights of high and low water during the progress of the survey, the period to extend over a definite number of lunations

Under any circumstances the period must extend over at least one lunation

This latter method will give only a rough value for mean sea level

Trigonometrical Interpolation

In rapid triangulation during military operations or on an expedition or with a boundary commission many artifices have to be employed which it would be improper to use on a regular topographical survey in the execution of which rapidity is not a governing condition A very valuable expedient in such cases is **interpolation**, or the determination of the observer's position by means of observations taken to previously fixed points

There are three methods of interpolation —

(a) By observing the horizontal angles from 1 unfixed point to 3 known points
(b) „ „ „ 2 points to 2 „
(c) „ 1 „ point to 2 „
with the help of an azimuth

FORM C—(S. M. I. Topographical Survey Form II.)

Computation of Trigonometrical Heights

DATA

A is the Station of which the height is known
B " " " " required
c = Distance in feet between A and B
D_a = Vertical angle of depression at A
D_b = " " " B
E_a = " " elevation A
E_b = " " " B
i_a = Height of instrument above ground at A
i_b = " " " B
q_a = Height of signal A
q_b = " " B
k = Correction for curvature and refraction (*see* Table below)

GENERAL FORMULÆ

$$\text{Height of B} = \text{Height of A} + c \tan S + \frac{q_a - q_b + i_a - i_b}{2}$$

(I.) Where S or Subtended Angle $= \dfrac{D_b - D_a}{2}$

If either angle is an elevation it is to be treated as a . . . .
. . . required

(II.) If one angle only is observed—
$S = K - D$ or $K + E$

DATA		
Station (A)	Patiya	
Known height of A	=	612 2
Station (B)	Sung	
Length c (in feet)	=	50 540
I. $+E_a$ or $-D_a$	=	− 1° 17′ 20″
$+D_b$ or $-E_b$	=	+ 0 57 30
	2	0 19 50
½ Algebraic sum (± S)	=	− 0 9 55
II. $+E_a$ or $-D_a$	=	
k (from Table)	=	
Algebraic sum (± S)	=	
Log tan S	=	$\bar{3}$ 4600930
Log c	=	4 7036352
Log h	=	2 1637282
h (±)	=	−145 79
$\frac{q_a - q_b + i_a - i_b}{2}$ *	=	+ 8 3
Known height of A	=	612 2
Deduced height of B	=	504 71

DATA		
Station (A)	Chiltan	
Known height of A	=	453
Station (B)	Murdah	
Length c (in feet)	=	53 420
I. $+E_a$ or $-D_a$	=	° ′ ″
$+D_b$ or $-E_b$	=	
	2	
½ Algebraic sum (± S)	=	
II. $+E_a$ or $-D_a$	=	0° 5′ 10″
k (from Table)	=	0 3 48
Algebraic sum (± S)	=	+ 0 8 58
Log tan S	=	$\bar{3}$ 4163581
Log c	=	4 7277039
Log h	=	2 14406[illegible]0
h (±)	=	+139 93
$\frac{q_a - q_b + i_a - i_b}{2}$ *	=	+ 4 25
Known height of A	=	453 0
Deduced height of B	=	596 61

TABLE—Value of K

These values are obtained by using a coefficient of refraction of 07

Distance (c) in feet	K	Distance (c) in feet	K	Distance (c) in feet	K
1 000	0′ 4″	8 000	0′ 34″	60 000	4′ 15″
2 000	0 9	9 000	0 38	70 000	4 58
3 000	0 13	10 000	0 43	80 000	5 4[illegible]
4 000	0 17	20 000	1 25	90 000	6 23
5 000	0 21	30 000	2 8	100 000	7 5
6 000	0 26	40 000	2 50		
7,000	0 30	50 000	3 33		

* Where only one angle is observed at A to the *ground line* at B the correction will be $+ i_a$

produced by curvature and refraction combined The labour saving table on p 31 has been computed for a coefficient of 070 Its use is as follows —

If a vertical angle has been observed at only one of the stations if this is a depression call it D if an elevation E

Then look out the value of K in the Table, corresponding to the distance between the stations

Let

$$S = K - D \quad \text{or} \quad K + E$$

then the difference in height between the stations

$$= \tan S \times \text{distance between the stations}$$

To get the exact height a correction must be applied for the heights in feet above their station dots of the instruments at A and B i_a and i_b and of the signals at A and B g_a and g_b

This correction to be applied to the already obtained difference of height

$$= + \frac{g_a - g_b + i_a - i_b}{2}$$

Two examples of the computation of heights are given on Form C, one where reciprocal angles have been observed and the other where only one angle has been observed

Datum Level—All heights on a map should be referred to mean sea level The mean sea level for the Ordnance Survey of the United Kingdom was determined at Liverpool The datum level of the Survey of India is the mean sea level at Karachi

Mean Sea Level—This is the mean height of the surface of the sea during an indefinitely long period As determined by the Survey of India the mean sea level is the mean of the hourly heights extending over a period of 5 years Five years is chosen so as to cover the cycle of recurrence of astronomical phenomena producing tidal effect This is the mean sea level which should be used in all civilized countries having an organized survey

It will often however be impossible on a topographical survey to determine mean sea level so accurately In such a case if there are any tidal records at the port chosen, the mean of high and low water heights throughout a year will give a good value

If there are no tidal records fix a graduated pole in the sea and observe the heights of high and low water during the progress of the survey the period to extend over a definite number of lunations

Under any circumstances the period must extend over at least one lunation

This latter method will give only a rough value for mean sea level

TRIGONOMETRICAL INTERPOLATION

In rapid triangulation during military operations or on an expedition or with a boundary commission many artifices have to be employed which it would be improper to use on a regular topographical survey in the execution of which rapidity is not a governing condition A very valuable expedient in such cases is **interpolation** *or the determination of the observer's position by means of observations taken to previously fixed points*

There are three methods of interpolation —

(a) By observing the horizontal angles from 1 unfixed point to 3 known points

(b) " " " " 2 points to 2

(c) " " " " 1 point to 2 with the help of an azimuth

Form C.—(S.M.E. Topographical Survey Form II)

Computation of Trigonometrical Heights

Data

A is the Station of which the height is known
B " " " " " required
c = Distance in feet between A and B
D_a = Vertical angle of depression at A
D_b = " " " B
E_a = " " elevation A
E_b = " " " B
i_a = Height of instrument above ground at A
i_b = " " " B
q_a = Height of signal A
q_b = " " B
k = Correction for curvature and refraction (see Table below)

General Formulæ

Height of B = Height of A + c tan S + $\frac{q_a - q_b + i_a - i_b}{2}$

(I.) Where S or Subtended Angle = $\frac{D_b - D_a}{2}$

(II.) If one angle only is observed—
S = k − D or k + E

Data		
Station (A)	Pabbas	
Known height of A	=	612.2
Station (B)	Sung	
Length c (in feet)	=	50,540
I. $+E_a$ or $-D_a$	=	− 1 17 21
$+D_b$ or $-E_b$	=	+ 0 57 30
	2	0 19 50
½ Algebraic sum (± S)	=	− 0 9 55
II. $+E_a$ or $-D$	=	
k (from Table)	=	
Algebraic sum (± S)	=	
Log tan S	=	3.4600930
Log c	=	4.7030352
Log h	=	2.1637282
h (±)	=	−145.79
$\frac{q_a - q_b + i_a - i_b}{2}$ *	=	+ 8.3
Known height of A	=	612.2
Deduced height of B	=	474.71

Data		
Station (A)	Chiltan	
Known height of A		453
Station (B)	Murdah	
Length c (in feet)	=	53,420
I. $+E_a$ or $-D_a$	=	° ′
$+D_b$ or $-E_b$	=	
	2	
½ Algebraic sum (± S)	=	
II. $+E_a$ or $-D_a$	=	0 5 10
k (from Table)	=	0 3 48
Algebraic sum (± S)	=	+ 0 8 58
Log tan S	=	3.4163591
Log c	=	4.7277039
Log h	=	2.1440630
h (±)	=	+139.93
$\frac{q_a - q_b + i_a - i_b}{2}$ *	=	+ 4.25
Known height of A		453.0
Deduced height of B	=	596.61

Table.—Value of k

These values are obtained by using a coefficient of refraction of .07

Distance (c) in feet	k	Distance (c) in feet	k	Distance (c) in feet	k
1,000	0 4	8,000	0 34	60,000	4 15
2,000	0 9	9,000	0 38	70,000	4 58
3,000	0 13	10,000	0 43	80,000	5 40
4,000	0 17	20,000	1 25	90,000	6 23
5,000	0 21	30,000	2 8	100,000	7 5
6,000	0 26	40,000	2 50		
7,000	0 30	50,000	3 33		

* Where only one angle is observed at A to the ground line at B the correction will be + i_a

(*a*) **Fixing the position of a point by angles taken from it to 3 previously fixed points**, in this there are three cases as below.

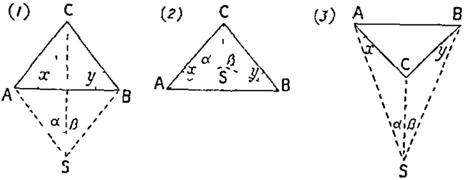


Fig 16

Let S be the observer's position, α and β observed angles, then

$$x + y = 360 - (\alpha + \beta + C) \text{ in (1) and (2)},$$

$$x + y = C - (\alpha + \beta) \text{ in (3)}$$

Let

$$\tan \phi = \frac{\sin \alpha}{\sin \beta} \times \frac{a}{b},$$

then

$$\tan \frac{x - y}{2} = \tan (\phi - 45^\circ) \tan \frac{x + y}{2}$$

NOTE —In case (1) if S, A, B, C, are on the circumference of the same circle the problem is insoluble

(*b*) **Fixing the position of 2 points by angles taken from them to 2 previously fixed points**

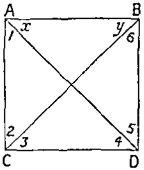


Fig 17

A and B are known points The angles 2, 3, 4, 5 are observed 1 6 are deduced Then if

$$P = \frac{\sin 1 \sin 3 \sin 5}{\sin 2 \sin 4 \sin 6},$$

$$\tan \tfrac{1}{2} (y - x) = \tan \tfrac{1}{2} (3 + 4) \frac{1 - P}{1 + P}$$

(c) **Fixing the position of a point by angles taken from it to 2 known points and an azimuth**

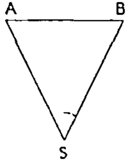


Fig 18

Let A and B be 2 known points At S observe the angle ASB and the azimuth of SA or SB (by the sun will do See p 148)

Get some approximate value for the position of S from the plane table, or by traverse

Then compute reverse azimuths of AS and BS, as below

$$dA = dL \sin \lambda,$$

where dA = increment of azimuth in seconds,

dL = difference of longitude in seconds of arc,

λ = mean latitude

The way to apply the quantity dA can easily be seen by considering in which direction the meridians converge Then knowing the azimuths of AS, BS, and that of AB (which is a known line) the differences give the angles A and B, and in the triangle ASB we know the three angles and one side (AB)

NOTE.—Method (a) and method (b) are much strengthened if an azimuth is observed The use of an azimuth enables two solutions to be found in each case and a check is thus obtained It is a general maxim in interpolation that azimuths should be observed when possible

CHAPTER IV

TRAVERSES

Definition —A traverse consists of a connected series of straight lines (on the earth's surface) of which the lengths and bearings have been determined

Different Forms of Traverse —Traverses may be carried out in different ways, according to the accuracy required

Where great precision is aimed at, the linear measurements are made with a chain or steel tape and the included angles are measured with a theodolite

Where rapid work is desired and where accuracy is not so important, the angles may be measured with a prismatic compass, and the distance either chained, paced, or taken by means of a perambulator, or by time

These two classes of traverses are known as —

(1) Theodolite traverses,
(2) Compass traverses

The latter are dealt with in Appendix VII

Theodolite Traverses —The principal varieties of theodolite traverses are —

A Traverses between points already fixed trigonometrically
B „ used on the line of march during an expedition
C „ as a substitute for triangulation in a country unsuited to the latter

A Traverses between Trigonometrical Points *

This class of traverse is used —

(a) On the Ordnance Survey of the United Kingdom, for large scale town surveys
(b) In India for cadastral surveys
(c) For certain large scale engineering surveys
(d) For forest surveys
(e) For the detailed survey of special routes, roads, railways, river banks, &c, &c
(f) To fix points for detail survey which cannot be provided by triangulation

The starting and finishing points of these traverses are trigonometrical points already fixed

Errors —Traverses are liable to errors of two kinds —

(1) Errors of direction These are due partly to inaccurate observations of the angles, and partly to bad centering of the theodolite, and also of the signals over the points marking the traverse stations
(2) Linear errors, due to faulty chaining

Traverse Stations —These stations should be as far apart as possible consistent with the ultimate object of the traverse, that is to say, in the case of traverses along winding roads the positions of the stations are governed to a great extent by the fact that offsets from the traverse line to detail along the road should not exceed a certain length

All stations should be carefully marked, and where possible a picket of about 2 × 2 section should be driven in flush with the ground A tintack should then be driven into the picket to show the exact centre point The further the stations are apart, the less will be the effect produced by bad centering of the theodolite, or signal over these points

* Including the case where there are no trigonometrical points but the traverse closes on the starting point

Signals—The sight vane commonly in use consists of a staff about 6 feet long and about 3 × 1 section, having a cross vane (a black cross on a white ground) which can be clamped at any height The difficulty is to hold this staff vertically over the centre point It is best therefore in order to minimise the errors in direction when taking horizontal readings, to make the cross wires of the theodolite bisect the bottom of the staff at the ground level In this case the horizontal reading should first be taken and recorded, after which the vertical angle to the cross vane clamped to the height of the instrument may be read Where the distance between the traverse points is small, it increases the accuracy of the work, if the horizontal readings are taken on the tintack itself, if visible, or else on an arrow or nail held over the same

There is an excellent sight vane in use on the Ordnance Survey, consisting of a tripod and staff with cross vane attached This last can be adjusted so as to be vertical over the centre points

Observations—We will assume that we have to run a traverse along a certain defined route, starting at A and finishing at B where A and B are stations mutually visible (fig 19)

Proceeding to A the theodolite is carefully centred over the station mark in the ground, and then levelled The horizontal plates are now clamped together so that the reading on the A vernier is anything between 0 0 0 and 0 10 0 (it is best not to try to set it to an exact reading) and with the plates so clamped the theodolite is directed on point B (the R O) and the signal bisected by the cross hairs The bottom plate is clamped and the horizontal verniers are read and recorded

Now release the top plate and direct the theodolite on to traverse station No I whither a man has been sent on with a sight vane clamped to the height of the theodolite The intersection of the cross hairs must then be carefully directed on the sight vane The horizontal verniers are now read and the verniers of the vertical are also the vertical level

A complete observation has now been taken on one face of the instrument

Now reverse the instrument and setting the A vernier at about 35 0 proceed in precisely the same manner as just described

This completes the observations at point A The theodolite is then taken to traverse point I where after careful centering and levelling a similar set of observations is made Here the trigonometrical point A becomes the R O Each traverse station is then visited in turn the station the observer has just left becoming the R O in each case On arriving at the point B the R O will be the last traverse station and forward readings taken on point A (*vide* fig 19)

Form D shows how the readings at traverse point VI would be recorded in the field book It is unnecessary to observe the vertical angles to the back station The details at VI correspond with those on Form F

Linear Measurements—The linear measurements are made with a chain or steel tape, preferably the latter The distance between traverse stations should be taped twice, to ensure accuracy A pull of 12 lbs is to be applied when taping this is done by means of a spring balance

Where the slope between stations are practically continuous, surface chaining should be employed The reduction to the horizontal is made afterwards by means of the vertical angles observed

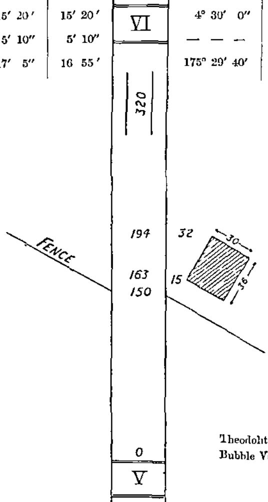
Where the ground is broken horizontal chaining must be resorted to In this case the observation of vertical angles is unnecessary

Computations of Co ordinates—In order to plot a traverse correctly it is necessary to work out the co-ordinates of the station points referred to some origin For the class of traverse under consideration rectangular co ordinates are always used It is found convenient to calculate the ordinates (y) for north and south, and the abscissæ (x) for east and west

Form D

Traverse Book.

		(Horizontal)		(Vertical)			
		A	B	C	D	E	O
F R	V	0° 3′ 20″	3′ 30′	— — —	— —	—	—
	VII	122° 15′ 20′	15′ 20′	4° 30′ 0″	30′ 10	7	5
F.L	V	35° 5′ 10″	5′ 10″	— — —	— —	—	—
	VII	157° 17′ 5″	16 55′	175° 29′ 40′	29′ 50′	4	8



Theodolite No 101
Bubble Value 1 Div 20′

Form I is the form used for the computation of these co-ordinates The traverse is that shown in fig 19, the starting and finishing points A and B, being mutually visible The observations taken at point VI are shown on Form D

Now A and B being points of a triangulation, the horizontal distance AB and also the true bearing of AB will be known

The legs of the traverse together with the line AB form an enclosed figure, and the sum of all the interior angles should be equal to twice as many right angles as the figure has sides less four right angles We thus have a check on the observed horizontal angles

It should be carefully noted that the included angles at the stations are always measured, either all to the right or all to the left of the back station

This is an invariable rule In the case of fig 19 they are all measured to the left of the back station Take point VI for instance Form D shows that the mean of the differences of the readings on V and VII is 122 11 52 5 The horizontal plate of a theodolite however, is always graduated in the same direction as a watch Hence we must deduct our result from

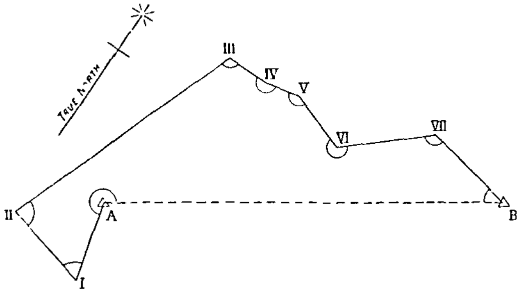


Fig 19

360°, which gives 237 48 7 5 The same must be done with all the angles before entering them in column 2 Form I. It is obvious that this last operation is unnecessary where the required angles are all measured to the right of the back station

Angular Error—The sum of all these interior angles then should be equal to $(2n \times 90 - 360)$ degrees where n is the number of sides to the figure Any discrepancy between these two is called the "total angular error" This error must then be distributed equally among the observed angles For example, if there are 6 sides to the figure and the sum of the observed angles comes out to 719 59 0 , then the error to be distributed is 1 Hence 10 must be added to each observed angle

Columns 2, 3, and 4, Form I , are self-explanatory

In this class of traverse it is quite unnecessary to take into account the convergence of the meridians, owing to the distance AB being short Hence the bearing of each leg is a "false bearing," as computed in column 5

It may here be stated that, before making any computations at all, it is best to draw a rough diagram of the traverse, such as in fig 19

Column 5 is computed thus —the bearing A to I is 360° − (249° 17 14 2) + (59° 4 33′)

FORM F —(S M T Form N)

Traverse

COMPUTATION OF CO-ORDINATES

Starting Point △ (A) } Mutually visible
Closing Point △ (B) }

Data —Hor Dist AB – 1995 78 ft
True Bearing of AB – 59° 4 33

1	2	3	4	5	6	7	8	9	10	11	12	13	14		15		16		17	
Stations	Angles observed at Stations	Angular Error	Corrected Angles	Total angle of Lines	Reduced Angles	Distances in Feet	[illegible]	Deduced Co-ordinates North	South	East	West	Errors of Co-ordinates of Closing Point	Corrected Co-ordinates from Origin Corr	N	Corr	S	Corr	E	Corr	W
T																				
B	0 0 0		°	° 9 4 83			199 8	10 5·64		1 1 ·08				10 64				1717 09		
													–		+		+		–	
I	°19 16	+1 6	°19 17 14 °	109 47 19	E 3 10 11	4 9 9	473		477 4	6 09				„	1	4 ..	36	6 4		
II	60 °1 0	+1 6 7	60 6 7	99 1	E 0 10 0	470 0	4 6	1 7 87			449·06				31	61 84			°	3 ° 2
III	89 5	+1 6 7	86 3 9	9 13	F 0 41 40	136 1	1361 9	1 1 0		31 °			46	39 00			1 04	1 9 3		
IV	111 49 7 5	+1 6 7	111 50 14 °	91 7 9	E 0 0	°10 1	10		4 1	°09 0°			6	°81 7			1 44	309 91		
V	191 1 7 5	+1 6 7	101 5 14 °	0 42 41	E 0 3 0	161 0	164	9 9		1 1 6			7	1013 83			1 8	31 63		
VI	144 6 4	+1 6 7	144 7 1 7	11 34 13	D 1 33	3 0 0	310 8		178 17	88 63				8 I			° 15	8 0 61		
VII	°37 43 7 5	+1 6 6	°37 49 11 1	4 9	D 4 30 0	·01	·00	66 3		4 ° 01			1 0	114° 08			° 5	1°11 8		
R	133 47 0	+1 6 8	133 48 8 8	103	E 4 ·8 35	483 9	48° °		110 3	4 7 85			1 1	10° 61			86	1 1 08		
A	44 1 5	+1 6 6	44 52 9 1																	
								1 07 89	681 01	°1 8 8	449 06									
Total	1 9 0 0		1 60 0 0					691 01		449 06										
... t	1 60 0 0																			
							B (deduced)	10 6 68		1 09 °										
9)	0 10 0																			
	0 1 6 6	to be added					Error	+ 1·°4		– ° 80										

Closing errors are + 1 °1 N and – 80 E
corrections S { + and W { –

NOTE —There are 9 sides to the figure sum of included angles = × 0 × 9 – 60 1 60

or 10[illegible] 47 18 8 The next bearing is got by adding 180 to this last, and then subtracting the corrected angle at II, and similarly for all the rest An examination of the figure will always reveal the correct method of computing the bearing

Column 6 gives the vertical angles of elevation or depression column 7 the distance chained along the surface The horizontal distances in column 8 are obtained thus —

$$\text{Hor dist} = \text{surface dist} \times \text{cosine E (or D)}$$

The coordinates Form L are computed with A taken as the origin From the triangulation data we know the length AB to be 1995 78 feet and the bearing of AB 59 4 33 The coordinates of B will then be 1995 78 × cos 59 4 33, or a north co-ordinate of 1025 64 and 1995 78 × sin 59 4 33 or an east co-ordinate of 1712 08

The next step is to compute in this manner by using the bearings in column 5 and the distances in column 8, the co-ordinates of each point of the traverse referred to the last point as origin

We have now a set of deduced coordinates referred to parallel axes throughout in columns 9 10 11 and 12 Thus the coordinates of III are 1254 0 N and 531 52 E referred to II as origin Also the coordinates of II are 157 87 N and 449 06 W, referred to I as origin

It is easily seen that where the bearing of the line lies between certain values, the deduced co-ordinates must be measured from the axes in certain directions thus —

Bearings		Deduced co-ordinates
°	°	
0 to	90	North and East
90	180	South and East
180	270	South and West
270	360	North and West

Now we see from the form that the co-ordinates of I referred to A are 422 4 S and 76 09 E, and the co-ordinates of II referred to I are 157 87 N and 449 06 W Hence the co-ordinates of II referred to A are—

422 4		449 06
157 87	and	76 09
264 53 S		372 97 W

Thus it is evident that the co-ordinates of any point referred to the primary origin (in this case A) are obtained by taking the algebraic sum of the deduced northings and southings and of the eastings and westings up to the point in question

These co-ordinates from the origin are entered in columns 14, 15 16 and 17 with certain corrections to be now described

On computing the co-ordinates of B by the algebraic sum of the deduced co-ordinates of the points throughout the traverse we find that they come to 1026 88 N and 1709 22 E Now, from the data the co-ordinates of B are 1025 64 N and 1712 08 E, hence the total northing at B is too large by 1 24 and the total easting too small by 2 86

In this class of traverse a rigorous adjustment of such errors is unnecessary The simplest way to distribute the errors of the co-ordinates is as follows —

If

n = number of legs of traverse,
y = error to be adjusted N and S,
x = error to be adjusted E and W,

then the several corrections to be applied to the points of the traverse in order will be

$$\frac{y}{n},\ \frac{2y}{n},\ \frac{3y}{n}\qquad\qquad \frac{ny}{n},$$

$$\frac{x}{n},\ \frac{2x}{n},\ \frac{3x}{n}\qquad\qquad \frac{nx}{n}$$

In the example on Form E the corrections to northings are all negative, and therefore, to southings all positive Similarly the corrections to all eastings are positive, and to westings negative

For example the corrections to be applied to the computed co-ordinates of VI are

$$\tfrac{6}{8} \times 1{\cdot}24 = {\cdot}93 \text{ on northing,}$$

and

$$\tfrac{6}{8} \times 2{\cdot}86 - 2{\cdot}15 \text{ on easting}$$

The algebraic sum of the deduced co ordinates up to this point gives 876 44 N and 818 46 E The corrections are entered in the columns provided on the left of columns 14 and 16, and applied to above results The corrected co-ordinates are then entered in columns 14 and 16

Thus the computed traverse co-ordinates of B have been made to agree with the trigonometrical values, and those of the other points have been adjusted in proportion

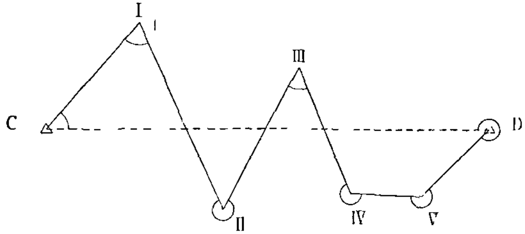


Fig 20

It oftens happen that the traverse crosses and re crosses the line joining the two terminal points as in fig 20

In such a case, where the traverse is started at C, the angle required at C must be under 180° In fig 20 this angle is measured to the left of the back station D As stated before all the other angles must be so measured

Then, as before

$$\Sigma\theta - 2n \times 90^\circ - 360^\circ$$

if the traverse crosses CD an *even* number of times

Where, however, the traverse crosses CD an *uneven* number of times

$$\Sigma\theta = 2n \times 90^\circ$$

The co-ordinates are computed in the manner already explained

Where it is desired to start and finish a traverse on the same point, the method is identical with that described, care being taken that only the interior angles of the figure are used in the computation, and that at the starting point the angle between the first and last legs of the

traverse is observed. If the starting point be taken as origin whose co-ordinates are 0, 0, then the co-ordinates carried throughout the traverse round to this point again must be 0, 0. If they do not come out to this value, the final co-ordinates must be made to have this value, and the remaining points adjusted as shown in Form I.

Chainage Errors.—In reducing the surface measurements to the horizontal, the computer should bear in mind the errors involved in chaining. The larger the scale to which the traverse is plotted the smaller must be the chainage error.

Thus for the Ordnance Survey maps of scales $\frac{1}{500}$ and $\frac{1}{1056}$, the error in ordinary chaining allowed is 1 in 1,000. For the $\frac{1}{2500}$ map an error of 1 in 500 is permitted.

In traverses measured with steel tapes much greater accuracy is to be expected.

B. Traverses on the Line of March (Subtense Traverses)

(See also p. 53.)

Often during an expedition in a tropical country, the nature of the ground and the time available render it impossible to execute a chain traverse. Under these circumstances, subtense methods may be employed.

Even without proper subtense instruments and subtense bars fitted with discs to be observed to, very fair accuracy can be obtained with the ordinary 5-inch micrometer theodolite and improvised signals placed at right angles to the forward ray.

With subtense work the subtended angle should be from 8 to 12 minutes, the base would therefore be some 30 feet for a ray of 10,000 feet. It is very important that the exact intersection of the signal posts with the ground should be measured, and it is consequently necessary that the ground round the signal posts should be systematically cleared of all grass; if the background is dark a white sheet or piece of cloth will be found very useful to show up the darkened signal post. Where it is impossible to observe the actual ground line, great care should be exercised in selecting signal posts as straight as possible and carefully plumbing them before measurement is made.

It is immaterial what the exact distance between the posts is so long as the subtended angle is somewhere within the limits specified above, and any suitable high ground or mounds should be selected in the vicinity of the traverse station, care, of course, being taken that the base is truly at right angles to the required ray.

This method of traverse was almost exclusively used in the Anglo-Liberian Boundary Commission of 1902-03, its advantages being that the computations were of the simplest and could be readily worked out in the field, enabling the necessary beacons to be put up at once.

The procedure is as follows:—

An assistant with a small clearing party and a compass is sent on ahead to a suitable hill as near the line of march as possible, and proceeds to clear the hill and fix his subtense posts at right angles to the direction of the post he has just left; this is done by any elementary method. A little practice will enable him to judge the distance apart the signals should be and to select a suitable place with a good background. He may or may not put up a separate signal as the traverse station proper, but it is preferable, perhaps, as on the appended sketch, to have a third signal traverse (theodolite) station.

As soon as his signals are fixed, he goes on to the next hill, selecting a hill by his compass as near the line as he can judge. He should be kept constantly informed how much he is distant from the required direction, so that he may select suitable hills.

The observer behind will take, say, three sets of 10 readings to the subtense base, see p. 42, and if they accord, having taken the angle between forward and back stations, and altered the subtense signals at his own station so as to be at right angles to the new ray, and carefully measured the distance in between them, will proceed to the forward station. Here he will at once, before there is any chance of the signals being disturbed, measure the length of base he has observed to. He will then set up the instrument over the new traverse station

and take the subtense angle on the traverse station he has just left so that he will have two values for the length of each ray, and if the two readings have been taken under equal conditions of light and clearness the mean of the two should be his accepted length for the ray

Owing to the difficulty of measuring the exact distance between the signals without the help of another white man and to the shimmer caused by the sun and heat except at certain times of the day great accuracy with small instruments cannot be expected To measure the distance in between the posts it was found that the best method was to drive a nail into the middle of one of the posts and fix the ring at the end of the steel tape on it pulling the tape tight to the middle of a pencil mark on the other signal post This was more satisfactory than trusting the other end of the tape to a native

The posts should be invariably guyed with rope or wire to make them as firm as possible

As regards the accuracy of the work it may be noted that during the Anglo Liberian Boundary Delimitation above quoted a difference of 20 feet in a ray of 10 000 feet was thought good *i e* a discrepancy of 1 in 500 At the end of the march when rays as long as 10 miles had to be taken the discrepancy in a length of 21 miles between the amplitudes of the latitudes deduced respectively from the traverse and from astronomical observations was 1

With these long rays as many as 60 or even 100 readings were taken of the subtended angle

As regards rate of working from 2 to 3 *legs* can be measured in a day each leg being 2 o 3 miles long

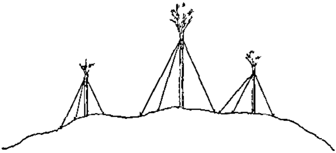


Fig 21 Subtense Station

Computations —In the instance quoted the latitude and longitude of each station of traverse was computed from the known length and azimuth of the ray by means of Form B, p 27

C Traverses as a Substitute for Triangulation in Countries Unsuited to the Latter and where Subtense Traverses are not Sufficiently Accurate

It is not suggested that traverses should ever be substituted for triangulation where the latter is possible but there are localities such as a dense forest or a flat grass covered country where the difficulties and expense of triangulation are such as to render it practically impossible In these circumstances **long traverses** will be found extremely useful in obtaining a framework of a high degree of accuracy on which topographical work can be based

In considering the relative accuracy of these two forms of survey, it must be remembered that after the triangles have been subjected to a rigorous treatment for the elimination of errors no side of a geodetic triangulation is likely to have an error of $\frac{1}{100\,000}$

The unsurmountable difficulty with regard to traverses in countries unfurnished with a Geodetic Survey is the impossibility of arriving at an accurate conclusion as to what errors have actually occurred In the absence of fixed geodetic points we are obliged to fall back on astronomical latitudes and telegraphic longitudes as a possible means of checking our measurements

Local Attraction.—Unfortunately, all astronomical observations are affected by "local attraction." The effect of this is that the plumb bob and the normal to the assumed geometrical surface of the earth do not coincide. The determination of the value of "local attraction" is one of the problems of a geodetic survey.

Those interested in the subject are referred to the "Account of the Principal Triangulation, Ordnance Survey," Vol. II; "Great Trigonometrical Survey of India," or "Geodesy," by Col. Clarke, C.B., F.R.S., R.E. Chapter X of the Account of the Principal Triangulation, Ordnance Survey, gives an account of the deflections produced by masses of different density and form. It may be taken that "the probable error of an observed latitude as due to local disturbance of gravity is certainly not less than 1·5. Several instances, however, exist where "local attraction" far exceeds this amount. At a station in Elginshire, determinations show it to be something like 8, and at a station in the plains of Lombardy—a fact all the more remarkable owing to the absence of any large range of hills—it is over 20.

Despite the most careful observations we cannot, therefore, say with any certainty that an astronomical latitude is correct within about 2.

This represents 200 feet on the earth's surface, and, dealing with 2 latitude stations, the error of amplitude probably amounts to $2\sqrt{2}$, or say 300 feet.

We cannot therefore put confidence in latitudes as a check on accurate traverses.

We may, perhaps, lay down the rule, that the latitude stations shall be so far apart, that the errors of the traverse and latitude amplitudes shall be about equal.

Distance Apart of Astronomical Stations.—Suppose that a traversing error of $\frac{1}{1000}$ may be expected. We have seen above that 300 feet is the probable error of two latitude stations due to "local attraction." The distance at which latitude observations should be made should, therefore, not be closer than 300×1000 feet—say 60 miles (of latitude). The distances between longitude stations determined telegraphically in a traverse running east and west should be even greater.

Latitude Observations.—Latitude observations should be made with a zenith telescope*—the mean of two nights' work, 6 pairs on each night, being taken. This should give a probable error due to observing of about 0·2.

Clearings.—The principal aim of long traverses is to establish points within the area to be surveyed on which topographical surveys can be based. In most countries where traverses will be used this involves clearing the bush or forest in order to obtain as long legs as possible and the preparation of the surface of the ground for the chain measurement.

It is probable that native labour only will be available for this class of work, but it is desirable, if possible, to put a European in charge of the "clearing party." The traverse direction having been decided on, it is best to keep as much as possible in the same straight line, and it is advisable to let the traverse touch the villages falling near the proposed line. These form good closing points for further topographical work.

Rate of Clearing.—The rate of clearing depends on the nature of the bush and the number of men employed. On the Anglo-German Boundary Commission on the Gold Coast of 1901, 16 men, under a native clerk, cleared on the average about 2 miles a day. The country was, however, not very difficult for clearing and differed totally from that in the Ashanti forest, where the Gold Coast Survey experienced the greatest difficulty in getting long straight clearings.

The tools required for a party of, say, 20 natives for six months would be—

50 matchets or billhooks,
5 shovels,
3 cross cut saws,
2 tapes,
1 magnetic compass.

* Or a theodolite suitably fitted.

It is best, where possible, to make the clearing fairly wide. This permits of a fair scope for the selection of the theodolite stations. About 10 to 12 feet is a useful width.

The "clearing party" should always be a mile ahead of the survey party which follows it.

Detail of Traverse.—The detail in carrying out these traverses is generally similar to that already described for traverses between fixed points. Owing, however, to the impossibility of obtaining any check on the results, it is important to use the greatest care in the measurement of the distance between stations. The angular measurements, too, must be made with every possible care although they are to a certain extent checked by means of azimuth observations taken at every 30 or 35 stations apart.

Causes of Error.—The chief causes of errors in traverses are given in the account on Trigonometrical Traverses, but may be repeated here. They are due to—

(1) Inaccurate centering of the instrument over the station mark;
(2) Inaccuracy in reading the verniers or micrometers and in the bisection of the station marks;
(3) Chainage errors;
(4) Insufficient length of traverse legs.

Of these (1) has been dealt with and requires no further explanation beyond emphasizing the fact that, having no fixed points to work between, it is of even greater importance in these long traverses to see that the centering is accurate.

(2) This can best be minimized by taking a sufficient number of arcs. The probable error of a mean varies inversely as the square root of the number of observations if all are equally good. As regards the bisection of the station marks, it must be borne in mind that the shorter the leg the more refined must be the method of bisection.

When the leg is over 1000 feet, it is sufficient to bisect a flag (at its junction with the ground).

When the leg is short, say under 200 feet, a nail or even a fine needle driven into a wooden picket forms the best station mark. Between these distances an arrow stuck in the ground can be observed to.

(3) Chaining should be done by steel tapes either of 100-feet or 300-feet lengths. All steel tapes should be carefully standardized before they leave England and it is advisable to have them tested at the same "pull," say 12 lbs., with which it is intended to use them in the field. The mean coefficient of expansion of steel* may be used and temperatures should be taken about every hour.

If available, Invar tapes should be used on account of their low coefficient of expansion and of their smaller liability to rust. See p. 8. The distance between stations should be measured twice, once in each direction. One tape should be kept solely for comparison purposes.

The measured lengths must be corrected for expansion of the tape as explained in Trigonometrical Traverses.

(4) The length of a traverse leg should not be less than 350 yards, but it is not always an easy matter to arrange this—depending as it does on the nature of the country and character of the bush. On the Anglo-German Gold Coast Boundary Commission the average length was considerably over 2000 feet, whereas in the Gold Coast Survey an average of 500 feet was extremely difficult to get.

Azimuth Stations.—Mention has been made of azimuth observations and these must be taken frequently in order to check any errors of angular measurement. They should be taken, with short legs, at every 30th or 35th station. Much however depends on local circumstances. In the Gold Coast Survey it is found almost impossible to adhere to this owing to the amount of clearing it would involve in order to see the sun or stars in a convenient

* ·0000065 of its length for 1° Fahrenheit.

position near the prime vertical Under these circumstances one has to be contented with taking observations for azimuth as native clearings in the vicinity of villages present themselves

As a rule it is sufficient to take the mean of three sets of observations to the sun The probable error should not exceed 20 When however, it is found impossible, for reasons above stated to get azimuth stations as close together as 35 legs, precautions should be taken to ensure a higher degree of accuracy and east and west stars, or better still, circum polar stars at elongation should be observed

The difference found to exist between an observed azimuth and the azimuth deduced through the traverse should be proportionally divided amongst the angles This difference is composed partly of the angular errors of the traverse and partly of the convergency of the meridians The proportional adjustment is sufficiently exact for practical purposes

Example An example is given on p 48 of a portion of a traverse carried out by the British Party of the Togo Gold Coast Boundary Commission The headings at the top of each column explain sufficiently what is required in adjusting the various errors in order to obtain the final true bearing of each leg

Computations The computations of the co-ordinates are similar to those already explained in the Trigonometrical Traverse, but in this case the sine and cosine of the *true bearing* are used to compute the co-ordinates

Latitude Deduced through Traverse—Having now obtained the value of the co-ordinates of each point it is an easy matter to deduce the latitude corresponding to the value of the co-ordinates in the north and south direction

In doing so it is better to calculate this for each azimuth station and to add the amplitudes in order to obtain the final difference of latitude

To determine the latitudes and longitudes of the azimuth stations of a long traverse —

If the legs are short—

Let n be the total northings or southings between two adjacent azimuth stations in feet, let M be the value in feet of 1 of arc at the mean latitude then the difference of the latitudes of the two stations is $\frac{n}{M}$ seconds of arc

The sum of all the amplitudes of the arcs between the azimuth stations will give the amplitude of the arc between the terminal stations of the traverse

Similarly if e is the total westings or eastings between two adjacent traverse stations in feet, P the value in feet of 1 of arc of longitude at the mean latitude the difference of the longitudes of the two stations is $\frac{e}{P}$ seconds of arc

If the legs are long (over 1 mile) the latitudes and longitudes should be determined by the method described on p 6

When the final latitude has been calculated it can be compared with that obtained astronomically, but, as previously remarked owing to our ignorance of the value of local attraction this is but an indifferent test of the accuracy of the traverse

To give a comparison between two latitude stations, and to show the method of deducing latitude through traverses, the following table is taken from the results of the British Party of the Togo-Gold Coast Boundary Commission (see p 47) —

Column 1 gives the names of the azimuth stations

Column 2 the distance in feet between these stations measured along the meridian These are the summation of the co ordinates in the north and south direction

Column 3 gives the amplitudes of arc corresponding to these distances

Column 4 gives the values of the latitudes obtained astronomically at the two stations

Column 5 gives the latitudes deduced through the traverse

It will be observed that the difference between the final observed and deduced latitudes is 0 84

The mean of the deduced and observed values is taken as the true latitude

Form H, p 49, gives a further illustration of work of this nature It is taken from the same Boundary Commission and shows the total differences between all the latitude stations observed and deduced in the two independent traverses carried out by both divisions of the Commission

Errors to be expected —To recapitulate, we see that the errors to which a traverse is subject are —

Errors of chainage and angular measurement, called traverse errors

Errors of observed latitude, longitude, and azimuth

Of these, traverse errors vary considerably (from $\frac{1}{1000}$ to $\frac{1}{50000}$), depending chiefly on the character of the country

On the Boundary Commission already quoted the following results were obtained as the measurement between two co-incident points by two quite independent traverses

	Feet N	Feet W
British	279 335	24 698
German	279 347	24,679
Difference	12	19

If the first of these be taken as representing principally the chainage errors (the traverses running almost due north) the resulting discrepancy was $\frac{1}{23,280}$

Astronomical observation errors (omitting "local attraction") should not exceed —

Latitude by zenith telescope	± 0″5
Longitude by telegraphic exchange of signals	± 3 0
Azimuth	± 20″

Reduction to Mean Sea Level —When the length of each traverse line has been finally computed (but not adjusted), it remains to reduce it to mean sea level This is done by the same method as used in the reduction of measured bases —

$$\text{Correction to length of traverse} = \text{length} \times \frac{h}{R},$$

R is the mean radius of the earth,

h is the mean height of the line

General Scheme for carrying out a Traverse Survey.—The following is a general scheme —

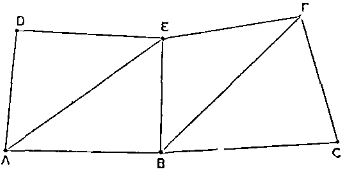


Fig 22

Each of the lines here represents a traverse of about 60 to 70 miles

Form F

Name of Station	Distance in Feet of y Co-ordinate	Amplitude of Arc	Observed Latitude	Deduced Latitude
			° ′ ″	° ′ ″
[illegible]			8 10 30 19	
	32171 13	5 19 2[illegible]		
Camp				8 15 49 30
	47[illegible]	7 4[illegible]		
[illegible]				8 23 38 79
	1[illegible]60 14	2 47 29		
[illegible]				8 26 26 08
	39723 85	6 31 13		
Salaga				8 33 1 21
	2[illegible]153 60	3 20 2[illegible]		
[illegible]				8 36 21 46
	3[illegible]0 80	6 17 6		
Daka				8 42 39 06
	[illegible] 61	[illegible] 88		
[illegible]				8 49 14 94
	28301 93	4 41 30		
[illegible]				8 53 56 33
	216[illegible] 44	3 34 85		
[illegible]				8 57 31 18
	1[illegible]372 88	2 32 [illegible]		
Pillar (near 9th degree)			9 0 2 88	9 0 3·72

FORM G

Road Traverse 9th Degree N L Pabiyá

Stations	Included angle from angle book	Deduced bearing	Azimuth observed	Error divided proportionally	True bearing	Horizontal distance in feet	Co ordinates		Co ordinates from origin (Station 1)	
							x	y	x E + W −	y N + S −
	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″					
1 2	223 40 6	342 18 44	342 18 44		342 18 44	1355 0	− 411 69	+1290 9	− 411 69	+ 1290 9
2 3	193 59 53	356 18 37		− 2	356 18 35	2020 4	− 130 04	+2016 2	− 541 73	+ 3307 1
3-4	179 58 18	356 16 55		− 4	356 16 51	930 8	− 60 37	+ 928 82	− 602 10	+ 1235 9
4-5	126 48 9	303 5 4		− 6	303 4 58	1347 9	−1129 4	+ 734 06	−1731 5	+ 1970 0
5-6	238 48 44	1 53 48		− 8	1 53 40	3200 4	+ 108 03	+3258 6	−1623 47	+ 8228 6
6-7	179 43 26	1 37 14		−10	1 37 4	2377 7	+ 67 13	+2376 7	−1556 34	+10605 3
7 8	180 7 1	1 44 15		−12	1 44 3	2905 6	+ 87 93	+2904 25	−1468 41	+13509 5
8 9	179 58 43	1 42 58		−14	1 42 44	3482 3	+ 104 05	+3480 7	−1364 36	+16990 2
9 10	252 20 55	74 3 53		−16	74 3 37	2320 4	+2331 2	+ 637 2	+ 966 84	+17627 4
10 11	170 8 7	64 12 0		−18	64 11 42	2699 1	+2430 0	+1175 0	+3396 84	+18802 4
11 12	179 58 29	64 10 29		−20	64 10 9	1757 5	+1581 9	+ 764 0	+4978 74	+19566 4
12-13	180 9 5	64 19 34		−22	64 19 12	2744 3	+2473 2	+1189 2	+7451 9	+20755 6
13 14	110 37 49	354 57 23		−24	354 56 59	1496 5	− 131 74	+1490 7	+7320 16	+22246 3
14-15	179 40 15	354 37 38		−26	354 37 12	3067 6	− 287 62	+3054 1	+7032 54	+25300 4
15-16	170 16 40	344 54 18		−28	344 53 50	2633 8	− 686 24	+2542 8	+6346 30	+27843 2
16 17	179 55 56	344 50 14		−30	344 49 44	3010 5	− 787 75	+2905 6	+5658 55	+30748 8
17-18	180 21 55	345 12 9		−32	345 11 37	2643 9	− 675 66	+2556 1	+4972 89	+33304 9
18-19	179 57 36	345 9 45		−34	345 9 11	1972 8	− 437 17	+1532 6	+4377 4	+34837 5
19 20	178 49 39	343 59 24		−36	343 58 48	1584 1	− 505 51	+1907 0	+3340 23	+36744 5
20 21	179 49 23	343 48 47		−38	343 48 9	3923 8	−1094 5	+3768 0	+2845 73	+40512 5
21-22	182 28 56	346 17 43	346 17 4	−39	346 17 4	4555 4	−1081 1	+4177 3	+1765 6	+44689 8

[illegible] II

Comparison of Traverse and Astronomical Amplitudes

Latitude station		Traverse amplitude	Observed latitude	Astronomical amplitude	Remarks
British	Padjai		8 10 [illegible] 19	°	Latitude mean of 2 pairs N and 18 stars p.e. = ± 0 37
		0 49 37 5[illegible]		0 [illegible] 12 [illegible]	
	Pillar		9 0 2 88		3 = ± 0 1[illegible]
		0 48 11 46		0 [illegible] 8 33	
	Patenju		[illegible] 48 11 21		2 = ± 0 33
		0 45 27 15		0 45 [illegible]	
	Gambaja		10 31 42 10		2 = ± 0 101
		2 21 11 14		2 21 11 [illegible]	Difference = 0″77
German	Dabamontu	°	8 7 49 86	°	Latitude mean of 5 pairs N and 18 stars p.e. = ± 0 73
		0 52 9 24		0 52 9 26	
	Pillar		8 59 59 11		7 = ± 0 31
		0 26 12 82		0 26 40 29	
	Yendi		9 26 3[illegible] 19		1 ?
		0 28 44 58		0 28 16 45	
	Gushiokho		9 55 25 85		1 ?
		0 21 11 1		0 21 10 05	
	Pion		10 16 35 90		1 ?
		2 8 47 7		2 8 46 04	Difference = 1 66

Observations for latitude are made at each of the stations A, B, C, D, E, and F, and, wherever possible, telegraphic signals are used as a means of obtaining differences of longitude

Approximate Adjustment—For any two stations such as A and D, we have an amplitude in latitude from the traverse, and one from the observed latitudes at A and D Take the mean of the amplitudes as the true amplitude and take as the true values of the latitudes at A and D, values which differ by the same amount from the observed latitudes, in opposite directions Treat the longitudes (if any have been determined telegraphically) in a similar way For a more rigorous method of adjustment, see Chapter XVII

Recapitulation—To recapitulate, the principal points to be observed are —

(1) The clearings should be as straight as possible,

(2)**The longer the legs of the traverse the better the result and the more rapid the work*,

(3) Great care must be taken in centering the instrument,

(4) The shorter the leg the greater must be the care in centering, observing and bisecting,

(5) An azimuth should be observed at every 30 stations, if possible,

(6) Chain or tape each distance twice,

(7) Read at least two arcs in taking the included angle,

(8) Compute as you go along, the clearing generally delays the observing and an opportunity then presents itself for computing

* This does not apply to *subtense* traverses

CHAPTER V

SUBTENSE METHODS AND TACHEOMETRY

The general principle of subtense work and tacheometry is the measurement of the angle subtended at the observer by a short measured length at a distance. In the simplest case suppose AB to be a bar placed horizontally, C the centre of the bar, O the position of the observer. Let AB be at right angles to OC. Let AOB $= \alpha$, AB $= l$.

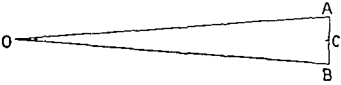


Fig. 23

Then
$$OC = AC \cot AOC$$
$$= \frac{l}{2} \cot \frac{\alpha}{2}$$

Hence if the length of the bar AB is known the measurement of the horizontal angle AOB is sufficient to determine the horizontal distance of the bar from the observer.

The angle AOB may be measured:—

- (*a*) On the arc of a theodolite.
- (*b*) By a movable micrometer wire in the diaphragm of a theodolite.
- (*c*) By fixed wires in the diaphragm of a theodolite, in which case AB must be a graduated bar and the graduations intersected by the fixed wires must be read.

The distance AB may be:—

- (*d*) A bar with 2 fixed discs on it at a known distance apart.
- (*e*) The length between 2 signals, the distance between which must be measured each time they are set up,
- (*f*) A graduated bar, such as a level staff,
- (*d*) and (*f*) may be either vertical or horizontal.

Eliminating the combinations (*c*) (*d*) and (*c*) (*e*), which are impossible, and (*a*) (*f*), (*b*) (*f*), which are unsatisfactory, it is seen that there are eight practicable and useful methods of determining distances by measuring the angle subtended at the observer by a known short length. To this it may be added that as regards (*c*) there are two distinct types of instrument which involve different computations.

Nomenclature

Tacheometry (called also in American books Tachymetry or Tachyometry) is a system of "rapid measuring" as above described and includes all the eight variations just mentioned. The system was first largely employed in Italy in 1820 but had been used in the eighteenth century in England. These various methods are clearly divisible into two main classes:—

- A. Those in which the angle at the instrument is a variable one and the distant base is constant or specially measured—those we shall call *subtense methods*,
- B. Those in which the angle at the instrument is constant and the base is variable, i.e., measured on a graduated bar—these we shall call *tacheometer methods*.

A Subtense Methods—(1) *With an ordinary Theodolite and a 10 or 20 foot Bar*

Let a bar over 10 feet long be supported horizontally on a tripod or on trestles, the bar being at right angles to the line joining its centre to the observer's theodolite. Let there be two discs on the bar whose centres are exactly 10 feet apart. Intersect the left disc with the vertical wire of the theodolite, record the horizontal readings. Now move the upper plate with the slow motion screw and intersect the right disc.

Leaving the upper plate clamped move both plates with the slow motion screw of the lower plate till the wire again intersects the left disc. The right disc is then again intersected by means of the slow motion screw of the upper plate. This process should be repeated say 10 times and the final reading of the right disc is recorded. The observer then has the initial reading of the left disc and the final reading of the right disc, the difference between these two readings divided by 10 will give him a fair measure of the angle subtended by the bar.

The two readings mentioned are the only ones really required, but as a precautionary

Fig 24

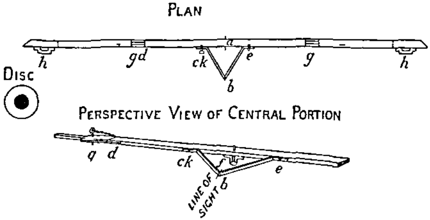


Fig 24* Subtense Bar used on the Survey of India

measure to give a check, it is a good plan to record also the reading of the right hand end of the bar the first time it is taken.

The difference between the readings of the two ends of the bar will give a rough value for the subtended angle sufficiently near to expose any error in counting the number of times the process has been repeated.

This method was introduced into the Indian Survey by Colonel Tanner, who used it largely in the Himalayas. Colonel G Strahan also employed it exclusively for the traverses of the Nicobar Islands. The mean linear error in the latter case which was very unfavourable, was about [1]

Bars have been used in India of 2 feet, 10 feet and 20 feet. In the Himalayas the 20 foot bar was in general use; this had 12 inch circular discs, and under favourable conditions a ray 3 miles long could be measured with reasonable accuracy. A 10 foot bar with 8 inch discs will give good results up to 1½ miles.

Description of fig 24—The bar is made of hard wood in three sections. The central section is square in cross section 1⅛ × 1⅛, with iron sockets 6 inches long *g g* at each end into which the outer portions of the bar fit being pinned into place by the pins *g′*. The outer ends of the bar carry iron sockets *h h*, which have the recesses in them accurately machined out. Into these sockets the discs fit by means of carefully fitted hooks on their backs. The

* Extracted by permission from Hints to Travellers

discs are of wood 10 inches in diameter, painted white with a black centre. Black cloth covers are also carried to fit tightly over the discs in case of working with a light back ground.

In the centre of the bar is a brass socket plate, by means of which the bar can be attached to a tripod.

The sighting arrangement consists of a light iron frame hinged at *e*, *l*, and *k*. The pin of the hinge *l* carries a point on the top and a similar metal point is fixed at *a*, in the centre of the bar, and the other end is fixed by a thumb-screw, *c*, in such a position that the line joining *ba* is at right angles to the line joining the discs. For travelling the thumb screw *c* is unscrewed and the frame is closed up against the bar, in which position the thumb screw screws into the hole *d* in a metal plate affixed to the bar. The bar is fixed in position by an assistant looking along the sights *a*, *b* and laying them on to the theodolite.

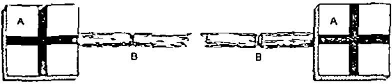


Fig 26 *

Fig 25 represents another form of rod and one more easily made, though not calculated to give such accurate results. A A are two boards 1 foot square painted white with a black cross on each. These are fastened on a bamboo B B in such a manner that the centres of the crosses shall be a known distance apart.

When using the rod in a vertical position it will often be found convenient to fasten a stick to it, extending about 2 feet beyond one of the boards. This when placed on the ground takes the weight off the rod and helps the assistant to keep it steady.

(ii) *With an Ordinary Theodolite and Two Signals*

(See also p 41)

The line joining the signals must be at right angles to the line joining the observer and the central point between them. The signals may be poles fixed vertically into the ground and stayed with rope or wire guys. It is desirable that the ground line should be visible from the observer otherwise the poles must be very carefully plumbed. In fixing the poles the right angle may be determined by compass or by tape, thus

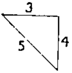


Fig 27

or, if available by a theodolite. The space between the poles should subtend at the observer an angle of from 8 to 12 minutes. The reason being that if we admit the possibility of an error of 1" this will in a total measurement of 8' be equivalent to an error of about $\frac{1}{500}$, which should not be exceeded. The distance between the centres of the poles should be carefully measured with a tape; this distance will vary from 10 feet to 100 feet, according to the length of the ray. For long rays conspicuous marks must be fixed at the top of the poles to attract attention: in India and W Africa wicker baskets about 3 feet in

* Extracted by permission from Hints to Travellers

diameter, have been found useful for the purpose The poles will be from 10 to 30 feet long One of the longest rays thus determined was one of 11½ miles, measured on the Sierra Leone Liberia Boundary Commission of 1903, the base (which on account of local conditions was somewhat small) subtended an angle of 5 47 and was 101 feet long, 5 sets of 10 repetitions of the angle were observed Experience shows that in such long rays it is desirable to use dark poles against a white back ground

The Observations—As a rule 3 sets of 10 repetitions should be taken for ordinary rays of 2 or 3 miles, for longer rays a larger number is necessary Each new set should begin on a different part of the arc, *i e*, if the left signal reads about 0° on the first set, it should read about 35° on the next set and so on Vertical angles should be taken also, 2 on F R, 2 on F L (Required for determination of heights)

The Computations—If α is the mean observed angle,
b the length of the base
d the required distance,

$$d = \frac{b}{2} \cot \frac{\alpha}{2}$$

(iii) *With a Theodolite fitted with an Eye piece Micrometer, and a Bar of Fixed Length*

The eye piece micrometer screw moves a wire in the diaphragm The wire is made to intersect the discs or marks at each end of the bar, and the difference between the two readings of the micrometer head gives the subtended angle in terms of the divisions of the head This process can be repeated many times with great rapidity and the mean, say, of 10 readings, will give a good value of the angle

A table should have been previously prepared giving the distances corresponding to a series of intervals of the micrometer head This table should be determined by *actual experiment* The reason is that owing to optical principles, the angles measured cannot be taken as those subtended at the vertical axis of the instrument, but at a point in front of this (see also p 55) It is easy to show, if the focus has not been altered,* that if M is the micrometer reading of a staff of fixed length, A and B constants to be determined d the required distance *from the vertical axis of the instrument*, $d = \frac{A}{M} + B$ A few experiments at various distances will be sufficient to determine the constants A and B A table should then be prepared

This method is sometimes very valuable and was used in the traverse in Abyssinia, the horizontal angles being measured with the same theodolite, where the route followed was in many places so rough and blocked with troops and baggage animals, that any other system of measurement would have been impossible The rapidity with which such a traverse can be carried on depends upon many considerations but chiefly upon the length of the shots that can be taken In the Abyssinian expedition of 1868, a traverse was carried with one of these instruments from Adigerat to Magdala without any break of continuity, the daily rate of progress averaging 5 miles, and being occasionally as much as 8 miles After this run of some 300 miles nearly due north and south, the observed latitude differed from that given by the traverse by about ¼ of a mile

The bar may be similar to those described on p 52 For preference the bar should be horizontal Rays a mile or two long are convenient

The horizontal angles between the stations should be observed as in a chain or tape traverse

The subtense methods just described are more generally useful to the topographer than the tacheometer methods, which will be next dealt with

* The instrument must be used at solar focus

B Tacheometer Methods—(i) *An ordinary Theodolite, with Two Fixed Wires in the Diaphragm and a Graduated Bar*

The theodolite is provided with two horizontal wires fixed in the diaphragm at equal distances from the axis of the telescope. The staff is held vertically, the divisions of the bar intersected by the fixed wires are read and these are sufficient to determine the distance of the bar from the observer.

Now, an ordinary theodolite has a bi-convex object glass and an eye piece and it is easy to show that the point at which the angle subtended by the fixed wires is constant is not on the vertical axis of the instrument, but at a distance $f + c$ in front of the vertical axis measured toward the bar, where

f is the focal length of the telescope (at solar focus),
c is the distance of the centre of the object glass from the vertical axis.

f can be measured by focussing the telescope on a distant object and measuring the distance from the centre of the object glass to the diaphragm.

Hence, if d is the required distance from vertical axis of theodolite to centre of bar, the bar being held vertically, the telescope being horizontal, if

a is the distance intercepted on the bar by the wires,
l a constant,

$$d = l\,a + (f + c)$$

l should be determined by experiment by taking several readings of the staff at 100, 200 and 300 feet distant, measuring from a point $f + c$ in front of the vertical axis.

If the rod is held vertically, but on a different level from the telescope, so that the latter is inclined at an angle α to the horizon when pointing at the centre of the rod, using the same notation as before,

$$d = l\,a\cos\alpha + (f + c)\cos\alpha$$

and the difference of height (h) between the axis of the theodolite and the mean reading of the staff is given by the expression

$$h = (f + c)\sin\alpha + a\,l\,\frac{\sin 2\alpha}{2}$$

The rapid and effective use of tacheometer methods thus clearly involves the employment of tables. There are several such tables in existence.* It will be seen that the method is not so straightforward as might at first sight appear.

American experiments appear to show that the limiting length of the rays should be about 800 feet. In English practice this limit is usually about 600 feet, but in exceptional cases rays of 1,200 feet may be measured. On account of the shortness of the rays this method is more suitable for large scales than for small, and generally it is more applicable to engineering than to topographical surveys. Rods from 10 to 15 feet long should be used—they may be specially graduated or may be level staves.

(ii)

By the introduction of a converging lens between the object glass and the diaphragm of a theodolite, measurements can be read directly from the vertical axis without the necessity of considering the quantity $f +$. The term tacheometer is best confined to instruments which have this optical arrangement. If there are two fixed wires in the diaphragm of such an instrument and a graduated staff is observed, the distance from the vertical axis

* See Theory and Practice of Surveying, S. B. Johnson, New York, Wiley and Sons. Messrs Troughton and Simms [illegible] instrument of this type.

to the staff varies directly as the number of graduations intercepted If the central l of vision makes an angle α with the horizon, if d is the required horizontal distance of the st from the vertical axis, if h is the height of the point at which the central line of vision strikes t staff, let n be the number of divisions between the wires,

$$d = k\ m\ \cos^2\alpha, \text{ when } l \text{ is a constant,}$$

$$h = d \tan \alpha$$

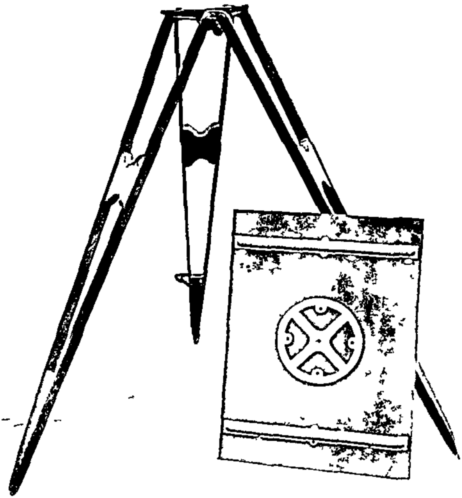
It is desirable in this case also to use tables The instrument is more generally suitable for large scale surveys than for topographical work

There is considerable literature on the subject of subtense methods and tacheometry Amongst other books may be mentioned —

"The Tacheometer," N Kennedy, M I C E
"Hints to Travellers"
"Encyclopædia Britannica,' vol XXXIII
"Theory and Practice of Surveying S B Johnson New York Wiley and Sons
"A Treatise on Surveying,' Middleton and Chadwick, Spon, London

There are many special instruments depending on the same principles, but they are not generally of much value in topographical surveying

PLATE IV



SERVICE PLANE TABLE 18 × 24

CHAPTER VI

PLANE TABLING

THE **plane table** is merely a portable table The table or board can be carried separately from the stand or legs the stand is usually a folding tripod For greater compactness, the stand is sometimes made of a collapsible pattern The adjuncts to the plane table are the sight vane, called also the sight rule or alidade and the box or trough compass, the plane table should also be provided with a waterproof cover

A convenient size is the Service pattern of which the board is 18 × 24 (see Plate IV)

The **sight vane*** should be about as long as the shortest side of the table, and the fittings should be of gunmetal or brass not iron or steel A common defect of sight vanes is that the slit in the rear vane is too small When working in hilly country the centres of the tops of the vanes should be joined by a string taut when the vanes are upright, to enable rays to be taken on steep slopes It is convenient as in the Service pattern for the sight vane to be graduated with useful scales notably 2 inches to 1 mile

The **trough compass** should not be combined with the sight vane The 6 inch trough compass in mahogany box with sliding lid which throws the compass off its pivot is the best Trough compasses should be stored horizontally They can be adjusted for "dip" by tying string round the elevated end and dropping sealing wax on to the string

Mounting a Plane table —With an ordinary plane table which has a plain board (and all arrangements of clips should be eschewed) if time presses the only thing to do is to pin the paper on to the board with drawing pins and if the paper is linen backed so much the better

But if time allows, "mount" the plane table as follows —

Get some fine linen about 8 inches larger in each direction than the board damp this slightly, and paste it on the underside of the board only The paste should be made of the finest flour available cornflour is the best Now take a piece of drawing paper the same size as the linen, damp it on the wrong side paste the linen all over and smooth the paper down on to the linen paste also the overlap of paper on the underside of the table Put in a few pins temporarily on the underside these can be removed in about 12 hours when the table will be ready to work on

When the work is finished and the field sheet is removed from the board its edges should be bound with tape

The reasons for mounting paper on linen are that it gives a good surface to work on and renders the paper less liable to be torn and somewhat less liable to be affected by expansion caused by wet and damp

SURVEYING AND SKETCHING WITH THE PLANE TABLE

The plane table is by far the most useful instrument employed in filling in detail

The following are **general maxims** which should always be observed —

(1) See that the leg screws and clamping screws are taut before beginning work
(2) Spread the legs widely especially with collapsible patterns the stiffness of the table largely depends on the spread of the legs In soft soil the legs should be well pressed into the ground The board should be horizontal
(3) Always keep the pencil sharp, there should be no error in essential features greater than the thickness of a pencil line With acute intersections which must sometimes be used, fineness of line is absolutely necessary Points required afterwards for

* [illegible] sights are [illegible] used in America and although they take away from the simplicity of the instrument they undoubtedly increase the accuracy of the work

interpolation should be fine dots surrounded by a circle No good results can be expected with a blunt pencil

(4) When it is required to draw a ray from a point, put the pencil upright on the table with the edge of the base just touching the point, and lay the sight-vane against the pencil Or instead of a pencil some expert plane tablers use the finger *Pins should never be used*

(5) When drawing a ray from a dot, set the point of the pencil into the dot first, so as to get the correct angle at which the pencil should be held when drawing it along the sight vane Without this precaution large errors creep in

(6) Whenever the plane table has to be set up or oriented by a previously drawn short ray it is important that the ray, when it is first drawn, should be extended by marking short lengths of it at or near the edges of the board These are called 'repere' marks and give a good line along which to lay the sight vane

(7) When drawing a ray to a distinct object mark its approximate position, as estimated, by a small circle on the ray Write the name of the object along the ray, to assist identification it is sometimes useful to draw a small sketch of the object at the end of the ray

The steps to be taken in making a plane table survey are the following —

1 **Plot the Graticule** —The method of doing this is fully described in Chapter XI Scales of latitude and longitude should be drawn on the board

2 **Plot the Trigonometrical Points** —The latitudes and longitudes of the trigono metrical points having been computed as described in Chapter III, the points are plotted with reference to the graticule lines and by means of the scales of latitude and longitude At the outset and before the graticule has had time to get distorted by expansion or contraction of the paper, it is perhaps somewhat more accurate to plot the points from the scale of inches used in constructing the graticule, by converting into inches the intervals in latitude and longitude from the graticule line

3 **Test the Plotting** by going to a trigonometrical station, lay the sight rule along the line joining this station and any distant visible station Move the board until the distant station is intersected by the vertical wire of the sight-rule, then clamp the table See if the other points are in their correct positions by aligning the sight rule on them Any point incorrectly plotted will easily be detected

Now place the box compass on the table (away from the area to be surveyed) move the compass until the needle points to the centre graduation, and draw a pencil line along each side of the compass box These lines will mark the magnetic north, and by their means the table can subsequently be set up approximately Remove the compass

4 **Fix Points by Intersection** —At this station, and at every other point where the plane table is accurately set up rays should be drawn to all points which can be easily identified or are likely to assist in the execution of the work Along all such rays should be written a brief description occasionally it is useful to draw a small sketch of the object at the end of the ray and a small circle should be drawn on the ray in the supposed position of the object

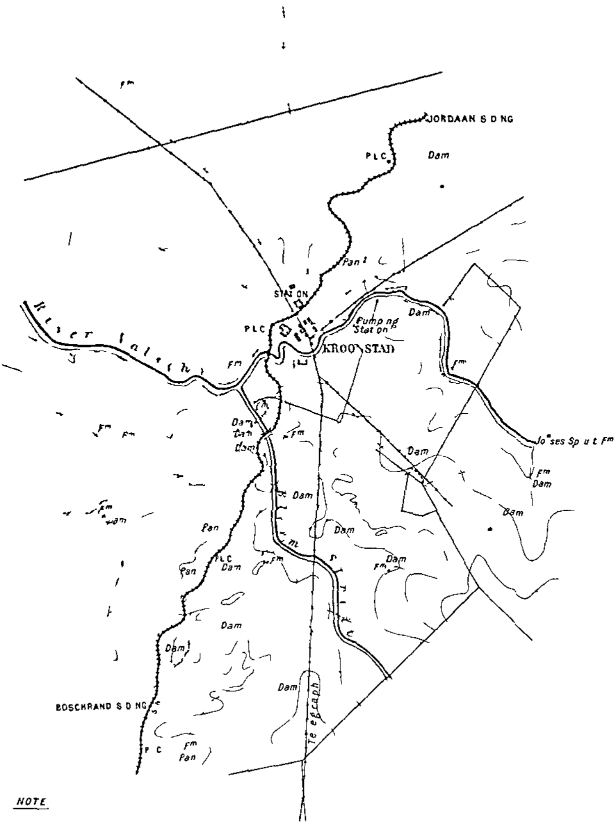
A position fixed by intersection of 3 rays may be relied on If a point has been fixed by only 2 rays it should be considered doubtful, and should not be used afterwards for interpolation

5 **Fill in the Detail, mainly by Interpolation** —Interpolation, or fixing by resection from three known points, sounds a very formidable process, but is in reality very simple It is the principal means by which the bulk of the detail should be fixed The method is as follows —

Set up the plane table at any point from which three previously fixed points can be seen These should be trigonometrical points if possible Orient the table roughly by the trough

Plate V

EXAMPLE of SMALL SCALE PLANE TABLE WORK
(IN UNDULATING COUNTRY)



NOTE

compass, and from the three fixed points draw back rays. If these three rays pass through a point, this point is the required position. If they do not pass through a point, the rays will form a small triangle called the "triangle of error." The true position is now to be determined by the following rules:—

I. If the "triangle of error" is inside the triangle formed by the three fixed points, the position is inside the triangle of error; and if outside, outside.

II.* In the latter case the position will be such that it is either to the left of all the rays when facing the fixed points, or to the right of them all. Of the six sectors formed by the rays, there are only two in which this condition can be fulfilled.

III. Finally the exact position is determined by the condition that its distances from the rays must be proportional to the lengths of the rays.

In the attached example, for instance:—

By condition I the point must be outside the triangle of error;
" " II " " " in sector 6 or in sector 3;
" " III " " " in sector 6, since the distances from it to the rays must be proportional to the lengths of the rays, and by estimation it will be where shown.

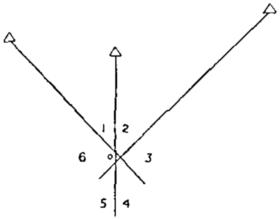


Fig. 28.

Having thus determined the position, place the sight rule along the line joining this and the most distant of the points used, set the sight rule on the point by shifting the plane table, clamp and test on the two other points. If there is still an error (which should, however, be much smaller) go through the process again.

The best position is inside the triangle formed by the three fixed points, of which two are near and one is distant. Accuracy of *position* is ensured by two points being near, accuracy of *setting* is ensured by aligning on the distant point. And in general, fix from near points, set by a distant point.

It is very necessary to be able to interpolate by this method both rapidly and accurately. It is the essential foundation of all good topographical plane tabling. The plane tabler in open country need not pace or measure a yard after he has fixed his base. In addition to the speed which results from a free use of interpolation there is also a gain in accuracy, since there is no piling up of errors and each fixing is dependent only on the main intersected or other well fixed points.

There is one case in which the method fails, viz., that in which the observer's position and the three points lie on or near the circumference of a circle. The plane tabler should be on the look out for this, and not interpolate from points so situated.

* Rule II obviously includes Rule I.

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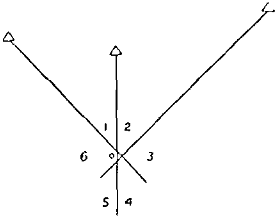


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There is one case in which the method fails, viz., that in which the observer's position and the three points lie on or near the circumference of a circle. The plane tabler should be on the look out for this, and not interpolate from points so situated.

* Rule II obviously includes Rule I.

It will have been noticed that the trough compass is only used to orient the plane-table roughly, the final fixing does not at all depend on the compass It is important that this should be remembered

Resection from Two Points—It will however, sometimes happen that only two fixed points or stations are visible, in this case the plane-table must be oriented by the compass, and rays must be drawn back from the two fixed stations intersecting in a point which must be taken as the required position It is clear that such a fixing depends entirely on the compass and may, therefore, be very inaccurate In parts of S Africa such a fixing would, on account of local magnetic attraction, be entirely worthless, and it is obvious that as a general rule compass fixings should be avoided

This method should, if possible, only be used for putting in detail, and not for throwing out points on which future work will depend

Whenever a fixing has been made the local detail should be drawn in, a combination of measurement and estimation should be used, the amount of estimation permissible will depend on the scale Thus on scales of 2 inches to 1 mile and smaller scales it is not necessary to pace the width of roads or size of houses, these are put in conventionally On a scale of 2 inches to 1 mile a distance of 8 yards is less than $\frac{1}{100}$ inch, so that any distance may be estimated which is not likely to be 8 yards in error, and so on When time presses, minute accuracy must be sacrificed

6 **Traverse Detail which cannot be Fixed by Interpolation**—It will sometimes happen even in open country that there are parts of the work, such as valleys and woods, in which plane table fixings cannot be made, it is then necessary to have recourse to plane table traversing Plane-table traversing has to be much used in flat country unprovided with prominent land marks and was so used in the mapping carried out during the operations in China in 1901

The method is as follows—Set up the plane table at some known point and orient it, draw a ray in the direction of some visible object (and the farther off this is the better), chain to this second point set up the plane-table there, place the sight rule along the ray previously drawn and turn the table until the sight-rule is on the first point, clamp, measure the chained distance along the ray and mark the second point on the plane table, draw a ray to some third point chain to it and go on as before Continue the traverse until it is possible to make an accurate fixing by interpolation, and then adjust the traverse between this fixing and the starting point Errors are inevitably generated in this sort of traversing which should therefore be pencilled in quite lightly, as it will have to be rubbed out when adjusted

The adjustment is to be carried out as follows—

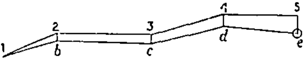


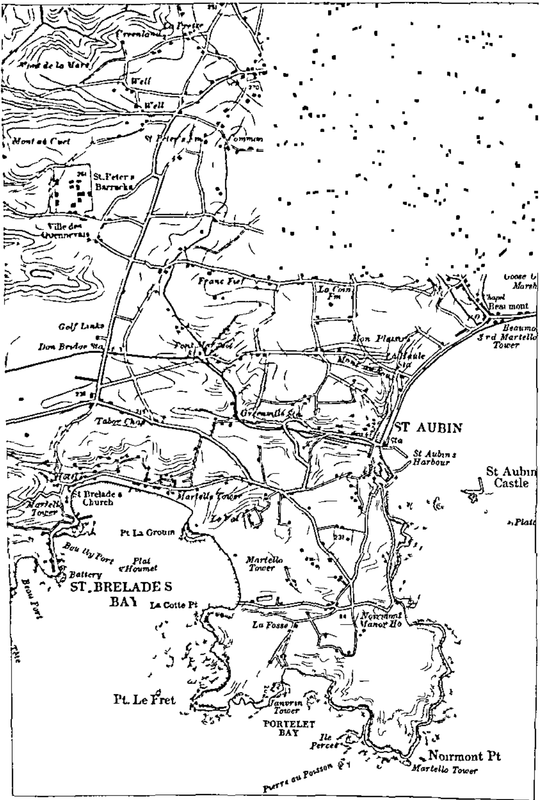
Fig 29

If 1 2 3 4 5 are the stations of a traverse, a fixing was made at 5, and the true position of this fixing was e draw lines through 2, 3, 4 parallel to $5e$, and make $2b$, $3c$, $4d$ bear the same proportion to 1—2, 1—3 that $5e$ does to 15 Then the traverse as corrected will be 1, b, c, d, e

Or, instead of orienting by back rays as above described, the plane table may be oriented by compass alone But it is not advisable to use this method where it can be avoided as a little consideration will show, for a compass may easily be ½° out, which is equivalent to a lateral error of $\frac{1}{120}$, now, in aligning by a back ray, it is very unlikely that the sight rule in a ray 1 200 yards long would be 10 yards off the object However, where the rays are very short, or the traverse is long or objects to line back on cannot be found, or where time is short it may be necessary to traverse with a compass

JERSEY

Scale – Two Inches to One Statute Mile $\frac{1}{31680}$



Ordnance Survey Southampton.

It is sometimes convenient to plot the traverse at the side of the board and subsequently to transfer it to its proper position when corrected.

7. Contouring.—In the preceding description of plane tabling no mention has been made of contouring. As described in Chapter X, contours vary in accuracy according to the method adopted for the survey. They may be fixed by level, theodolite, water level, or clinometer, or may be sketched in by eye. The method adopted depends on the scale of the map, the time and money available, and the purpose of the map.

Speaking generally, so far as the scale is concerned, contours would be determined as follows:—

4 inches to 1 mile and larger	level, theodolite, or water level,
2 ,, 1 ,,	water level or clinometer,
1 inch to 1 ,,	,, ,,
½ ,, 1 ,,	clinometer or sketch contours,
¼ ,, 1 ,, and smaller	sketch contours.

Where contours are determined by level or theodolite, the detail should be surveyed before the contouring is commenced. Where the contours are approximate contours put in by clinometer or sketch contours or form lines put in by eye, the contouring should be carried out during the survey of the detail.

To fix the ideas we will now suppose that it is intended to contour a 1-inch field sheet with the aid of the Indian pattern clinometer.

The Indian Clinometer is 9 inches long and has two vanes which are upright when in use; there is a sight hole in the rear vane and the front vane is graduated in degrees on the left side of its central slit and in natural tangents on the right. The sight hole is brought to a level with the zero of the front vane by a screw behind the rear vane, a level attached to the horizontal arm showing the position. The scale of natural tangents can be read by estimation to 3 places of decimals; the extreme range of elevation or depression is 23°.

Suppose now that the plane tabler has fixed his position by means of resection from three known points of which the heights are known. He sets the clinometer on the table, levels it, and directs it on one of the known points, preferably the nearest. He looks through the sight hole and notes against which reading on the scale of natural tangents the point appears. He now measures the distance from his fixing to the point, multiplies the distance in feet by the natural tangent obtained, and the result is the approximate difference of height in feet between the plane table and the fixed point.

Thus, if the natural tangent is ·043 depression, the distance *2000* yards, difference of level = ·043 × 6000 feet = 258 feet; and if the height of the distant point was 401, and the plane table board was 3 feet above ground, the height of the ground at the plane table is clearly 258 + 401 − 3 = 656, and this height is marked against the fixing. It is not desirable to use this method for long rays on account of its comparative roughness; about 2 miles is a satisfactory limit. If values for the height can be got from more than one fixed point, it is advisable to take them and use the mean.

In place of actually computing the height a diagram may be used (such as Colonel Wahab's height indicator), from which the difference of height may be measured off.

Having now the height of the ground at the fixing, the next step is to determine the position of the nearest contour. Suppose that in the case quoted the fixing is on the top of a gentle rise, and that the ground slopes down uniformly in one direction at an angle of ½° as measured by the clinometer. Now a rule that all topographers should remember is that a slope of 1° is equivalent to a rise or fall of 1 in 57·3; for ½° this is 1 in 114·6, or 1 foot in 38·2 yards.

The height of the fixing is 656 feet, and if the contour intervals are 50 feet the nearest contour will be 650, 6 feet below the fixing, or a distance of 6 × 38·2 yards or 229 yards

away This point is marked on the table, and so are other surrounding points on the same contour

We must now define the terms vertical interval and horizontal equivalent

The **Vertical Interval** is the vertical distance between any two successive contours, and is usually given in feet

The **Horizontal Equivalent** is the distance on plan or on the map between any two successive contours, and is usually given in yards

The plane-tabler should prepare a scale of horizontal equivalents for the particular interval in use Thus suppose the scale to be 1 inch, and the vertical interval to be 50 feet, the scale would be prepared as follows —

1°	is equivalent to a rise of		1 foot in		19.1 yards,	
1°	,,	,,	,,	50 feet ,,	955	,,
2°	,,	,,	,,	50 ,, ,,	478	,,
					&c, &c	

We have therefore, this list of horizontal equivalents for a vertical interval of 50 feet

Degrees	Yards	Degrees	Yards	Degrees	Yards	Degrees	Yards
1	955	6	159	11	87	16	60
2	478	7	136	12	80	17	56
3	318	8	120	13	74	18	53
4	239	9	106	14	68	19	50
5	191	10	96	15	64	20	48

If the slope is uniform, the positions of the other contours will be found by stepping off the horizontal equivalent for the particular slope in question, from the first fixed contour, along the ray down which the slope was taken

But slopes, as a rule are not uniform throughout and this method should only be used for the distance up to which the slopes may be considered sensibly the same, the plane-table being then moved to the next change of slope and the same procedure carried out again there In mountainous country it is best to contour the top of a ridge or hill first, then when the valley is reached the lower contours can be put in

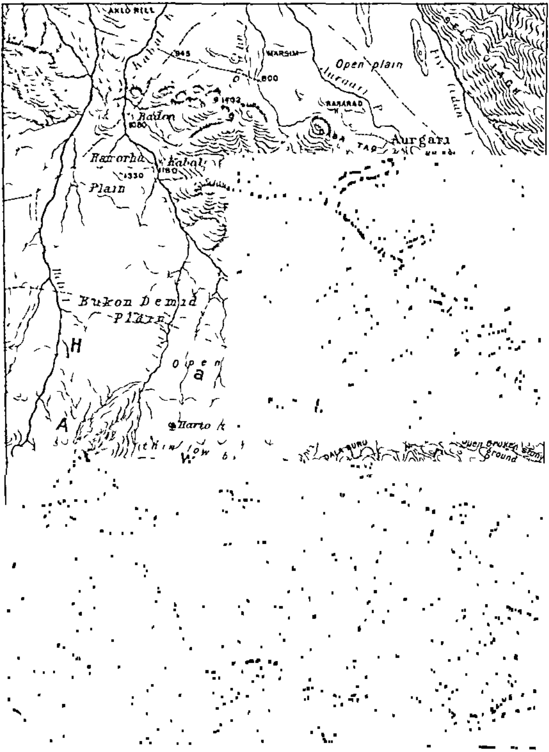
But if time presses, and the bottom of the valley is visible, the difference of level, and hence the number of contours, between the fixing and the bottom may be determined, and the contours spaced by eye, closer on the steep portions of the slope than elsewhere This is rapid approximate contouring

Generally in approximate contouring it is desirable to work in the first instance along ridges spurs and tops of hills putting in the valley contours subsequently

For a more elaborate method of contouring with the clinometer, see Chapter X

SPECIMEN OF SMALL SCALE PLANE-TABLE WORK

(A portion of a survey in Somaliland during operations 1903 4)



Scale $\frac{1}{250\,000}$

FORM LINES
FOR SMALL SCALE WORK



Scale $\frac{1}{126\,720}$

Facsimile of a portion of a plane-table field sheet
Baluchistan Survey 1886

CHAPTER VII

SURVEYS UNDER ACTIVE SERVICE CONDITIONS

Rapid Triangulation.—When [illegible] are called on to undertake surveys, either on active service [illegible] or when as explorers they map country with a [illegible] operations the scene of their labours [illegible] country and extend over a considerable area. On such [illegible] opportunity for more accurate work than is obtainable by mere [illegible].

The [illegible] a rapidly executed map of an extended area [illegible]

(I) [illegible] from a measured base
(II) [illegible] with astronomical observations,
(III) [illegible]

[illegible] regarded as only suitable for rigorously accurate surveys [illegible] It has however been proved on [illegible] the limitations of active service it is possible, in a favourable [illegible] which though falling short of the accuracy to be [illegible] can still be of immense use in eliminating errors and in facili[illegible] the subsequent compilation of different men's work by providing fixed points on to which to hang the detail surveys.

The [illegible] in this class of work are want of time, uncertainty as to when and [illegible] will be made, the possibility that ground once passed may never be [illegible] the risk of going far from camp without a strong escort.

To [illegible] the procedure to be followed though generally similar to that of regular triangulation requires certain modifications in practice, of which the chief are:—

(1) The use of smaller and more portable instruments.
(2) Less refinement in measuring the base.
(3) Less regard for symmetry in the triangles.
(4) Disregard of the rule of always observing all three angles of main triangles,
(5) The selection of natural marks to observe to, owing to the impossibility, as a rule, of making a preliminary reconnaissance and erecting artificial marks.

(1) It is obviously advisable to carry the lightest instrument that will give sufficiently accurate results.

Excellent small microscope theodolites are now made, i.e. theodolites in which the divisions on the limbs are read by micrometers instead of verniers; time is saved in reading and the readings are more accurate.

With a 5-inch microscope theodolite the micrometer heads are graduated to 10 seconds and read to 1 second. These instruments are especially valuable when astronomical work has to be done. They will stand a good deal of rough usage. If this type of instrument is considered too heavy, a Vernier theodolite must be taken. The smallest that has been used is a 3-inch theodolite reading to minutes, in two small boxes with camera stand. Weight with stand and boxes 19½ lbs.

(2) The base would be measured as described in Chapter II.

In selecting a site for a base it is necessary to consider the facilities afforded by the natural features of the surrounding ground for the extension of the triangulation. A stretch

of perfectly flat open ground, though admirably suited for the actual measurement, may be most unfavourably situated as regards extension, and, as a general rule, in surveys of the class now described, it is better to lay out the triangles symmetrically, and to run the risk of an error of a foot or so per mile in the measurement of the base, than to have a more accurately measured base and badly shaped triangles

In the first case the error will be carried through the work consistently, and may be eliminated later by measuring a check base on more favourable ground, in the second case, it is probable that still larger and discordant errors will be introduced, which will cause endless trouble to the computer and, perhaps throw out the detail surveyors

Bases of this description are seldom more than one or two miles in length, and time rarely allows of the measurement being made more than twice

As level ground is naturally selected for the base, and as high natural features are, if possible, chosen for the first stations fixed from it, it follows that the surveyor, when observing back from those stations to the ends of the base, will see them against a back ground of the surrounding plain or hills The result is that any artificial marks erected on the ends of the base will be very hard to see unless they are flashing instruments, such as heliographs Whenever therefore, it is likely that a base may have to be measured, it is desirable that the surveyor should have a couple of heliographers with their instruments at his disposal (N B—Camp looking glasses can be made very useful)

It may occasionally happen that there is no ground suitable for measuring a base at the starting point of a triangulation In such a case, if it is convenient to start from a long base, running nearly north and south, and if there are suitable hills some 10 or more miles apart and nearly on the same meridian an approximate value for the distance between their summits can be obtained by taking a careful series of latitude observations at each, and by observing the true bearing of one from the other See Chapter XV

Still, on account of instrumental errors and 'local attraction,' the resulting distance can be regarded as only approximate, and, although this method gets rid of the troublesome base measurement and extension of the triangulation from the base, the result obtained can never be regarded with confidence, and the earliest favourable opportunity should be taken of measuring a check base with a steel tape

(3) As mentioned previously, the ideal shape for a triangle is the equilateral Such triangles are unfortunately, seldom obtainable in rapid work, and though it is worth while expending some labour to avoid having any angle of the main triangulation less than 20°, still the work must not be regarded as valueless if angles of 15°, or even 10°, occur occasionally Distant intersected points can often be fixed within 200 or 300 feet by triangles whose apices do not exceed 4° or 5°

(4) The more rapid the work the less chance there is of the surveyor being able to observe all the angles of his principal triangles This need not, however, seriously impair the value of his work, if he makes a skilful use of his intersected points as described on p 69

The fact of all three angles not being invariably observed renders it far more necessary in rapid than in regular triangulation to fix each point by more than one triangle

(5) Whenever time and circumstances permit, a preliminary reconnaissance should certainly be made for the purpose of selecting and marking all the stations and intersected points Unfortunately in rapid work it is seldom possible to do so, and consequently natural objects must in the main be relied on In these circumstances the measure of success with which rapid triangulation can be carried out must depend enormously upon the nature of the country The greater part of Afghanistan, Persia, and Baluchistan, abounding as it does with lofty, well marked peaks, which, owing to the clearness of the atmosphere, are visible from very great distances offers an ideal field for this class of work The forest clad hills of Upper Burma and the Shan States and of parts of tropical Africa, on the other hand, present very much greater, but not insuperable difficulties, and excellent work of this class has been

done in those regions, though with less rapidity than in the first mentioned countries. When, however, the surveyor is called on to work in the forests of the Gold Coast, or on the rolling veldt of parts of South Africa, rapid triangulation becomes an impossibility. Though at the first start, in a favourable country, it may be possible to make a small preliminary reconnaissance, and to erect marks on the first stations that are to be observed, still as a rule, the triangulator will be compelled to utilise for his observing stations and intersected points any natural marks that offer themselves, such as tops of hills, trees, rocks, &c., and it is in the selection of these and in their identification from different points of view, that his skill and forethought will be most severely tried. *Always observe to and from the highest point of a hill only.*

General Sketch of the Method of carrying out a Rapid Triangulation.—Assuming that the surveyor with a military expedition has arrived, with his instruments at the nearest point to the scene of operations to which he can proceed, and that he has a certain number of days in which to start his work before the force advances, he would probably proceed as follows:—

Having taken a general look round from the nearest suitable elevation, he would proceed to select a site for his base, and mark the two ends provisionally. He would then measure its distance approximately by pacing, or by any other rough method, and plot the two ends on a plane table on any suitable scale, 1 or 2 inches to the mile for choice. From the two ends of his base he would intersect all the surrounding points that appear suitable for the first stations of his triangulation. If time permitted he would extend his reconnaissance till he had embraced all the apparently suitable points in the required direction that he could fix approximately without moving his camp, and he would at the same time have cairns of stones or poles erected wherever the natural features were not sufficiently clearly marked for observing purposes. As a general rule, these cairns should be of the smallest size compatible with their being clearly visible through the telescope at the required distance. At very short distances a pole is better than a cairn, but if the latter is built, it need not be more than 2 feet high, and should have a sharp-pointed stone sticking up on its top.

It is now possible to decide where to place the first stations, and an inspection of the figures on the plane-table may show that an alteration is advisable in the length or position of the base, in order to make the triangles more symmetrical. The ends of the base are now finally chosen, and marked with wooden pegs covered with small cairns of stones or mounds of earth, so as to be easily recognisable.

The next step is to measure the base as accurately as possible under the circumstances, and to observe the angles from both ends of it. If time permits, these observations should be *computed, and the lengths of the sides of the triangles fixing the first stations should be* obtained. The true bearing of the base and the latitude of one end of it should also be determined astronomically.

Next, the projection should be drawn on the plane-table (p. 93). The latitudes and longitudes of the points should be computed and plotted, or if time does not allow this, the points must be plotted by bearing and distance.*

On arriving at one of the near points that he has already observed, the cairn must be taken down and the theodolite placed over the spot. In order to make sure of finding this spot when the stones are removed, it is well to lay a couple of sticks or lines of stones so that their prolongations cut the highest point of the cairn at right angles. When the cairn is removed, there will then be no difficulty in finding the exact point where the theodolite should stand. When the observing is concluded, similar referring marks should be placed, to ensure that, when the cairn is rebuilt, its top is exactly over the spot where the theodolite stood.

In addition to observing the ends of the base and the points observed from the base, fresh objects must be selected for observation in the direction in which the surveyor expects to move, with a view to their use as theodolite stations. All natural features, too, which are

* The *triangulation chart* should not, as a rule, be on a larger scale than 4 miles = 1 inch (say $\frac{1}{250,000}$) or the triangulator will run out of it much too soon. The topography may be on any scale.

likely to be of service as intersected points must be observed, in order that, when fixed, they may facilitate the delineation of the topography.

In a regular survey, when the triangulator starts work he is provided with a plane table, on which all the points he has to observe are plotted with sufficient exactness to render it impossible for him to mistake or omit one of them. In rapid work, on the other hand, owing to the absence of a previous reconnaissance, the triangulator has to construct the diagram of his triangulation on his plane table as he goes along.

Once the triangulation gets started clear of the base line and its surrounding stations—which, being necessarily rather close together, are comparatively easy to recognise—the work of identification becomes more difficult, and to effect it with success the triangulator must trust to a skilful use of his plane table in the following manner —

Having oriented his plane table correctly, and having ascertained by calculation, intersection or resection the spot on it corresponding to the station from which he is observing, the triangulator draws lines to each of the natural features that he has observed, and writes against each of them the name that he has given it in the angle book. To this he adds an outline sketch of the hills as they appear to the naked eye and their estimated distances, together with any descriptive remarks that are likely to aid in their recognition from other points of view. When the telescope discloses any peculiarities of shape that are invisible to the unaided eye, a sketch of the hill top as seen through the telescope should be made in the angle book.

At the next observing station the same process would be repeated.

From these two initial stations (the line joining them being as nearly at right angles to the line of probable route as possible), several hills that appear likely to serve as the next observing stations, besides other objects to be used as intersected points, will probably be fairly well fixed, but others will remain still undetermined either on account of the acuteness of the two rays or because they have been observed from only one of the two stations.

The force of circumstances, such as the direction of the next advance during military operations or the position of water, the direction of the route, or the best site for a camp in other cases, will generally decide which of the points observed to is to be made the next observing station. When this point is a conspicuous hill there will be no difficulty in recognising it, but the task is far from easy when, as often happens in mountainous country, on descending from the initial stations on to the low ground along which the line of march runs, the observer finds that most, if not all, of the points to which the observations have been taken have disappeared from view.

It is in such a case that the plane table becomes of the greatest value. By its means the position of the observer on the line of march with reference to all points previously fixed, whether visible or invisible, may always be ascertained by resection, provided two of these are visible and sufficiently well situated to admit of their being used for this purpose. From such resection rays should be drawn to other points, and by this means a new series of points is obtained, by the help of which the observer will be able to continue his advance without losing the means of finding his position.

At the same time it will be necessary to ascend some sufficiently commanding hill near the line of march, the position of which can be resected, in order to take second rays to such points as have not yet been sufficiently well fixed, before the altered point of view changes their appearance beyond recognition.

Supposing now that a hill has been reached which is believed to be a point previously observed to, and from which it is now intended to observe, the accuracy of the surmise can be tested as follows —

The plane table having been set up, its position is resected from the fixed points that are most suitably situated. If the fixing thus obtained falls upon the previous intersection, or upon the single ray should only one have been taken, there is little doubt that the hill has been rightly identified. Once all doubt as to this has been removed, all the other already intersected points can be easily recognised, if visible, by directing the ruler on them in turn

For the identification of the actual spot observed (the highest point of the hill for choice), with the theodolite the observer must trust to his recollection, aided by such notes and sketches as have been previously made

It will be seen from these observations how large a part of the difficulty in executing a rapid or reconnaissance triangulation consists in selecting suitable points, and in recognising them from different points of view, and how necessary practice with the plane table is, in order to make a proper use of its aid

This process will be much facilitated if circumstances allow of the surveyor visiting a hill some two to four miles on the right or left (with reference to direction of route) of his observing station, and of approximately the same height, from which he can get second plane table rays to his newly-observed points As the point of view will be so similar, he should have little difficulty in recognising them and though the intersections will be very acute if the points are distant still the approximate positions they will give will be most useful in aiding him to recognise his points

The same principle can be very advantageously applied to triangulation (see Fig 30)

Here the stations C and D have been fixed from A and B On visiting D, the triangulator selects F and G as his next forward stations, and observes to them There is

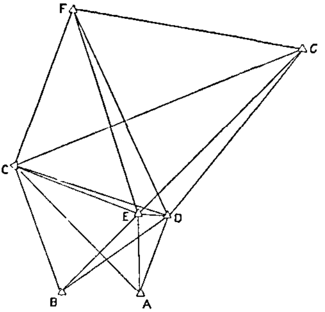


Fig 30

however, a considerable risk that when he visits C he may not be able to recognise these points with certainty He therefore makes a station on E, a hill near D, which he should have observed from A and B, and from it he observes with the theodolite to F and G If these hills are at all clearly marked, the triangles DEF and DEG, though very badly shaped, will suffice to enable him to fix the positions of F and G so nearly to the truth that, when they are plotted by their computed distances along the plane table rays from D, he cannot possibly have any difficulty in recognising them from C

In this way, too a second triangle is secured to fix each of the points F and G Should circumstances not allow of the theodolite being used at E, the intersection of plane table rays drawn thence with those drawn from D will probably fix the positions of F and G sufficiently accurately for subsequent identification from C

The length of sides permissible in the triangles depends upon the height of the stations and the extent of view from them, the nature of the objects observed, the clearness of the

atmosphere, and the maintenance of sufficient symmetry. In some countries, as far as view is concerned, triangles can be laid out of 30 and 40 mile sides, and intersected points as far distant as 100 miles may be fixed.

The difficulties connected with this class of work, however, generally make it advisable to keep the sides of the triangles between 10 and 20 miles in length.

Theodolite Observations.—Unless the points to be observed are extremely well defined it is useless to multiply repetitions of the same angle. When observing to natural objects under Service conditions, one observation on each face is as much as can generally be obtained, but the triangulator should satisfy himself, before taking down his theodolite, that the observations on the two faces agree within small limits. If only one observation is taken to an object, no great reliance can be placed on it in view of the many possibilities of error in the observation, the reading of the verniers, and in the recording. When, however, two accordant values have been obtained on different faces, it is extremely unlikely that there is anything wrong. With regard to the selection of objects it is a good rule to observe everything that is observable (*i.e.*, every point that is sufficiently lofty to appear suitable for a station, and sharp enough to be intersected with accuracy), whether it lies in the probable direction of advance or not. It constantly happens that observations to points far away from the direction in which the triangulator expected to move turn out not only useful but indispensable.

In choosing intersected points, the great object is to fix such as will be useful for the topographers. As they do not always require to be fixed with the same accuracy as the stations, it is not necessary that they should be so sharply defined, but they should be conspicuous points, about which, when seen with the naked eye, there can be no mistake. Many a hill appears to be an admirable point to observe to until it is examined through a telescope, when it may turn out to have a flat, rounded top. Such a hill, unless previously marked with a pillar or pole, would make a bad station, but an excellent intersected point for plane tabling purposes.

Generally speaking, the highest point of a hill should always be observed; as a rule, there is little chance of making a mistake as to which of a group is the highest, but when there are several close together of apparently equal height, they should all be observed. Occasionally it happens that the highest point, though suitable for the theodolite to stand on, and easily marked by a cairn when visited, offers no point, when first seen, that can be intersected with any accuracy. In such a case, if a little below the top there is a conspicuous rock or bush, it should be observed to, and the angle can be corrected afterwards to the actual position of the theodolite.

As each point is observed for the first time, its outline, as seen in the theodolite, is carefully sketched in the angle-book. If possible it is entered under its real name, but as this is seldom obtainable for a distant hill, it is usual to give it some descriptive name, such as black rock, jagged peak, &c., or else to give it a distinguishing letter or number. Whatever designation is given should be adhered to throughout the angle-book and computations. If eventually a reliable name is obtained, all the entries can be changed together.

Computations.—The method of computing rapid triangulation is similar to that already described, except that the computer must allow himself more latitude in dealing with any excess or defect on 180°, when the three angles of a triangle are observed, as it is seldom the case that all three are equally trustworthy, owing to the varying nature of the marks observed. If the computer is familiar with the method of calculating the latitudes and longitudes of the triangulated points, he will probably find it best to make it his practice to compute them in spite of the additional labour involved. Plotting by latitude and longitude is far more accurate than building up triangles by distances, and the advantage for compilation of having all the topographical sketches properly located as regards latitude and longitude is alone sufficient to repay the labour of calculation. The distance between points

not directly connected can be readily calculated, see Appendix III This is particularly useful for interpolations The advantage is perhaps most apparent when it is the triangulator's business to provide fixed points for several sketchers more particularly if they are working some distance apart

If his only means of communication with his assistants is by *letter or heliograph, it is no* easy matter to indicate to one of them how some newly fixed point should be plotted by distances supposing, as may well happen that the stations from which it has been fixed lie off his board On the other hand if the latitude and longitude have been computed and sent to the plane tabler he can have no difficulty whatever in plotting them correctly on his projected plane table sheet.

It has been mentioned previously (p 64) that in rapid work it may be sometimes impossible to observe all three angles of the triangle and that by a skilful use of intersected

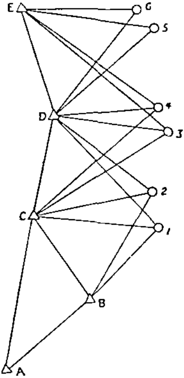


Fig 31

points triangulation may still be carried on without appreciable loss of accuracy Instances of this are given in figs 31 32

In these it is assumed that the observer finds it impossible from the nature of the country or for other reasons to visit any points better situated than ABCDE and that AB is the last side that he has been able to fix in the ordinary way by observing all three angles

A glance at the figures shows that BCDE are too nearly in a line for it to be possible to calculate with accuracy the triangles they form If there are no well marked points on either side rapid triangulation of a continuous nature becomes impossible but if there are well marked points on one flank such as 1 3 5 the difficulty can be surmounted If such points can be found in pairs as 1 2 3 4 5 6 the situation becomes much improved and the triangulation would be computed as follows —

Compute the side CB of the triangle ACB

Then with BC as base compute the triangles BC1 and BC2 the angles at 1 and 2 being the supplements of the angles at B and C

With the base 1C compute the value of the side CD. Repeat the same operation with the base 2C.

Take the mean value of CD as base, and compute D3 and D4. From these two bases obtain DE, and from DE calculate E5, E6, and so on.

The most favourable case for this sort of triangulation is when points can be obtained on both sides of the line of stations, as in Fig 32. Here the method of computing would be exactly similar, two values being obtained for each of the sides CD and DE.

In spite of all the surveyor's foresight and energy cases may arise, even in a favourable country, where continuous triangulation becomes impossible. For instance, after fixing one or more peaks on a range of hills, over which his route passes, he may be obliged to cross to the other side without having an opportunity of visiting any of these points. In such a case, if circumstances allow, the best course is to start a fresh triangulation on the further side and

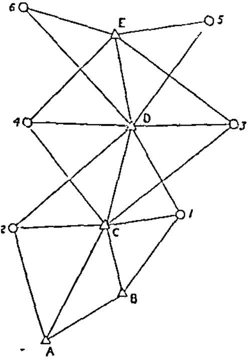


Fig 32

from it to intersect the peak or peaks already included in the previous triangulation. If this is done the values of latitude and longitude of the new triangulation can be computed from the old one, and all the points fixed by both can be plotted correctly relatively to one another.

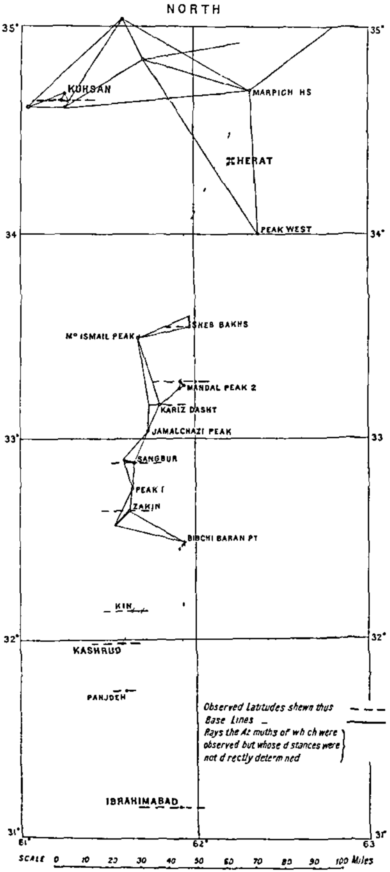
In this case also the methods of trigonometrical interpolation may be of great assistance (see p 3[illegible]).

If the march is very hurried indeed even this may be impossible, and if the route lies east and west the relative longitudes of the different points along the line can be ascertained only by astronomical observations, or by a traverse. Which of these two methods was the more reliable would depend upon the circumstances under which they were carried out.

If, however, the route runs approximately north and south, a combination of observed latitudes with the azimuths of conspicuous points and the measurement of a few base lines will enable the surveyor to maintain a good value of longitude as long as the country is of a suitable character. For, where the latitudes of two points and the azimuth of the line joining them are known, the difference between their longitudes can easily be computed (see Chapter XV).

CHART SHEWING THE TRIANGULATION AS EXECUTED ON THE MARCH OF THE AFGHAN BOUNDARY COMMISSION BETWEEN IBRAHIMABAD AND KUHSAN, FOR THE PURPOSE OF FIXING THE LONGITUDE OF KUHSAN

Th s portion of the March was 310 Miles long and took 19 days



For purely topographical purposes the same result could be obtained by plotting the initial point A and the azimuth from it to the second point, B. Then the point B will be at the spot where the azimuth cuts the parallel drawn through the observed latitude of B. A good instance of both this method and that above described, of starting a fresh triangulation when it is impossible to carry on the old one, is to be found in the march of the Afghan Boundary Commission to Herat in 1884, when it was considered of great importance to carry the true longitude from India up to the Afghan frontier (see Plate VIII).

By various methods the longitude had been carried with fair accuracy as far as Ibrahimabad. At this point the route turned north and a lofty hill, Bibchi Baran, became visible on the horizon in the direction of the line of march.

At Ibrahimabad a latitude was observed and an astronomical azimuth taken to the Bibchi Baran peak, some 93 miles distant almost due north. The column then marched through Panjdeh, Kahrud and Kim, at which places latitudes were observed and azimuths taken to the same Bibchi Baran hill; these were afterwards used to fix the positions of these three places. At Zakin a 7-mile base was measured with a perambulator along the line of march, and observations were taken to Bibchi Baran and other peaks from each end, the true azimuth of the base line and latitude of Zakin being also observed.

From these data the distance from Zakin to Bibchi Baran peak, the difference of longitude between the two places, and the latitude of the latter, were computed. Then from the latitudes of Ibrahimabad and Bibchi Baran peak, and the observed azimuth of the ray Ibrahimabad to Bibchi Baran, the true longitude of Bibchi Baran peak, and thence that of Zakin were obtained. From Zakin onwards to Sangbur, Karez Dasht, and Mandal, short bases were daily measured on arrival in camp, and triangulation executed, by means of which each day points were fixed on ahead, which were observed back to from the next camp.

At night azimuths and latitudes were observed, the latter serving merely as checks on the triangulation. By these means the latitudes and longitudes were computed on from Zakin to Mandal. At Mandal an azimuth was observed to 'Peak West,' a hill some 60 miles to the N.N.E. The march was then continued through various places to Kuhsan, bases being measured almost daily and triangulation executed as before. It was found afterwards to be unnecessary to utilise these latter bases for the purpose of bringing up the longitude, but they served their purpose as aids to the topography at the time. At Kuhsan a base was carefully measured, and a comprehensive system of triangulation commenced based on an observed latitude. This triangulation, which was afterwards extended to Mashhad in one direction and to Bamian in the other, included, among other points fixed, that called "Peak West," which had been observed from Mandal. The latitude of this "Peak West" and its difference of longitude from the Kuhsan base were then computed. With the latitudes of "Peak West" and Mandal and the observed azimuth of the line Mandal to "Peak West," the difference of longitude between these two points was now calculated, and the true longitude thus brought up from Mandal through 'Peak West' to Kuhsan.

A check was also obtained by an azimuth taken from Marpich Hill Station—a station of observation of the Kuhsan main triangulation—to Mahomad Ismail Peak, a point that had been fixed from the Karez Dasht base. The result on the whole was most satisfactory, the work standing rigorously any check applied to it by observations taken to various alternative points.

While on the march it was impossible to compute all the observations; in fact only those that were of immediate use to the topographers, such as the latitudes of the camps and the distances of some of the prominent peaks, were computed daily. Still, even with this limitation, the work could not have been carried through unless there had been a very strong party employed upon it, and it was found that the trigonometrical and astronomical observations, together with a plane-table and perambulator traverse, on the scale of 1 inch to 4 miles, provided full employment for three officers and three native surveyors.

As a measure of the rapidity with which this method can be conducted in a very favourable country, such as this was, it may be noticed that the whole distance from Ibrahimabad to Kuhsan, some 310 miles, was covered in 19 days including halts, during which the connection in longitude was kept unbroken In the diagram of the triangulation, Plate VIII, many of the daily local triangulations are omitted, as it was found unnecessary to use them when the triangulation was subsequently extended from Kuhsan As regards the accuracy, it was estimated that the error in the longitude of Kuhsan relatively to Ibrahimabad, computed as described above, probably did not exceed 5 seconds of arc, that is less than 200 yards If there had been no opportunity of starting an extensive triangulation from Kuhsan and working back, and the daily work of the march had been alone available, it is probable that the error in the longitude of Kuhsan would not have exceeded half a mile

Interpolations (See also p 30) —In rapid work interpolations play a more important part than they do in regular surveys By means of this method useful stations can sometimes be made in the middle of a valley where there is no marked spot to which it would be possible to observe

In interpolating a station it is always advisable and sometimes indispensable to observe the azimuth of one of the points from which the station is to be fixed

For instance, supposing that the surveyor finds, that, owing to the configuration of the country and the exigencies of the occasion, his triangulation ends in an intersected point A

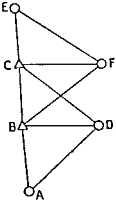


Fig 33

and a hill station B situated on a range of hills running parallel to the direction in which he proposes to travel, and that from B he has observed to C, another point on the same range The surveyor having crossed the range ABC is encamped at D on flat ground

Now, if a third fixed point were visible in a suitable position, it would be possible for him to interpolate the position of D as described on p 32, but as there are only two fixed points visible he must observe at D the azimuth of either A or B as well as the angle between them, as described on p 33

In the same way the position of the next camp, F, can be interpolated, and if the observer visits C and observes E from it the position of E can be fixed, and so on

In this way it would be possible to carry a triangulation along a single ridge of hills without ever observing from it to any points on a flank, and though the work would not be of very great accuracy, still, if the triangles were fairly well conditioned, the points on the hills were well marked, and the azimuth observed with accuracy, no great error could creep in

In all rapid triangulation, where it is always an object to observe from as few stations as possible, the important thing to remember is to keep a continuous series of azimuths This is attained by ensuring that from each station at least one back station is observed, from

which it has itself been observed. When it is impossible to do this, as in the instance just described, the azimuth must be observed astronomically.

Of course, the use of azimuths involves the calculation of the latitudes and longitudes of all the stations and intersected points of the triangulation, a procedure which has been advocated above on other grounds.

Perhaps the most successful application of the method of interpolation ever utilized in a new country was made when triangulation was carried across the "snowy range" of the Himalayas into the Pamirs for the Boundary Commission of 1895. The interpolation was made from peaks of one of the secondary series of the Himalayan triangulation.

When Triangulation is impossible, owing to the nature of the country or the extreme rapidity of the march, if north to south or diagonal rays can be obtained the next best thing is to provide a framework of points by means of latitudes and azimuths (see Chapter XV).

If, on an expedition, even this is impossible, positions must be fixed by rough compass traverses controlled by astronomical observations. This is the explorer's method of mapping, and is necessarily of a low standard of accuracy. If the march is generally north to south, observed latitudes will serve to check the distances traversed; if east to west, they will check the bearings observed; if the march is in a diagonal direction, the observed latitudes will serve to check both bearings and distances. For the methods of determining latitudes see Chapter XIV.

CHAPTER VIII

LEVELLING

Levelling is the art of determining the relative heights of various points referred to a hypothetical surface which cuts the direction of gravity everywhere at right angles This surface is consequently slightly irregular where the direction of gravity is influenced by local and abnormal attractions

Mean sea level, at some selected station on a coast is the usual datum to which such heights are referred though other arbitrary levels may be assumed if necessary

There are three principal methods by which relative heights may be determined

(i) The first and most accurate method is by spirit levelling

(ii) The second in order of accuracy is by trigonometrical or angular measurement This method has the advantage that it enables the relative heights of inaccessible points to be determined

(iii) The third method is by the comparison of the barometric or atmospheric pressure at the required points

The last two are indirect methods of levelling, and are described elsewhere in this book The first is the only method which is technically speaking, included in the term "levelling"

By levelling is meant the direct determination of the differences of height of various points from the readings of the heights at which graduated staves held vertically over these points are cut by the horizontal plane which passes through the eye of the observer

Direct levelling may be subdivided into —

(a) Geodetic, or precise levelling
(b) Ordinary, or engineering levelling

These two classes of levelling are similar in general principles, but differ chiefly in the details of their execution particularly as regards the degree of accuracy to which personal and instrumental errors are eliminated

Geodetic or Precise Levelling has for its object the determination of the heights above mean sea level of a number of fixed points known as "bench marks," scattered over the country The main object is to afford reliable points of reference for mapping and engineering purposes consequently every precaution should be taken to make these heights as accurate as is possible Geodetic lines of levels are run so as to cross and recross each other, forming a network over the country The crossing points, where there are bench marks common to two or more lines, afford a means of checking the accuracy of the work This class of levelling is of a special character and of a high order of accuracy

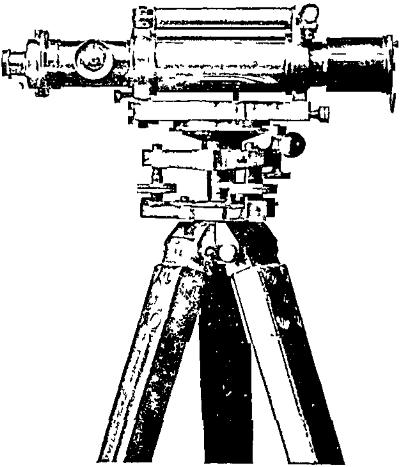
We are here only concerned with **Ordinary or Engineering levelling** such as is required for general engineering purposes, as the construction and laying out of irrigation works, roads railways, canals, buildings, &c , also for the accurate contouring of maps

INSTRUMENTS

The instrument generally employed is known as a **level**, though any theodolite which carries a spirit level attached to its telescope may be used for this purpose, and is, at times, more expeditious though less accurate than the regular level

The essential part of all levels is a telescope provided with a long spirit level and containing a diaphragm carrying a horizontal and (usually) two vertical wires

PLATE IX



COOKE'S REVERSIBLE LEVEL.

The telescope is capable of being revolved in azimuth about a vertical axis, and is secured to the usual tripod stand It is supplied with three foot screws for the purpose of making the axis round which the telescope revolves truly vertical

Some old pattern levels are mounted on four instead of three foot screws, this is a mechanically unsound method of support

A level is described by the focal length* of its telescope and sometimes, in addition, by the aperture of the object glass

Fourteen inches focal length is the size in most common use Ten inch levels are used for rough surveys, and 16 to 26 inch levels for very exact work

Patterns of Levels—There are several kinds of levels, of which the Dumpy, Cooke's Reversible and the "Y" pattern are the ones most frequently used The two latter are more readily adjusted than the Dumpy and are generally used for ordinary levelling operations

It will be sufficient here to describe Cooke's reversible level, which is the pattern most generally used in the Service and to explain the adjustments of all three above named patterns

Cooke's Reversible Level

An illustration of this instrument is given on Plate IX

Its construction is shown below, where fig 34 is the telescope (in longitudinal elevation) This telescope has two equal cylindrical flanges, F and F, turned on it concentrically with the tube

The cross wires, consisting of one horizontal and two subsidiary vertical wires, are lines ruled upon parallel glass, carried by a perforated block in the eye end which is adjustable vertically to correct for collimation by the two antagonising diaphragm screws *d* and *d*

The eye-piece is focussed by pushing it in or out The object-glass is focussed by turning the milled head M

The two flanges F and F fit closely inside two corresponding collars S and S, *vide* figs 35 and 36

When the cover of the object glass and the screw Sc are removed, the telescope can be introduced indifferently from either end of the socket SS (fig 35) and pushed home until the stop flange St (fig 34) comes in contact with the socket end S or S

Great care should be used in reversing the instrument, not to allow any grit to enter the sockets

One end S of the socket is furnished with a threaded bolt which passes through a hole in the base plate PP (fig 36), and is adjustable vertically by means of the two lock nuts N and N

1 Adjustment for Parallax—To make the focus of the eye piece and the image formed by the object glass coincide on the plane of the cross wires

First adjust the eye piece by carefully pushing it in or out in its tube, until the cross wires are seen as distinctly as possible, after which there should be no necessity to adjust the eye piece again, then focus the telescope on some well defined object at the average distance at which the levelling staff may be expected to be held (say 200 feet)

If, when the eye is moved gently up and down behind the eye-piece the image of the object does not move in the slightest degree with respect to the horizontal cross wire then the telescope is correctly focussed but if the image seems to follow the movements of the eye, that is, to tend to move above the cross wire when the eye is raised and to move below it when

* The focal length of a level is usually measured from the front shoulder of the object glass to the cross wires when the telescope is focussed for parallel rays It would be sufficiently accurate to focus on an object 2 miles distant This is near enough for most purposes though not theoretically correct the true focal length being measured from the cross wires to the optical centre of the object glass and the latter point is a varying distance (according to size) behind the front shoulder

the eye is lowered, then the image is not focussed exactly upon the cross wire, but lies on the side towards the object glass, which must consequently be made to approach the cross wires by means of the milled head M (*vide* fig 36) The reverse is the case when the image tends to move below the cross wire when the eye is raised and *vice versa*, that is, the error of parallax must be corrected by moving the object glass farther away from the cross wires

The above adjustment for parallax is the same for all levels and theodolites, and must be carried out before any correct observations can be taken This adjustment should precede all others, including even the levelling of an instrument for ordinary use

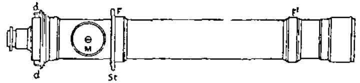


Fig 34

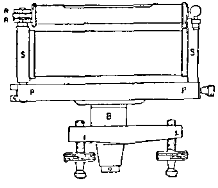


Fig 35

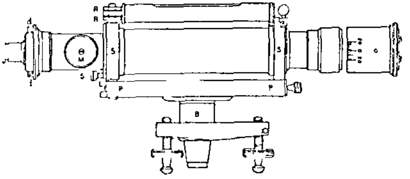


Fig 36

Generally, when once set for the day, there is no occasion for altering the focus of the eye piece but the focus of the object glass will obviously require alteration as the distance of the object varies

2 Adjustment for Collimation.—To make the line of collimation* coincident with the axis of rotation of the telescope in its sockets

* The *line of collimation* is the line joining the optical centre of the object glass and the intersection of the cross hairs

Having set the telescope over a foot screw, direct it on an object as in the last adjustment and bisect this object with the horizontal cross wire, then clamp the instrument, withdraw the fixing screw Sc (*vide* fig 36) and twist or rotate the telescope half way round in its socket, so that the screw hole, through which Sc passed, is brought to the top and the cross wire is again horizontal but reversed If the object is not still bisected, the error of collimation is corrected half by means of the foot screw lying under the telescope, and half by the antagonising diaphragm screws *d* and *d*

3 Adjustment for making the "Horizontal Axis of the Reversible Socket exactly at Right Angles to the "Vertical" Axis—The terms 'horizontal' and "vertical" are here used in an explanatory sense, as the two axes are not made truly horizontal and vertical until the final adjustment is carried out Referring to fig 36, this adjustment is to make the "axis of figure of the reversible socket SS exactly at right angles to the 'vertical axis B

First remove the cover of the object glass and the screw Sc (*vide* fig 36), then set the telescope over a foot screw, direct on and bisect an object, as in the last adjustment With draw the telescope carefully from its socket SS then turn the socket round end for end and carefully insert the telescope again keeping the screw hole in the stop flange St through which the screw Sc passes vertically below the centre of the tube when the telescope is pushed home The socket SS will now be reversed with reference to the telescope Again direct on the object, and if it be not now bisected by the horizontal cross wire correct half the error by the foot screw below the telescope and half by the lock nuts N N

This operation of reversing the socket on the telescope and the correction of the error half by the foot screw and half by the lock nuts, should be repeated until the bisection of the image remains undisturbed by the reversal of the socket When the adjustment is complete the telescope ought to lie in its socket with its stop flange St against the end S of the collar so that it may be secured by the screw Sc

Care should be taken that no dust is allowed to settle on the flanges of the telescope during its insertion in the socket

4 Adjustment of Vertical Axis of Rotation—That is, to make the vertical axis of rotation truly vertical, and to make the telescope level at right angles to it

This is the same adjustment which is gone through for levelling a theodolite on first setting up

Place the telescope parallel to two foot screws and with them alone bring the bubble approximately to the centre of its run, noting the reading at one end of the bubble (say the end near the capstan headed screws)

Turn the telescope through 180 degrees until it is again parallel to the same two foot screws, and again note the reading of the same end of the bubble By means of the two foot screws, bring this end of the bubble to the mean of the two readings Now turn the telescope until it is over the third foot screw and by it alone bring the same end of the bubble to this mean reading

Repeat until one end of the bubble reads the same in any position, when the upright axis will be truly vertical The bubble can then be brought to the centre of its run by means of the two lock nuts R R at one end of the level (*vide* fig 36)

It is obvious that if the third adjustment is carried out and both axes of the instrument are thereby made exactly at right angles to each other then the last adjustment necessarily makes the horizontal axis of the reversible socket truly horizontal

All the above adjustments should be repeated until the required degree of accuracy is attained

Slight errors in adjustment may in many cases be eliminated as explained later in the procedure for levelling

The "Y" Pattern Level

An illustration of this instrument is given in Plate X.
The following are the necessary adjustments:—

1. Parallax,
2. Collimation,
3. Telescope level,
4. Vertical axis of rotation.

The first and last adjustments are the same as those already described for the Cooke's pattern level.

Adjustment for Collimation.—Having set the telescope over a foot screw, turn the telescope by a rotary motion in the Y's until one pair of diaphragm screws is vertical and direct the intersection of the cross wires on some distant and well defined object. Now turn the telescope half round in the Y's and if the horizontal wire has moved vertically from the object bring it back half by the foot screw and half by the vertical diaphragm screws.

Adjustment of Telescope Level.—To make the telescope level parallel to the axis of revolution of the telescope in the Y's and therefore parallel to the line of collimation, bring the bubble of the telescope level to the centre of its run by means of two foot screws, reverse the telescope end for end in the Y's and if the bubble does not still remain in the centre, adjust by means of the screws giving vertical motion at the other end of the telescope level. Repeat until perfect.

The Dumpy Level

This is the usual pattern of non reversible level. An illustration is given in Plate XI.
The adjustments of the Dumpy level are for:—

1. Parallax,
2. Vertical axis of rotation,
3. Collimation.

The first two adjustments are the same as the first and last adjustments of the Cooke's reversible level already described. But, owing to the Dumpy level not being reversible it is impossible to correct for collimation in the same way.

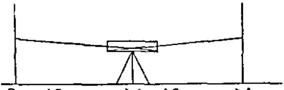


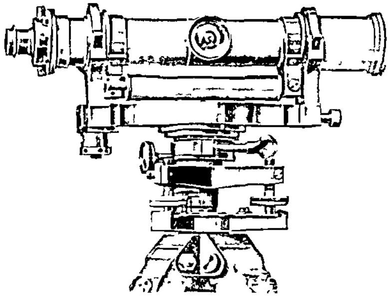
Fig. 37.

Therefore instead of this the line of collimation (i.e., the line joining the optical centre of the object glass and the intersection of the cross wires) is made at right angles to the vertical axis of rotation.

The method of carrying out this **adjustment for collimation** of a Dumpy level is as follows:—

Select a level piece of ground, set up the instrument in the centre, and carry out the first

PLATE X



Y LEVEL

is so adjusted that the staff is perpendicular in all directions when the pendulum point hangs in the centre of the ring

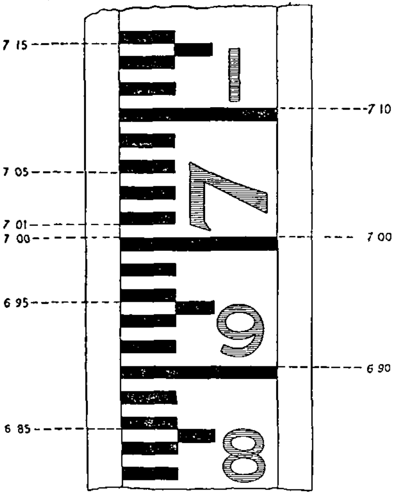


Fig 39

CURVATURE

By means of the level a line can be traced tangential to the surface of the earth at the point where the instrument is set up Owing to the sphericity of the earth it is evident

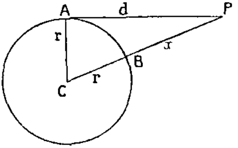


Fig 40

that the further this tangential line is produced the further it moves from the centre of the earth

In fig 40 representing a vertical section through the centre of the earth, let A be the

observer's position, P that of the distant object, situated in the plane tangential to the surface of the earth at A. Let C be the centre of the earth. Join CP, cutting the circumference at B. Call BP, x, and AP, d, and AC and BC, each representing the radius of the earth, r.

Then

$$(r + x)^2 = r^2 + d^2 \quad \text{or} \quad (2r + x)\,x = d^2$$

But the diameter of the earth $2r$ being so great in comparison with the quantity x at all distances to which a levelling operation usually extends, $2r$ may be taken for $2r + x$ without sensible error.

Then

$$2rx = d^2 \quad \text{and} \quad x = \frac{d^2}{2r}$$

The mean diameter of the earth may be taken as 7 916 miles, therefore if $d = 1$ mile, the excess x of the apparent over the true level = 1/[illegible] of a mile, or a little over 8 inches; at 2 miles it is four times that quantity, or 32 inches, and so on, increasing in proportion to the square of the distance. Hence the simple formula is deduced that the *correction for curvature in feet is two-thirds of the square of the distance in miles*. That is to say, if a man were swimming in a perfectly smooth sea with his eyes on a level with the surface of the water, he would be just able, as regards curvature, to see the top of an object floating in the sea a mile off if it protruded 8 inches above the surface.

Curvature and Refraction

The distance at which objects are visible is, however, augmented by the effects of refraction, which makes objects, as a rule, appear higher than they really are.

Taking the normal amount of refraction in the middle of the day, it would increase the distance at which an object of a certain height could be seen, taking curvature only into consideration, by about 1/13.

This gives the approximate formula for both curvature and refraction, that the height in feet of the distant object equals 4/7 of the square of the number of miles that it is distant from the observer, or $h = \frac{4}{7} d^2$.

The effect of refraction, however, varies so much with the place, season, and time of day, that it is necessary to adopt some method of observing that will eliminate it. This is attained by placing the instrument at an equal distance from any two points to which observations are taken. If this is done, the errors in the readings of both staves will be equal in amount, even if the telescope is not perfectly collimated, and consequently the differential reading will be correct.

This method also obviates the necessity of changing the focus of the telescope, and so avoids the possibility of introducing an instrumental error caused by the focussing tube not moving truly parallel to the visual axis of the instrument on a change of focus.

Procedure when Levelling

Suppose the spirit level to be set up and adjusted at A (fig. 41), directed on the levelling staff at 1, and the figure noted which is cut by the horizontal wire. This figure would be the difference of level between 1 and A, minus the height of the horizontal cross wire of the instrument above the ground at A (which might be measured and deducted). Suppose the staff was now held at 5 and the telescope directed on it, the difference between the readings at 1 and 5 would evidently be the difference of level between these two points, and is independent of the height of the instrument above the ground, which is common to both, assuming that

points 1 and 5 are equidistant from the instrument at A, they would be equally affected by curvature,* refraction, and instrumental errors, which might therefore be ignored

The levelling staff might also be held at intermediate points 2, 3, 4, &c , and the readings noted It would then only be necessary to measure the horizontal distance between these points in order to obtain the data for drawing a section of the ground between them, as shown in fig 41

It is evident that, unless the intermediate points 2, 3, and 4 are the same distance from the instrument at A as 1 and 5, the accuracy of the determination of their heights will be slightly affected by curvature, refraction, and instrumental errors

These errors will not be cumulative if entries of level readings are made as shown in the form of field book described on the next page, in which the levels of successive sets of readings depend only on the heights of the forward and back stations

If it were required to continue the levelling beyond 5, the spirit level would be carried forward and set up in such a position B and at such a height that it might intersect some part

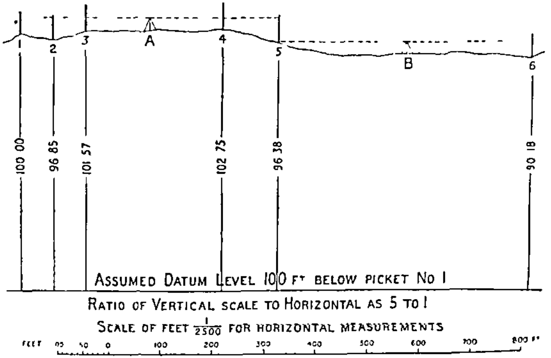


Fig 41

of the staff at 5, care being taken not to alter the position of the staff meanwhile, but merely to turn the required face towards the level

As regards the length of sights, it is advisable not to exceed 100 yards for accurate work with a 14 inch level For relatively unimportant work, the length is only limited by the power of the telescope used

In the operation represented in fig 41, where the levelling commenced at 1 and proceeded forward to the right, the reading of the staff at 1 would be called the "back reading" and the reading of the staff at 5 the "forward reading" The readings at 2, 3, and 4 are called "intermediate readings"

On setting up the level at B, the point 5 becomes the back station and point 6 the

* This is only strictly correct when the readings on the staff are taken at the same height above homogeneous ground, as refraction varies with the altitude of the ray and is also influenced by the nature and temperature of the soil

FORM J

Number of Picket	Horizontal Distance in Feet	*Back* Reading	*Intermediate* Reading	*Forward* Reading	Height of Instrument	*Reduced* Level	Remarks
1	0	{ 5 98 6 [illegible]			10[illegible] 98	100 00	Assumed
2	63		{ 9 13 9 43			96 8[illegible]	
3	1[illegible]		{ 4 41 4 71			101 [illegible]7	
4	3[illegible]		{ 3 23 3 [illegible]			102 7	
5	[illegible]04	{ 0 [illegible] 0 91		{ 9 [illegible]0 9 90	9[illegible] 9[illegible]	96 38	
6	1008			{ 6 81 7 11		90 18	
&c	[illegible]					&c	

Totals	13 78	33 42
		− 13 78
For double reading divide by		2) 19 [illegible]
Total amount of fall		9 82

Heights from above Column of Reduced Levels—

Picket 1	100 0[illegible]
Picket 6	90 18
Total fall	9 8[illegible]

forward station Similarly each forward station becomes a back station when the instrument is moved forward The stations are called "back" and "forward" with reference to their relation to each other on the line of levels, and not as regards the instrument which might be to the side and before or behind both

For ordinary levelling operations the form of field book on p 83 will be found generally convenient the entries refer to fig 41 —

In this case two readings have been taken at every picket one on each side of an Ordnance Survey pattern staff This insures greater accuracy, as the difference of two readings at any picket must be a constant if the readings are correctly taken and if the staff holder is careful not to knock the picket or press it into the ground while swaying the staff on it

For rough work it is sufficient to take single readings on a staff, but when it is graduated on both sides, great care must be taken to ensure that the same side is always made to face the instrument

In the above field book form the "reduced level" of picket No 1 is assumed as 100 feet and the heights of all the points along the line of levels are referred to a hypothetical datum plane passing 100 feet below picket No 1

It is convenient that the datum level should be lower than any part of the section so that all measurements may be taken above it

The horizontal distance of each picket from the starting point is entered in column 2 of the field book form, and is measured by chain or tape the deduction for the degree of slope being made by holding the chain horizontally when measuring up or down hill

The entries in columns 3, 4, and 5 are the readings of the levelling staff at the "back," "intermediate," and "forward" stations respectively

The "height of the instrument" is entered in column 6 and is obtained by adding the "back reading" to the "reduced level" of the back station or starting point thus the height of the instrument at A = 100.00 + 5.98 = 105.98 and at B = 96.38 + 0.61 = 96.99

The "reduced level" of an intermediate or of a forward station is entered in column 7, and is found by adding the "back reading" to the "reduced level" of the station on which the back reading is taken (shown in column 6 as "Height of instrument") and subtracting the intermediate or forward reading from this Thus the "reduced level" of picket No 4 would be 105.98 − 3.23 = 102.75 feet and of picket No 5, 105.98 − 9.60 = 96.38 feet above datum level

When it is only required to find the difference of level between two terminal points, it is only necessary to record the back and forward readings at the various stations, and the difference of their sums will give the required difference of level, if the sum of the back readings is greater than the sum of the forward readings, then the difference of level is a rise and a [illegible] If the levelling operation is carried back to the starting point, it is clear that the sum of the back readings will be equal to the sum of the forward readings, if the work is accurately performed

This method of finding the difference of height between two terminal points is usually called "check levelling" and is useful for checking a series of levels taken in greater detail or in closing a detailed series on to the starting point or on to the nearest known level such as a bench mark of the Ordnance Survey

In extensive levelling operations it is usual to leave permanent marks, called "bench marks" on objects such as milestones and walls, and to record their exact heights for future reference

In Ordnance Survey maps the levels of various bench marks are figured with their height in feet and decimals of feet above mean sea level at Liverpool which is the datum to which all altitudes in this survey are referred

The Ordnance Survey bench marks are of this pattern $\overline{\wedge}$, cut about half an inch deep into the vertical face of brick or stone structures the exact point of reference being the centre of the V shaped cut in the horizontal line

LEVELLING WITH THE THEODOLITE

Sections of ground may be taken with the theodolite more expeditiously, though less accurately, than with the regular level. Thus:—

Let the theodolite be set up as shown in the figure, and its telescope directed parallel to the general slope of the hill. Observe this vertical angle with all precautions for accuracy proper to the operation.

Keeping the telescope clamped in this position, note the successive readings of a levelling staff held by an assistant at the various points which will best define the irregularities of the

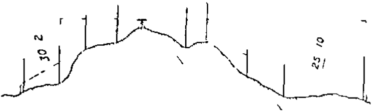


Fig. 12

section, the height of the theodolite and the horizontal distance between these points being measured, a section can be drawn thus:

Draw the horizontal line passing through the telescope. Protract the observed angle of inclination (as shown in the figure). On the horizontal line set off the measured distances to scale. Draw vertical lines from these points cutting the hypotenuse. Beneath each point thus found set off the staff reading, being the level of the ground where the foot of the staff stood for each observation.

Should the staff be so distant that the markings are illegible, a system of signals can be agreed upon, so that the assistant can move a visible mark along the face of the staff until signalled correct; the assistant can then note the reading.

CHAPTER IX

HEIGHTS

THE datum to which all heights should be referred is mean sea level For the determination of M S L see p 30

Heights may be determined by any one of the methods in the following list, the methods are given in order of their accuracy —

(i) By levelling
(ii) Trigonometrically (by vertical angles with a theodolite)
(iii) By clinometer (for short distances)
(iv) By barometer (mercurial or aneroid)
(v) By hypsometer (boiling point thermometer)

As regards accuracy (i) a line of good (not geodetic) levels should not produce an error of 0 1 foot per mile or 1 foot in 100 miles or 2 feet in 400 miles (The error may be assumed to increase as the square root of the distance)

(ii) In principal and secondary triangulation it is found that for any given ray of 30 miles or so an error of say 1 or 2 feet may be expected, hence we might expect an accumulated error of about 4 to 8 feet in 500 miles In topographical triangulation the accuracy of the heights will largely depend on the nature of the signals used In rapid triangulation under favourable conditions the error might be 10 feet in 200 miles

(iii) Clinometer heights are only used for short rays up to 2 miles and depend on neighbouring trigonometrical heights, each clinometer height will be correct within 2 or 3 feet

(iv) Barometer heights should be used in dependence on level or trigonometrical heights, judiciously used heights may be determined with an aneroid barometer within 10 or 20 feet "Absolute" determinations with a barometer will give large errors similar to those resulting from the use of the hypsometer

(v) Hypsometer heights depend on the actual air pressure at the moment, the hypsometer is necessarily a rough instrument, an error of $\frac{1}{10}$° Fahrenheit corresponds roughly to an error in height of 55 feet When every precaution has been taken, we may assume that a hypsometer height taken inland in a new country is liable to be at least 100 feet in error

For determinations of level heights see Chapter VIII
trigonometrical heights, see Chapter III
clinometer heights, see Chapter VI

Barometer Heights Barometer heights are frequently of great use in places where no trigonometrical station can be fixed such as in forests, valleys and deep ravines Barometer heights should always, where possible, be based on level or trigonometrical heights Thus suppose a trigonometrical station to be on the top of a 2,000 foot hill, suppose, as often occurs that the adjoining valleys are thickly wooded it is nevertheless very desirable to fix some heights in the valley This is best done by reading the barometer and thermometer on the top of the hill then reading them at various points in the valley, and then if possible reading them again on the top Aneroid barometers are most convenient for the purpose

Use of the Aneroid Barometer—Barometers should not be used near the limit of their range thus a barometer graduated to 5,000 feet should not be used much above 4,000 feet

Barometers should be tapped gently with the nail or a pencil before reading

Barometers take some little time to record the correct reading if a change of altitude has been rapidly made they should therefore be given a rest of a few minutes

They should always be read in the same position either when horizontal, or vertical with the needle on a level with the eye, which should be always in the same relative position to the needle to avoid the effect of parallax

The thermometer (in the shade) should be read at the same time as the barometer

If possible get a Kew certificate for each barometer in use or compare with a mercurial barometer to ascertain index error

For determining differences of height with an aneroid barometer see Tables V and VI

Use of Tables—

If β be the reading of barometer at lower station in inches,
β' " " upper " ,
t " Fahrenheit thermometer at lower station,
t' " " " upper "
A the number in Table V corresponding to t and t',
C " " VI " latitude,

make $$D = \log \beta - \log \beta',$$

then $$\log D + A + C = \log \text{ difference of altitude in feet}$$

Example—In lat [illegible] the aneroid barometer at the lower station read 29 81 inches at the upper station 25 55 The readings of the thermometers in the shade were 63° and 50 5 F

What was the difference of altitude?

$$
\begin{aligned}
\text{Log } 29\ 81 &= 1\ 47436 \\
\text{Log } 25\ 55 &= 1\ 40739 \\
D &= 0\ 06697 \\
\text{Log } D &= \bar{2}\ 82588 \\
A &\quad 4\ 80339 \\
C &= 1\ 99973 \\
\text{Log } h &= 3\ 62900 \\
h &= 4256 \text{ the required difference of height (in feet)}
\end{aligned}
$$

When time does not allow of this computation a good aneroid will give fair indications of differential heights taken within a short interval of time, if the scale of heights given in feet is used

Hypsometer Heights—The hypsometer, or boiling point apparatus, consists of a thermometer graduated from 180° to 215°, a spirit lamp boiler, and a telescopic tube which screws on to the top of the boiler, and in which the thermometer, supported by a rubber washer, is placed when in use The boiler should be about a quarter full, and the telescopic tube so adjusted that the bulb of the thermometer is just clear of the water When the water boils the steam ascends through the tube and causes the mercury to rise When the mercury becomes stationary, read the thermometer, and at the same time take the air temperature with an ordinary thermometer

To compute the difference in altitude between two stations by boiling point observations, take from Table VII the numbers opposite to the boiling points at the two stations The difference between these numbers is the difference in height when the mean temperature of intermediate air is 32 F When the temperature differs from 32°, the difference between the

numbers obtained from Table VII must be multiplied by the multiplier given in Table VIII for the given temperature

The third column of Table VII gives a ready means of comparing the readings of the aneroid with those of the hypsometer

When observations are only taken at the upper station and its absolute altitude has to be determined the mean barometric reading for the time of the year reduced to the sea level in the locality must be taken as the reading of the lower station The best approximations to the true values will be found in the isobaric charts given in 'Hints to Travellers

Whenever the observations at the two stations under comparison are not simultaneous, or when the mean barometer is taken in place of the lower station, the correction for the diurnal variation must not be omitted In the tropics there is a regular diurnal oscillation amounting to one tenth of an inch approximately, the maximum daily readings occuring at about 9 a m and 9 p m, and the minimum readings about 3 a m and 3 p m In other respects the barometer is very steady

Determinations of height by the hypsometer can only be looked upon as very rough approximations The determination is affected by atmospheric changes by errors in graduation of the thermometer, by errors in reading, by the purity of the water used, and by other factors

As an illustration of the discrepancies to be expected in the determination of the height of an inland station by this means, the following list of hypsometric values of the height of Lake Tanganyika is given —

Baumann	2 886 feet
Stanley	2,755 ,
Hore	2,749
Cameron	2 709 ,
Stairs	2,690 ,
Wissmann	2,670 ,,
Popelin	2,663 ,,
Reichard	2,558

the mean value being 2,713 and the range 328 feet It is not known what the true value is

CHAPTER X

CONTOURING AND HILL SKETCHING

A Contour is the representation on a map of an imaginary line on the ground joining all immediately adjacent points which are the same height above mean sea level For small areas a contour may be considered to be the outline of the intersection of the surface of the ground with a horizontal plane

Contours vary in accuracy from deliberately surveyed contours determined by instrumental levelling to sketch-contours drawn in by eye, such sketch contours are called **form lines**

In determining the accuracy necessary for the contouring of a map the principal factor is the scale of the map Suppose, for instance, that it is required to show the hill features in a plane table survey on a scale of ¼ inch to 1 mile, it would be waste of time to attempt any instrumental contouring Hill features on such a scale must be generalized, and it is sufficient to show them by form lines controlled by scattered heights on the hills and in the valleys On the other hand a 1 inch map such as the Ordnance Survey 1 inch map can show contours with great exactness and they may be fixed by instrumental levelling or by using the water level or may be approximate contours determined by clinometer

Methods of Contouring —The principal methods of contouring, using the term in its widest sense are —

(1) With theodolite or level
(2) With water level
(3) With clinometer
(4) With aneroid barometer
(5) By eye (sketch contouring)

(1) **A small Theodolite** (used as a level) and a contouring staff 8 feet long provided with a sliding vane are required and pickets as in the operation of levelling

The contourer begins by choosing a bench mark or known height as near the required altitude as possible and levelling from it up or down till he has established a point on the *contour and marked it by a picket driven in the ground at the correct height The instrument* is then set up nearly on the same level as the picket and at a distance of about 10 chains, beyond which a correction would be required for the curvature of the earth the staff is held on the picket and the vane raised or lowered till it is intersected by the cross hairs of the telescope, it is then clamped, and the staff with the vane fixed at this height is then moved along the contour and set up at every change in the form of the ground the assistant who holds it being directed into position by a signal from the contourer at the instrument a mark is made at each station, and when a point is reached at about 10 chains from the instrument (or nearly the same distance as the first picket) another picket is driven, and carefully adjusted to the correct height The instrument is now moved and set up again in a similar position to the second picket as it was to the first the vane of the staff is re adjusted on the new picket, and the process of marking out the contour continued

Where the vertical interval is small more than one contour may often be marked out without shifting the position of the instrument if the staff is long enough Too much should not, however, be attempted at one time

In marking out it often happens that "isolations, or small detached contours on the level of the main contour, are passed, and care should be taken that they do not escape

observation examination of the horizon in all directions with the telescope will point out ground where isolations will probably be found Should this be close to the main contour the isolation may be marked from one of the ordinary positions of the instrument, but when it is distant a point on it must be found by levelling it in the ordinary way

When a sufficient length of the contour is marked the contourer plots it on the map and fixes the points marked on the ground either by traverse or by chaining as in ordinary chain survey or on a plane table by intersection and resection, the latter method being more suitable for the smaller scales

(2) **The Water Level** is an instrument of great simplicity which never requires any adjustment As it has no telescope it is impossible to take long sights with it and it is of course, not susceptible of minute accuracy On the other hand, no gross errors can creep into work with this instrument It is easy to improvise a water level

Plate XII shows the pattern of water level in use on the Ordnance Survey The distance between the bottles is 1 foot the length of tripod when closed is 3 feet 1 inch The tripod has one leg which can be lengthened for setting up on a hill side A prismatic compass can be fitted to a projection between the bottles The bottles are partially filled with coloured water through a small opening at the top of the tube frame, a cap fits over the opening The bottles are connected above and below with hollow tubes Weight of level 4 lbs 2 ozs

To use the instrument it is best to look diagonally across the two bottles say from the right edge of the near bottle to the left edge of the far bottle, and get the bottles in such a position that the distant object comes between the two edges

The level is to be placed at the starting point of the contour which has been determined previously by levelling or by careful clinometer work The sight vane on a staff is to be adjusted to the height of the water in the bottles, and the staff man sent on with it some 400 or 500 feet or to any change in direction of the contour, the level man aligns him right or left until the vane is on a level with the top of the coloured water in the bottles A picket may now be driven in by the staff man and the position of the picket is determined either—

On a plane table by forward bearings and resection
By the prismatic compass and chaining or pacing

The level man now moves up to the picket and proceeds as before In contouring round a hill an excellent check is obtained by closing on the starting point

(3) **Deliberate Method of Contouring with the Indian Clinometer and Plane Table**—To carry out this method two men are required viz one plane tabler and one flagman and it is necessary that three fixed points should generally be in sight and that the plane tabler should always set up his table at the same height above the ground

A convenient way of doing this is for the plane tabler to always set up his table to the level of some point on his person say, for instance one of the buttons of his coat, but if this is not considered sufficiently accurate he can carry a plumb line of fixed length The flagman carries a small disc or board and notes the height of the table so as to hold up his disc always at that height

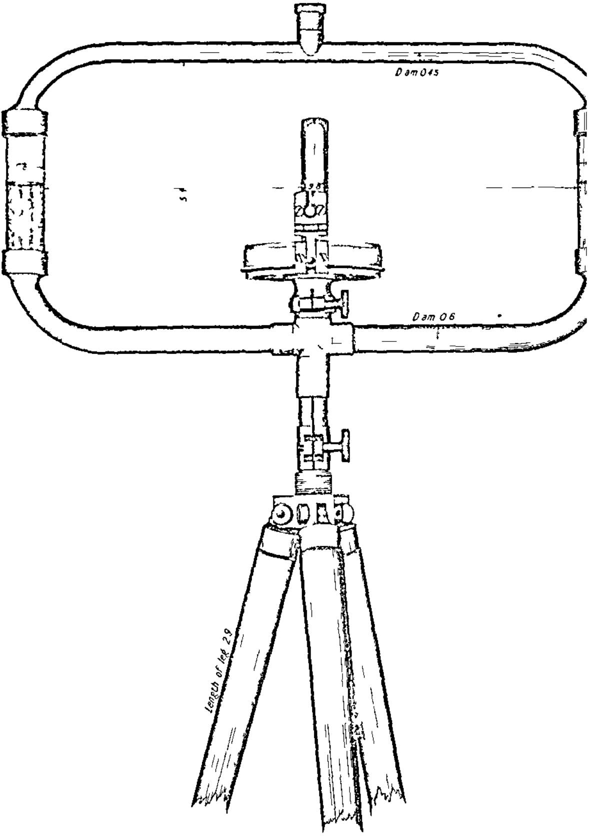
The plane tabler sets up his table at station 0 the starting point of the contour, and interpolates his position He then levels his clinometer and sends on the flagman approximately along the contour line and when the latter has gone sufficiently far ahead he aligns him by means of his clinometer until he is on the same level as himself

The plane tabler takes a ray on the flagman and moves up to him He sets up his plane table at that point and again interpolates his position, while the flagman goes on ahead as before and so the work proceeds This will be found a very convenient method on large scales

Rapid Method of Approximate Contouring with Indian Clinometer and Plane Table—This is described in Chapter VI

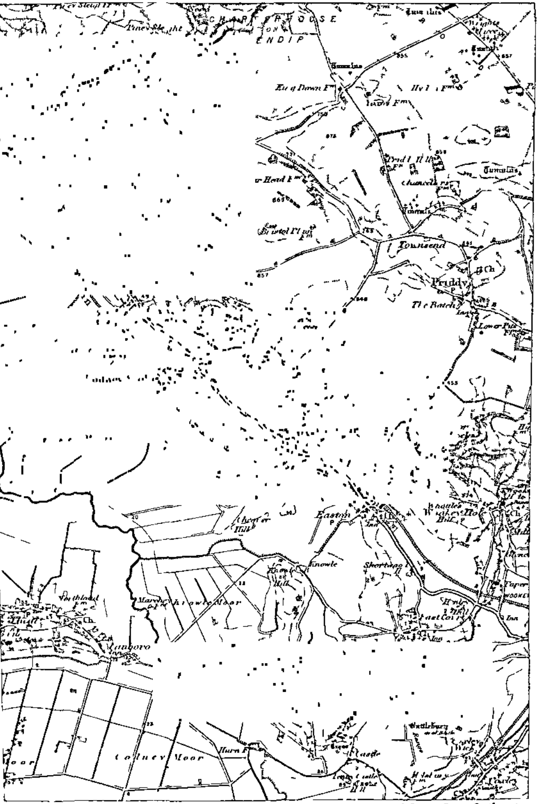
WATER-LEVEL

Ordnance Survey Pattern



ORDNANCE SURVEY

Scale - 1 Inch to 1 Mile



Ordnance Survey Southampton.

SHOWING THE COMBINATION OF CONTOURS AND VERTICAL HACHURES

(4) Contouring with Aneroid Barometer—Where the vertical interval of the contours is large say at least 25 feet, approximate contours can be put in by means of the aneroid barometer This is a particularly convenient method of approximate contouring for scales of ½ inch to 1 mile, and a vertical interval of 100 feet, or 1 inch and an interval of 50 feet In variable weather this method cannot be used

The barometer should be carried loose in the pocket

Before reading it should be lightly tapped, in reading it should be held with the face vertical, the eye should always be in the same position relative to the needle to avoid the effect of parallax

A 2½ inch barometer is a convenient size The scale of feet should be used The barometer should not be used near its extreme range, thus a barometer graduated to 5000 feet should not be used much above 4000 feet

In using the barometer to determine the approximate positions of contours, it is necessary to work between points of which the heights have been previously fixed Thus supposing the height of the top of a hill is known and also a height in the valley below, at the lower station set the instrument to the proper height, move up along any convenient spur, stopping where the barometer reading shows the required contour levels, fix these points on the plane table sketch in the neighbouring features, and close on the top of the hill If the reading differs from the known height of the top of the hill, a proportionate correction must be made to all the contours drawn in It is desirable that the distance between the fixed heights should not, as a rule, exceed a few miles and it is clearly necessary that the elapsed time between starting and closing should be as short as possible

Aneroid barometers do not at once take up the reading of a new height if a change of altitude has been rapidly made, a few minutes rest is necessary

(5) Sketch Contouring or Hill Sketching—As has been explained, on a small scale such as ¼ inch to 1 mile, it would be waste of time and money to determine contours instrumentally

The hill features are, therefore, sketched in by eye by means of **form lines** Form lines serve to show the shape of the ground rather than its exact height at any point, and on small scales it is obvious that the hill forms must be generalized In fact hill sketching on small scales is not a mechanical science with exact rules but partakes somewhat of the nature of an art, the hill sketcher must of course show the main features with accuracy, but it is impossible to show all the minor features

The summits of the hills, the ridges spurs, valley lines and streams will be accurately fixed, and the sketching will be controlled by a few trigonometrical (and in some cases barometrical) heights

In drawing the form lines it is desirable that the sketcher should observe a rule as to the number of lines to fit in for a given difference of level A very convenient rule is that the interval (in feet) between two adjacent form lines = 50 × number of miles to the inch Thus on the ¼ inch scale the vertical interval between successive form lines would be 200 feet Supposing for instance, that the sketcher was at a point which he knew or judged to be 1000 feet above the valley, he should draw five form lines below the one representing the crest of the hill

In hill sketching it is important to get on to the tops of hills ridges, and spurs The positions of these tops ridges and prominent points are fixed by plane tabling in the usual way, and the sketching of the hill features is carried on at the same time as the fixing of the detail The form lines need not be continuous Plates VI and VII are examples of hill sketching with form lines

CHAPTER XI

MAP PROJECTIONS AND GRATICULES

A map projection is the system on which the terrestrial meridians and parallels are represented on paper

A graticule is the net work of lines representing meridians and parallels on paper, the term is usually applied to the net work of meridians and parallels plotted for topographical work in the field

The term *projection* though sanctioned by long usage, is an unfortunate one It gives the beginner the idea that map projections are arrived at in the same way as the projections treated of in solid geometry Now the great majority of useful map projections are not obtained in any geometrical way A map projection is to be treated as the representation on a plane by any law of the terrestrial meridians and parallels in fact any definite system of drawing the meridians and parallels is a map projection

Before dealing with the general subject of map projections a subject which every topographer should have some acquaintance with we will describe the projections used in the field

It is obvious that since the surface of the earth is curved no portion can be represented on a plane with entire truthfulness It is evident also that the smaller the portion to be represented the more nearly will it be represented truthfully If for instance we represent



Fig 43 *

on paper a portion of the earth's surface extending 30 from north to south and 30 from east to west and draw meridians and parallels as straight lines, then the portion ABCD of the earth's surface is represented by the trapezium *abcd* in which *ab bc, ad cd* are the rectified lengths of AB BC AD CD

Now it can be shown that if AB BC CD AD do not exceed 30 then the surface ABCD can be represented by the rectilinear figure *abcd* on scales of ¼ inch to a mile and under without causing the topographer to make sensible errors If the parallels were drawn on paper as portions of circular arcs cutting the meridians at right angles, the result would be still more accurate But in the field it is inconvenient to plot circular arcs and the topographer must as a rule be content to plot his graticules with straight lines

A graticule should be drawn on *every small scale map* which as a rule, *should be bounded by meridians* and *parallels* The object of the graticule is to identify positions to enable new points to be plotted to enable maps to be compared with each other, and to facilitate joining maps of the same scale together

It is not very important what the intervals are The following would be suitable —

On a scale of ¼ inch to a mile the meridians and parallels should be drawn for every ½ degree
On a scale of ½ inch to a mile the meridians and parallels should be drawn for every ¼ degree
On a scale of 1 inch to a mile the meridians and parallels should be drawn for every ⅛ degree

* The convergence of the meridians is much exaggerated in this figure

Example of Plotting a Graticule —It is required to plot the graticule of a map on a scale of ¼ inch to 1 mile, between 10° and 11° N lat and 25° and 26° E long

From Table X., by simple proportion, take the values in *miles* of the 30′ lengths of meridians and parallels Divide these by 4 to get the lengths in inches

The diagonals of the squares are obtained as shown at the foot of Table X

Thus

$$BE = \sqrt{(8\ 59^2 + 8\ 51 \times 8\ 50)}$$
$$= 12\ 09 \text{ inches}$$

Now, having all the necessary data, take the plane table or map and draw the straight line AB (fig 44) for the central meridian

Measure AC CB the lengths of 30′ of latitude, then with centre B and radius BE

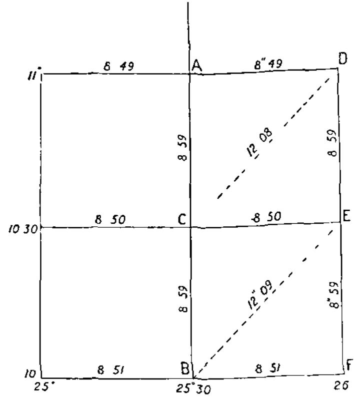


Fig 44

(diagonal) intersect a circle with centre C and radius CL (30′ of longitude) Similarly for the other trapeziums

The result will be a graticule in which the meridians converge, and the parallels are concave to the pole

Second Example of Plotting a Plane Table Graticule —Suppose it is required to plot a graticule on a scale of ¼ inch to 1 mile Limits 30° and 31° S 25° and 26° E

Proceed as follows (fig 45) —

Draw a straight line ABC down the middle of the paper to represent the central meridian 25° 30′ E

Look out in Table X the length of 30′ of latitude from 30° to 30° 30′ This is 34 440 miles, or on this scale 8 61 inches, mark this length CB From 30° 30′ to 31° is found in the same way to be 8 61 inches, mark this BA

Next look out the length of 30 longitude on parallels 30° and 30° 30, viz, 7 50 and 7 46 inches

Then the length of the diagonal of the northern quadrilaterals

$$= \sqrt{(8\ 61)^2 + 7\ 50 \times 7\ 46} = 11\ 41 \text{ inches}$$

In a similar way it is found that the diagonal of the southern quadrilateral is 11 37 inches

Plane Table Graticule

Dimensions in inches for a scale of 4 miles to 1 inch

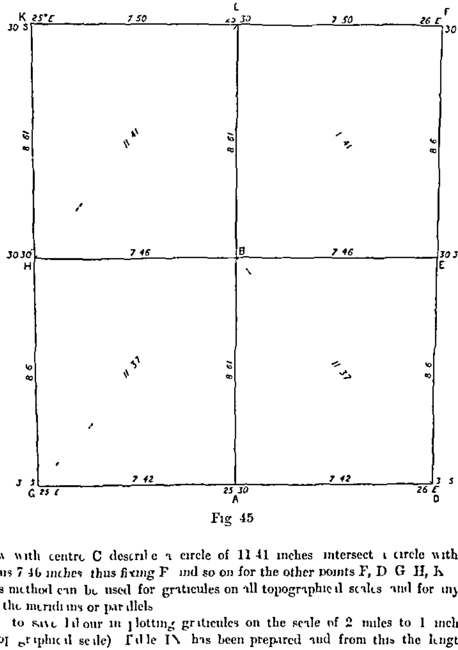


Fig 45

Now with centre C describe a circle of 11 41 inches intersect a circle with centre B and radius 7 46 inches thus fixing F and so on for the other points F, D G H, K

This method can be used for graticules on all topographical scales and for any intervals between the meridians or parallels

But to save labour in plotting graticules on the scale of 2 miles to 1 inch (a very useful topographical scale) Table IX has been prepared and from this the lengths of 15 spaces on meridians and parallels and the lengths of the diagonals can be taken directly This table is not to be used for other scales or other intervals

The field projection above described is to be considered as a rectilinear approximation to a conical projection It is all that a topographer absolutely requires in the field, but if it is

possible it is better (though not actually essential) to plot the parallels as portions of circular arcs. Labour saving tables for the purpose are 'The Construction and use of Marginal Lines for Maps' J. O'Farrell 1862, a War Office pamphlet 'On the Projection of Maps' by Major L. Darwin R.E., and 'Tables for a Polyconic Projection of Maps' published by the United States Coast and Geodetic Survey.

We now proceed to the general discussion of the subject of map projections. The remainder of this chapter need not be read by the beginner.

General Discussion of the Subject of Map Projections

The term map projection is sometimes misunderstood to mean a projection by means of straight lines from the earth's surface on to a plane, cylindrical or conical surface. It is actually a term of much wider meaning. Any representation on a plane by any law of the network of terrestrial meridians and parallels is a map projection, and the great majority of useful map projections are not arrived at in any obvious geometrical way.

It is necessary to bear in mind that, in general, a map projection is not a simple geometrical projection but the result of any definite system of drawing the earth's meridians and parallels.

There is a common idea that because there can be no perfect plane projection, all projections are equally bad or good, an idea encouraged by the customary geometrical way of looking at the question. Now the choice of a projection for a map is in general by no means a matter of indifference. Projections may indeed be considered as graphic representations of certain properties of the curved surface, and if we know the special purpose of the map we also know the special property or quality which it is most important to represent truthfully.

Useful theories of map projection are based on this fact, that although it is impossible to represent on a plane surface any portion of a spherical or spheroidal surface with entire truthfulness, yet we may select some quality or property of the curved surface and represent that accurately.

For instance, there is a whole group of projections in which areas are correctly represented, another group in which there is no local distortion, another group which preserves the azimuths from a central point, one projection which preserves distances and azimuths from a central point, one projection in which circles are still represented by circles, whilst in most projections we can represent at least one line correctly in addition to other properties.

There is also a class of projection in which no general property is accurately represented, but all properties are represented fairly well, a sort of golden mean.

Field Projections

All map making is divided into two parts—work in the field and work indoors. The former includes surveys, explorations and reconnaissances, the latter is a matter of drawing, reducing, or compiling.

Field Projections.

First. Large scale cadastral surveys, such as $\frac{1}{2500}$ maps of the Ordnance Survey, or the 16 inch to the mile maps of the Survey of India, are always based on a triangulation plotted by rectangular co-ordinates. Latitude and longitude are not usually marked; if they were, the resulting projection would be that known as Cassini's. Each sheet represents such a very small part of the earth's surface, and the boundaries are so near the central meridian, that the matter need not be gone into further.

Secondly. Rapid explorations usually result in maps of a rough and ready order, most interesting but not most accurate. The intrepid explorer draws a thin red line across a

continent, and there is little on each side of it If his march is east and west, he should use a simple conical projection, if north and south, a polyconical, and if in a diagonal direction, a so-called secant conical projection

Thirdly What projection should be used in the field by the topographer who works on scales varying from $\frac{1}{10000}$ to $\frac{1}{500000}$? As the difficulties evidently increase as the scale decreases, we can deal with the smaller scale

When the plane-tabler looks along the sight rule, any points which are bisected by the central vertical thread are considered to be on the same straight line That is, great circles are represented by straight lines Hence, considering this condition only, the plane-table is on the gnomonic projection

But the plane-tabler also expects that the scale shall be the same all over the map

If we only consider one point, and set up the plane table at this point, all rays from this point are to represent great circles, and the scales of all the rays are to be the same This gives the equidistant zenithal projection

If the plane-table is set up at other points, we get conflicting conditions, which, in theory, are best reconciled by Airy's projection, but in practice are approximately satisfied by some form of conical projection, provided that the area of the plane table does not exceed about 1 square degree

In the field it is inconvenient to plot such flat arcs as are required for the parallels, it is therefore customary to substitute for them rectilinear inscribed polygons, or, which comes to much the same thing, to plot, instead of a small portion of the curved parallel, a straight line of the same length

The resulting projection may be known as the field rectilinear projection, it should not be used where the extent of latitude is more than 2 degrees

There is a case on record in which a survey party commenced work on Mercator's projection, which is, of course, entirely unsuited for land surveys

List of Projections

We can now deal with the more general and interesting question of the projections used to represent much larger portions of the earth's surface, or even the whole of it

Projections have been termed —

a **Orthomorphic** (or *conform*, or *conformable*) —In these the scale is the same in all directions round a point in its immediate neighbourhood. It follows that very small areas are similar in outline to the originals on the earth's surface It also follows that the meridians and parallels intersect at right angles, though the fact that they intersect at right angles does not show that a projection is orthomorphic Mercator's is a projection which belongs to this group, so does the stereographic

b **Equal Area** (or *equivalent*, or *surface-true*) —In these, equal areas on the map represent equal areas in reality, i e, areas are correctly represented, though distances and angles are not Bonne's modified conical projection belongs to this group

c **Perspective,** a name which explains itself The eye may be imagined to be inside, outside, or on the surface of the earth The gnomonic projection is perspective

d **Zenithal.**—In these the azimuths round a central point to all other points are correctly represented and equal distances from this point represent equal distances in reality Perspective projections are zenithal though all zenithal projections are not perspective The equidistant projection belongs to this group

e **Conical,** which can be imagined as drawn on the surface of a cone (not necessarily *projected* on to the cone)

f **Cylindrical,** which can be imagined as drawn on the surface of a cylinder. Mercator's is cylindrical

7 **Conventional** —Those which are projected by arbitrary rules for convenience of drawing The globular projection is conventional, so are polyconics

These terms do not strictly represent classes of projections as they are not all mutually exclusive nor do they comprise all possible projections and to treat them as classes would be to fall into his error who divided the human race into Frenchmen red haired people, and cannibals

Thus we may have an orthomorphic cylindrical projection (Mercator's) or an equal area cylindrical projection (Lambert's) or an equal area zenithal projection (Lorgna's), and so on (But a projection cannot be both orthomorphic and equal area which would imply a perfect representation) The seven terms given are in fact just descriptive terms

The following is a list of the thirty principal projections, a description of each is given on pp 100 to 111

Orthomorphic —

1 Mercator's (cylindrical orthomorphic),
2 Transverse Mercator's (Lambert's first),
3 Conical orthomorphic (Lambert's second),
4 Circular orthomorphic (Lagrange's) Also the stereographic (17)

Equal Area —

5 Cylindrical equal area (Lambert's third),
6 Transverse equal area (Lambert's fourth)
7 Modified conical equal area (Bonne's)*,
8 *Sinusoidal equal area (Sanson Flamsteed)*,
9 Polar equal area (Werner's),
10 Simple conical equal area (Lambert's fifth),
11 Conical equal area (Albers'),
12 Zenithal equal-area (Lambert's sixth—called Lorgna's)
13 Rectilinear equal area (Collignon's),
14 Homalographic equal-area (Mollweide's)

Perspective —

15 Orthographic
16 Central or gnomonic,
17 *Stereographic*,
18 Minimum error (Clarke's),
19 James's,
20 La Hire's

Zenithal —

21 *Equidistant zenithal*
22 Projection by balance of errors (Airy's)
Also the zenithal equal area (12)
And perspective projections (15 20)

Conical —

23 *Simple conical*,
24 Conical with rectified meridians and two standard parallels sometimes wrongly called 'secant conical
25 Simple polyconical
26 Rectangular polyconical,
Also Nos 3, 7, 10, and 11

* Sometimes though wrongly called Flamsteed's Modified

Cylindrical —

27 Simple cylindrical,

Also Nos 1, 2, 5, 6 and 29

Conventional —

28 Globular

29 Projection by rectangular co ordinates (Cassini's),

30 Field rectilinear

Also Nos 25 and 26

This is not a complete list of all the projections which have been in use, there are a number of others (especially perspective and conventional projections) which have been proposed or used, but they are not now of any importance, and their interest is only historical

Those interested in the subject will find a very complete account, both technical and historical, in an admirable work, entitled "Traité des Projections des Cartes Géographiques," A Germain, Paris A Bertrand, Editeur, Librairie de la Société de Géographie

The Choice of Projections

The choice of a projection falls under one of the following cases —

(1) *For field work* This has been discussed

(2) *For a single sheet map*,

(3) *For a series, which is to be capable of being accurately fitted together*,

(4) *For a series without the last condition*

(2) *Single sheet maps* —

This second case includes any map, though it may be made up of several parts, which is meant to be used as a unit The Cape Colony $\frac{1}{800000}$ map is an example This case also includes all atlas maps All possible projections are available though all are not equally suitable for the special purpose in view For instance, statistical maps should be on equal area projections great circle charts on gnomonic projections and so on

A single-sheet map of Africa may be taken as an example and the following is a preliminary selection —

Equal area	Sanson Flamsteed or Zenithal Meridian (Lambert's)
Zenithal	Balance of Errors (Airy's) or Equidistant meridian
Conical	Simple polyconical or Rectangular polyconical
Orthomorphic	Mercator's or Stereographic meridian
Perspective	Minimum error (Clarke's) or Orthographic meridian

Of these Mercator's is out of the question for any land purpose

If the map is a political or statistical map, one of the equal area projections should be taken and of these the zenithal is the best, and Sanson Flamsteed the easiest to draw

If the map is for general purposes we may reject equal area and orthomorphic projections as being extreme cases and the polyconic projections as conventional, and finally choose from

Balance of errors (Airy's)	No 22
Minimum error perspective (Clarke's)	No 18
Zenithal equidistant	No 21

These are all good projections for the purpose

(3) *A One Projection Series.*—

The third case is that of a series of maps which are to be capable of being accurately fitted together, *i.e.* which are to be on one projection. This case, which is practically very convenient, can only be used for a comparatively small part of the earth's surface and on medium scales (*i.e.* smaller than topographical and not so small as atlas scales).

We may take as a typical country for this treatment Africa south of the Zambesi, and as a typical scale $\frac{1}{1,000,000}$. It is not absolutely necessary, but it is convenient, to bound the sheets by meridians and parallels. All this points to the use of a conical projection, and one of the best for the purpose is that form of conical projection in which the meridians are their true lengths and in which the errors of scale along the extreme parallels and parallel of maximum error are made equal. In the case of S. Africa the errors on the parallels mentioned are less than $\frac{1}{2}$ per cent., whilst the scale along the meridian is correct throughout.

(4) *A Topographical Series.*—

The fourth case is that of a series of maps of which each sheet is independent; the sheets are not meant to fit accurately together, and the accuracy of each sheet is the principal consideration, *i.e.* the case of ordinary topographical map.

Each sheet will be constructed on its own central meridian, and the projection (as for the field) will be a conical projection, but on account of the smallness of the area dealt with simple or rectangular polyconic give sensibly the same results. The present War Office system is the rectangular polyconic (p. 110), and does very well for topographical scales.

Note.—When the country is of small extent in either latitude or longitude, topographical maps may be projected on one projection as in Case 3 above. Ordnance Survey one inch maps of England, for example, are a series on Cassini's projection, and the extreme error due to the projection does not amount to $\frac{1}{800}$, *i.e.* less considerably than may be expected from the distortion of the paper.

$\frac{1}{1,000,000}$ *Map.*—

The question of the projection to be chosen for the universal $\frac{1}{1,000,000}$ map is interesting, and Africa may be taken as an example.

First, it is clear that a general projection for the whole of Africa is out of the question on such a scale.

Next, we *may* construct each sheet independently, and this may sometimes be the best system, but some difficulties would be found in fitting the sheets together, since each will represent some 24 square degrees.

Perhaps the best system is a compromise: special areas, such as Egypt, Nigeria, S. Africa, &c., should be treated as units, each on the special projection best suited for its particular case. This course would have the balance of local advantage, which is the chief thing to consider, and the projection can be so chosen that the resulting error is negligible. The only objection to this occurs when it is required to fit on sheets of an adjoining block. This can be obviated by arranging that the sheets shall well overlap the boundary.

Projections Used for Some Important Maps

Conical Orthomorphic (Lambert's second). Generally known as Gauss's. (No. 3.)

Used for a map of Russia published by the Geographical Society of St. Petersburg in 1862.

Modified Conical Equal area, known as Bonne's. (No. 7.)

Adopted in 1803 by the Depôt de la Guerre for the map of France.

Ordnance Survey 1 inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch of Scotland and Ireland.

Ordnance Survey 10 mile maps of Scotland and Ireland
Indian Atlas 4 miles to 1 inch
Cape Colony $\frac{1}{800\,000}$

Orthographic (No 15)
French map of Africa $\frac{1}{2\,000\,000}$, 1895, where its defects are very marked

Central or Gnomonic (No 16)
Some charts have been published on this projection by the U S Hydrographic Department

Zenithal Projection by Balance of Errors (Airy's) (No 22)
Ordnance Survey 10 mile map of England

Conical with rectified Meridians and two Standard Parallels (No 24)
Ordnance Survey $\frac{1}{1\,000\,000}$, 1904
Survey of India India and Adjacent Countries, $\frac{1}{1\,000\,000}$, 1904

Simple Polyconic (No 25) U S Coast Survey maps

Rectangular Polyconic (No 26) Intelligence Division, War Office maps

Projection by Rectangular Co-ordinates or Cassini's (No 29)
Ordnance Survey 1 inch, ½ inch, and ¼-inch of England
" " 6 " of The United Kingdom
" " $\frac{1}{500}$ " "

Brief Description of the Thirty Principal Projections

Orthomorphic Projections

1 **Mercator's**,* or cylindrical orthomorphic

In this projection the meridians and parallels are straight lines at right angles to one another

The meridians are spaced at their equatorial distances

The distance of any parallel from the equator is —

$$\frac{a}{M}\left\{\log\tan\left(45^\circ + \frac{l}{2}\right) - e^2 \sin l - \frac{e^4 \sin^3 l}{3} \qquad \right\},$$

where a and b are the semi axes,

$$e = \frac{\sqrt{a^2 - b^2}}{a},$$

M the modulus of common logarithms, l is the latitude

The scale at any point is the same in all directions, and small outlines are represented without distortion A straight line represents a loxodrome or line of constant true bearing, the projection is therefore universally used at sea, it is of no value for land surveys A loxodrome continuously pursued describes an endless spiral round the pole

2 **Transverse Mercator's**, or transverse cylindrical orthomorphic (Lambert's first) †

Mercator's projection may be considered as developed on a cylinder touching the earth along the equator, imagine this cylinder turned round so as to touch some chosen meridian The lines which formerly represented meridians now represent great circles at right angles to the chosen meridian, and the lines which represented parallels now represent small circles parallel to that meridian It is not necessary to consider this further

3 **Conical Orthomorphic Projection** (Lambert's second), generally called Gauss's

* Mercator, born A D 1512, died 1594
† Lambert German mathematician born 1728 died 1777

In this projection the parallels are represented by concentric circles and the meridians by their radii. Two selected parallels are of their proper lengths, but the lengths of the other parallels and of the meridians are determined by the following law, treating the earth as a sphere —

If z_1, z_2 are the co-latitudes of the selected parallels

$$\text{let } \lambda = \frac{\log \sin z_1 - \log \sin z_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}},$$

then the angle two meridians make with each other

$$= \text{true difference of longitude} \times \lambda,$$

then if r is the required radius of a given parallel (of co-latitude z), l a constant which defines the scale

$$r = l\left(\tan \frac{z}{2}\right)^{\lambda}$$

This is a remarkable and useful projection.* Small outlines are correctly represented, and the scale is the same in all directions round any point. The scale along two parallels is correct and is consequently correct in their neighbourhood in every direction. Between the selected parallels the scale is very slightly too small; outside them it is too large. It has been used for a map of Russia and is very suitable for large areas having a great extent in longitude.

4 **Circular Orthomorphic Projection** (Lagrange's)

In this both meridians and parallels are represented by circular arcs. More interesting to the mathematician than useful to the map maker.

The stereographic projection is orthomorphic. See No. 17.

EQUAL-AREA PROJECTIONS

5 **Cylindrical Equal area Projection** (Lambert's third)

If the earth is considered a sphere this is the simplest of all projections. Imagine a cylinder to circumscribe the earth touching it at the equator and from every point on the earth let a straight line be drawn at right angles to the earth's axis and produced backwards to intersect the cylinder. When the cylinder is unrolled the meridians and parallels are clearly straight lines at right angles to one another and the distance of any parallel from the equator is equal to a sin lat, when a is the radius.

It follows from the elementary geometry of sphere and cylinder that the projection is equal area.

This projection is only useful for maps which do not extend more than a few degrees from the equator.

6 **Transverse Cylindrical Equal area Projection** (Lambert's fourth)

If the cylinder touches the earth along a meridian and points are projected as above then, treating the earth as a sphere if x, y are the co-ordinates of a point l, a its latitude and longitude

$$x = \sin a \cos l,$$
$$\tan y = \tan l \sec a$$

The meridians and parallels can be drawn by giving a and l respectively constant value.

This projection can be used for countries having a great extent in latitude and but little in longitude. It is not, however, the best for the purpose.

* This last expression may be taken as giving the whole group of orthomorphic conical projections of which this is a particular case. Mercator's is another.

7 **Modified Conical Equal area Projection** (Bonne's), sometimes called the modified conical or Bonne's projection and known in France as "projection du dépot de la guerre This, with its two modifications (8 and 9), is an important projection

Imagine a cone having the same axis as the earth to touch it along any selected parallel, imagine the cone spread out and a radius of this parallel taken as the central meridian Along this central meridian on each side of the parallel, measure the true rectified distances to the other parallels Through these points describe circles having the same centre as the chosen parallel and along each parallel measure the true distances to the successive meridians

The projection is clearly equal area, since all of the small elements between any two parallels bounded by two infinitely close meridians are equal to each other, and to the corresponding elements on the earth's surface

The true spheroidal shape of the earth has been taken account of

If l_1 is the latitude of the selected parallel, and l of any other parallel, the radius of the latter is r cosec l_1 + the rectified arc of meridian between l and l_1 Where $r = \frac{180}{\pi} \times$ length of 1° of chosen parallel This can be found from geodetic tables

The projection has the following properties —

(1) It represents areas correctly
(2) Parallels are their true lengths
(3) The meridians cross the chosen parallel at right angles

The meridians are not simple curves, though so easily constructed

The projection has been much used in France, and the $\frac{1}{800\,000}$ map of Cape Colony published by the Surveyor General is also on this projection also the Ordnance Survey 1 inch maps of Scotland and Ireland

The less its extent in longitude the better It is frequently used in ordinary atlases for countries for which it is not suited—Asia, for example

8 **Sinusoidal Equal area Projection** (Sanson's) —Sometimes, though wrongly called Flamsteed's or Sanson Flamsteed

If Bonne's cone (No 7) touches the equator, it becomes a cylinder, and each parallel becomes a straight line at right angles to the central meridian cutting it at the true distance from the equator Along each parallel distances are measured from the central meridian equal to the portion of the arc of the parallel represented

Assuming the earth a sphere each meridian becomes a sine curve, hence the name of the projection But it is clearly equally easy to plot the true spheroidal lengths The general remarks on the last projection apply to this It is a good equal area projection for the whole of Africa

9 **Polar Equal area Projection** (Werner's)

If Bonne's cone touches the pole, the cone becomes a plane The parallels are then incomplete concentric circular arcs with the pole as centre, the central meridian is still a straight line, which is cut by the parallels at true distances The meridians as before pass through points on the parallels at true distances from the central meridian measured along the arc

This projection though interesting is of little use It is not possible to use it for Polar regions and is clearly inferior to the two previous projections for other parts of the earth's surface

10 **Simple Conical Equal area Projection** (Lambert's fifth)

In this projection the parallels are concentric circles of which the pole is the centre the meridians are radii The radius of each parallel is

$$2a \sec\frac{z_1}{2}\sin\frac{z}{2},$$

when a is the earth's radius, θ the co-latitude, and θ_1 the co-latitude of a selected parallel, which is then correctly represented. The angle any two meridians make with each other is $\alpha \cos^2 \frac{\theta_1}{2}$ when α is the difference of longitude.

The cylindrical equal-area projection is *not* a particular case of this because here we have the condition that the pole is the centre of the circular parallels.

The properties of this projection are:—

(1) It represents areas correctly;
(2) The meridians cut the parallels at right angles;
(3) The selected parallel is correctly represented;
(4) It is easy to draw.

The objection to it is that it misrepresents distances both along the parallels and along the meridian. It cannot be used near the poles nor for countries having a great extent in latitude.

If in the above we put $\theta_1 = 0$ the radius of each parallel is $2a \sin \frac{\theta}{2}$, the meridians are *then inclined at their true angles to each other.* (*See No. 12.*)

11. **Conical Equal-area Projection** (Albers).

In this projection any two selected parallels are correctly represented. The projection consists of concentric circles with radii for the meridians, but the pole is not now the centre.

If l_1, l are the chosen latitudes, r_1, r the radii on paper, put $k = \dfrac{1}{\sin \frac{l_1 + l}{2} \cos \frac{l_1 - l}{2}}$,

a being the earth's radius.

Then $r_1 = ka \cos l_1$

$r = ka \cos l$

the inclination of two meridians $= \dfrac{\text{angular difference of longitude}}{k}$ and if r is the radius of any other parallel of latitude l, $r^2 = 2a^2k (\sin l_1 - \sin l) + r_1^2$.

The pole is now represented by the arc of a circle. Two parallels are correctly represented, and the properties are otherwise the same as those of the preceding projection. Except for very large areas it does not differ materially from No. 24, where representation of areas is important it should be used in preference. It should not be used near the pole. It would make a good projection for Central Asia.

12. **Zenithal Equal-area Projection** (Lambert's sixth). Sometimes though wrongly called Lorgna's.

This is a particular case of 10. Meridians are represented by straight lines at their true angles to each other. The parallels are circles of which the meridians are radii. If θ is the co-latitude of any point and a the earth's radius, or better the radius of curvature at the pole, the radius of any parallel is

$$2a \sin \frac{\theta}{2}$$

As described it may be called the polar equal-area projection.

The same projection can however be applied to any part of the earth's surface if we *imagine great circles to radiate from some selected point in the same way as the meridians radiate from the pole.* The meridians and parallels can be drawn by a transformation of spherical co-ordinates. In the special case where the central point is on the equator we have (assuming the earth spherical)—

If θ is the angular distance of any point from the centre, l, α the latitude and longitude of this point,

$$\cos \theta = \cos \alpha \cos l$$

If A is the azimuth of this point at the centre, $\tan A = \sin x \cot l$, A is then truly drawn and the distance θ is represented by

$$2a \sin \frac{\theta}{2}$$

The properties of this projection (which is on the spherical assumption) are—

(1) It is zenithal, *i e* azimuths are correct at the central point and equal distances are represented by equal distances ,

(2) It is equal area

(3) There is but little deformation for 30° from equator or central meridian

It is a very good projection for a single sheet equal area map of Africa

13 **Rectilinear Equal area Projection** (Collignon's)

The meridians are straight lines which radiate from the pole and the parallels are parallel straight lines cutting the central meridian at right angles

Interesting but of little use

14 **Homalographic Equal area** (Mollweide's) Sometimes called Babinet's

The parallels are parallel straight lines and the meridians are ellipses, the central meridian being a straight line at right angles to the equator which is equally divided

The distance of any parallel from the equator is $\sqrt{2} \sin \delta$ where $\pi \sin \text{lat} = 2\,\delta + \sin 2\,\delta$

The whole sphere can be represented in an ellipse of which the major axis is twice the minor

Perspective Projections

Generally speaking these projections are of no value for land surveys

The principal consideration is the position of the point of sight The projection plane is always supposed to be at right angles to the line joining this and the earth's centre, the distance of this plane from the centre only affects the scale

15 **Orthographic Projection** —In this the point of sight is supposed to be infinitely distant, and the perspective rays consequently at right angles to the projection plane If the plane of projection is the equator, the result is the polar orthographic, if a meridian, a meridian orthographic and if any other plane, a horizontal orthographic

In the latter case which is the most general, it is clear that the central meridian is a straight line and all other meridians and parallels are ellipses, obviously only a hemisphere can be represented From the nature of the case it is a pictorial projection, and from its simplicity it is a good deal used in popular works It is not the best perspective projection for large areas and for practical purposes is of little use The laws of its construction are very obvious

It has been used for the French $\frac{1}{2\,000\,000}$ map of Africa The distortion on the extreme sheets, *e g* St Louis, is very marked It is a projection to be avoided

16 **Central or Gnomonic Projection** —In this projection the eye is imagined to be at the centre of the earth As before we may have polar, meridian, or horizontal, central projections In the latter case, which is the most general, the meridians are straight lines which meet at the pole, the parallels are conic sections The term gnomonic was applied in ancient days to this projection because the projection of the meridians is evidently a similar problem to the graduation of a sun dial If θ is the angle any meridian makes on paper with the central meridian,

λ the inclination of the projection plane to the earth's axis,

x the difference in longitude of the meridians,

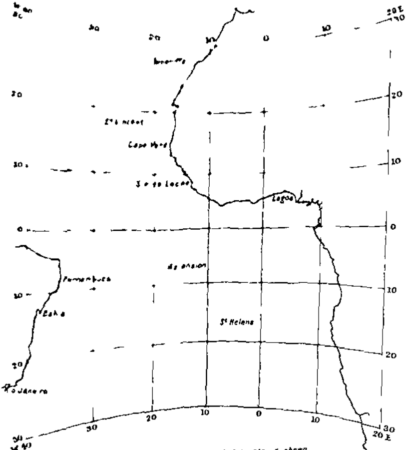
$$\tan \theta = \sin \lambda \tan x ,$$

Part of the

ATLANTIC OCEAN

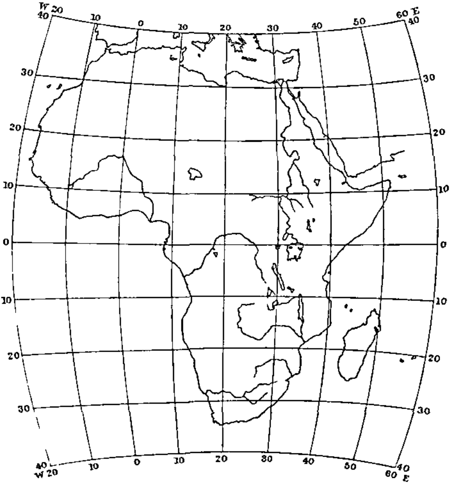
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MERIDIAN GNOMONIC PROJECTION



The shortest path between two points is shown on this projection by a straight line

AFRICA
on
CLARKE'S PERSPECTIVE PROJECTION



The distance of the point of sight from the centre of the earth is 1.625 × radius

and if l is the geocentric latitude of any other point, the distance from the equator along the meridian is

$$n \frac{\sec \alpha \sin l}{\sin (l + r)}, \text{ where } \tan r = \cot \lambda \cos \alpha, \text{ and } n \text{ is a constant defining the scale}$$

The equator is of course a straight line at right angles to the central meridian

The special property of the projection is that all great circles on the sphere (*i.e.* shortest lines) are represented by straight lines. There is no projection which accurately possesses this property for the spheroid but one which does so very nearly is that which results from the intersection of terrestrial normals with a plane. In this case the normal sections between two points, either on the same parallel or the same meridian are strictly represented by straight lines

The central projection is useful at sea for star charts and for the study of direct routes in great countries

An example is here given of part of the Atlantic Ocean on a meridian gnomonic projection (Plate XIX)

17 **Stereographic Projection**—In this the eye is imagined to be on the surface of the sphere which is supposed to be hollow and the surface represented is that of the opposite portion of the sphere. The projection is orthomorphic, and has the remarkable property (on the spherical assumption) that circles are represented by circles. It has of course, three aspects, polar meridian and horizontal. In the latter case which is the most general, the central meridian is a straight line the others are circles intersecting at the pole. The parallels are circles but are not concentric. The graphic construction is simple. As it is only used for general maps of the hemisphere on very small scales and not much for these, there is no need to consider the compression of the earth

18 **Minimum Error Perspective Projection** (Clarke's)*—In this projection (due to Colonel Clarke) the point of sight is outside the sphere and the distant surface is that to be represented. The position of the point of sight is chosen so that at each point the sum of the squares of the errors of scale in two directions at right angles to each other shall be a minimum over the whole surface. In other words the total misrepresentation is a minimum. The surface to be represented is that cut off by a plane parallel to the projection plane, and the boundary is therefore a circle

If u is the spherical arc between any point and the adopted centre, h the distance of the point of sight from the centre, k a constant defining the scale, then the distance at this point from the centre (on the map) is

$$\frac{k \sin u}{h + \cos u}$$

The following table shows the value of h for several values of β (the spherical radius of the map) —

β	h
40°	1 625
54	1 61
90	1 47
108	1 40
113½	1 367

19 **Sir Henry James's Projection**—This is a particular case of Clarke's projection in which the spherical radius of the area to be represented is 113½°, in this case $h = 1\ 367$

* Colonel A R Clarke, C B, R E, F R S

(*As originally described by Sir H James, h was taken as 1 5, which does not give the minimum error*) The total area represented is about two-thirds of the earth's surface

20 **La Hire's Projection** —A perspective projection in which the point of view is outside the sphere at a distance from the centre equal to $\left(1 + \frac{1}{\sqrt{2}}\right) \times$ radius The middle point of the radius of the map of a hemisphere, therefore, represents the middle point of the opposite quadrant It is used for maps of a hemisphere, but has no great merit It is sometimes confused with the globular projection, which is a conventional one (No 28)

ZENITHAL PROJECTIONS

21 **Equidistant Zenithal Projection** —In this projection, any point on the sphere being taken as the centre of the map, great circles through this point are represented by straight lines of the true length to scale, and intersect each other at the true angles There may thus be polar, meridian and horizontal equidistant zenithal projections

The position of the intersection of any meridian with any parallel is given (on the spherical assumption) by the solution of a simple spherical triangle

Thus if z_1 is the co latitude of the centre of the map (longitude 0),

z ,, ,, any other point

α , *longitude of* ,, ,,

A ,, required azimuth

c ,, ,, spherical distance

Let $\tan \theta = \tan z_1 \cos \alpha$, then $\cos c = \cos z_1 \sec \theta \cos (z - \theta)$ and $\sin A = \sin z \sin \alpha \operatorname{cosec} c$

The projection is neither orthomorphic nor equal area, the scale in the direction of the centre is always correct and at right angles is less than 5 per cent in error at 30° from the centre It would make a good projection for a single sheet map of Africa, and is especially useful for polar maps *

22 **Zenithal Projection by Balance of Errors** (*Airy's*) —*At any point of a zenithal* projection we may consider two scales one along the radiating straight line which represents a great circle and one at right angles to this

If these two scales are equal at each point, though they differ from point to point, the result is an orthomorphic projection—the stereographic (No 17)

*If the product of the two scales is constant, the result is the zenithal equal area pro*jection—Lambert's sixth (No 12)

But we may deal with them in a different way, and make the sum of the squares of the errors of the two scales at each point a minimum over the whole surface to be represented The result is Airy's projection, which we are now dealing with, in this the total misrepresentation is a minimum

If β is the spherical arc from proposed centre to proposed circumference, u the spherical arc of any point from the centre, then the distance of this point from the centre on the map is

$$2 \cot \frac{u}{2} \log_e \sec \frac{u}{2} + C \tan \frac{u}{2},$$

where

$$C = 2 \cot^2 \frac{\beta}{2} \log_e \sec \frac{\beta}{2}$$

All this is on the assumption that the earth is spherical

* The earliest known instance of the use of the Polar Equidistant Zenithal Projection occurs in a treatise by Glareanus written about A D 1510

This projection is, so to speak, the golden mean between an orthomorphic and an equal area projection. It has been used for the Ordnance Survey 10 mile map of England.

The zenithal equal-area projection, Lambert's (No. 12), and perspective projections (15 to 19) have been discussed.

Conical Projections

These are really developments on a conical surface, not projections on to the surface.

23. **Simple Conical Projection.**—Imagine a cone to touch the earth along any chosen parallel, the cone and the earth having the same axis. Unroll the cone; then the radius of the chosen parallel is $\nu \cot lat$, where ν is the normal terminated by the minor axis. Along this parallel measure the true lengths of the degrees of longitude (for the parallel in question), and join the apex of the cone and these points by straight lines—these are the meridians. The other parallels are circular arcs, concentric with the chosen parallel, and distant from it the true lengths of the rectified arc of the meridian.

This projection is unsuited for a country having a great extent in latitude, but it may stretch any extent in longitude. When the chosen parallel is the equator, the projection becomes the *equidistant cylindrical projection* (27), a projection of no value. When the chosen parallel is the pole, the projection becomes the *polar equidistant zenithal projection*, a very valuable projection already described (No. 21).

Note.—It is possible, though it has never been done, to construct transverse conical projections in which the cone touches some parallel to a meridian. In the same way it is possible to have transverse secant conical projections.

Again, these cones may be diagonal, the pole of the small circles being neither at the earth's pole nor on the equator.

24. **Conical Projection, with 2 Standard Parallels.*** This is one of the most useful of all projections. (It is sometimes wrongly called "secant conical.")

Two selected parallels are represented by concentric circular arcs of their true lengths; the meridians are then radii. The degrees along the meridians are represented by their proper lengths, and the other parallels are circular arcs through points so determined, concentric with the chosen parallels.

This construction evidently fixes the position of the centre of the parallels, for if $p_1 p$ are the true lengths of the chosen parallels, m the length of the intervening meridian (all of which can be taken from geodetic tables), then $h = \frac{p - p_1}{m} \times \frac{180}{\pi}$, where h = ratio between the inclination of two meridians on paper, and their true difference of longitude.

The parallels may be chosen arbitrarily from a consideration of the country to be mapped, but it is better to adopt one of the following methods:—

24*a*. The absolute errors for a given difference of longitude along the centre and extreme parallels may be made equal. This is Euler's projection. For the purpose of choosing the projection it will be sufficient to treat the earth as a sphere, and in the construction to plot the true spheroidal lengths of the meridian arcs.

Or (24*b*) the errors of *scale* on these parallels may be made the same as follows:—if c, c^1 are the colatitudes of the extreme parallels, the distance in degrees of the centre of the parallels from the pole, h the ratio between the inclination of two meridians and their difference in longitude:

$$\frac{h(\ \ +\ \ )}{\sin c} \quad 1 - \frac{h(\ \ + c^1)}{\sin c^1} - 1 \quad 1 - \frac{h\left(\ \ + \frac{c+c^1}{2}\right)}{\sin \frac{c+c^1}{2}}$$

from which h and   are easily found.

* The earliest known instances of the use of one of the cases of this projection occurs in maps of the two hemispheres by Ruysch dated A.D. 1508.

Applying this to South Africa south of the Zambesi, taking the extreme parallels as 18° and 34° S, $h = 439$, $= 52° 7$, and the error of scale on the extreme and mean parallels is 0 0049 i e less than ½ per cent or less than may be expected from ordinary contraction of paper

Or (24c) as suggested by Colonel Clarke, the mean length of all the parallels may be made correct still keeping the meridians their true length

This is equivalent to equating the total areas of the zones, and must be combined with another condition for example, that the errors of scale on the extreme parallels shall be equal (This is not the same as Albers' projection No 11 in which the meridians are not kept their true length) We then have

$$z = \frac{c^1 \sin c - c \sin c^1}{\sin c^1 - \sin c}, \quad h = \frac{\cos c - \cos c^1}{(c^1 - c)\left(- + \frac{(c + c^1)}{2}\right)}$$

in the case of South Africa, as above,

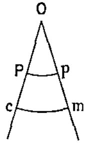
$$h = 440, \quad z = 52° 7,$$

i e practically the same result as 24b

It is easy to imagine other variations, for instance, we may consider the maximum error instead of that of the mean parallel (24d) This projection was used for the $\frac{1}{1\,000\,000}$ Ordnance Map of the British Isles There is no difficulty in working out the projection for the spheroid

As an example of the choice of a conical projection we may take the case of South Africa, south of the Zambesi the limits being 15° S lat to 35° S lat

To save labour, in the preliminary calculations the earth will be treated as a sphere



Let c be the co latitude of any parallel which is represented by cm, let P correspond to the pole let OP = and the angle com hu, where u is the true difference of longitude of the meridians oc om, Pc pm are their true lengths

Then the error of scale along any parallel is $1 - \frac{h(z + c)}{\sin c}$ This is a maximum when $\tan c - c$

Now, let it be a condition that the scale errors on the extreme parallels shall be equal, then the largest error will be a minimum when these two errors are equated to that on the parallel of maximum error

Then

$$\frac{h\left(+ 75 \times \frac{\pi}{180}\right)}{\sin 75} - 1 - \frac{h\left(+ 55 \times \frac{\pi}{180}\right)}{\sin 55} - 1 \quad 1 - \frac{h(z + c)}{\sin c},$$

where

$$\tan c - c = z$$

From these equations we get

$$z = 56° 62$$
$$c - 64\ 66$$
$$h - \quad 424,$$

and the maximum error of scale is less than 0 7 per cent It occurs on the parallels of 15°, 25 20 and 35 S *The errorless parallels, to the nearest degree, are those of 18° and 32 S*

To plot the projection of which the elements have been thus determined we shall use the true spheroidal lengths, making the parallels of 18° and 32° S errorless, this will slightly alter the value of h as follows —

The true length of the meridian between 18° and 32 S is 963 70 miles

The true length of 1° on the parallel of 18° S is 65 808 miles

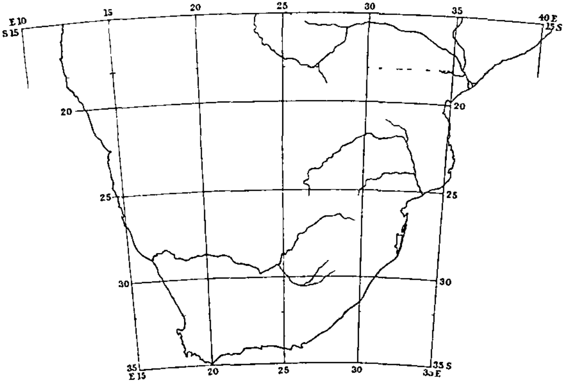
" " " " , 32° S is 58 720 "

whence $h = 421$, which does not differ materially from that found above

SOUTH AFRICA

on

A CONICAL PROJECTION WITH RECTIFIED MERIDIANS AND TWO STANDARD PARALLELS



Scale 400 miles to 1 inch

The meridians and two parallels (18° and 32° S) are true to scale

The radius for parallel 32° is now easily found to be 7983.7 miles.

The radius for parallel 35° is thus—the length of the meridian between 32° and 35°, 7983.7 − 206.7 = 7777 miles.

The length on the map of 1° of longitude on 35° S. is now

$$7777 \times .421 \times \frac{\pi}{180} = 57.143$$

The true spheroidal length is 56.728, an error of about 0.7 per cent.

The radii of the other parallels are obtained in the same way.

Next to calculate the coordinates for plotting. Take the parallel of 35° S.

The radius of this parallel is 7777.0 miles.

Take the meridian of 25° E. as the axis of y and the intersection of this meridian with the parallel of 35° S. as the origin.

To determine the coordinates of the point 35° S. 30° E.; difference of longitude = 5°.

Angle between meridians on map θ = .421 × 5° = 2° 6′ 18″.

Then

$$x = 7777 \times \sin 2° 6' 18'' = 285.63 \text{ miles}$$
$$y = 7777 (1 - \cos 2° 6' 18'') = 5.25 \text{ miles}$$

and so on.

The following table gives the results.

Conical projection for S. Africa.
Coordinates in miles.
Origin point 35° S. 25° E.

Latitude S.		Longitude E. 25°	30°	35°	
35°	x	0	285.7	570.1	The parallels of 18° and 32° S. [illegible]
	y	0	−5.3	−21.0	[illegible] miles
30°	x	0	298.3	596.2	
	y	344.5	339.0	324.6	
25°	x	0	310.9	621.5	See Plate XVI.
	y	688.7	683.0	665.9	
20°	x	0	323.6	646.7	The projection is symmetrical and the coordinates are to be laid off for points W. of 25° E.
	y	1032.7	1026.8	1008.4	
15°	x	0	336.2	672.0	
	y	1376.5	1370.3	1351.8	

25. Simple Polyconical Projection.—In this projection, which is a conventional one, any parallel is represented by a circular arc of radius $\nu \cot lat$, where ν is the normal terminated by the minor axis. The central meridian is a straight line through the centres of the parallels, which are then true distances apart measured along this meridian. The other meridians are curves which cut the parallels at the true distances, measuring along the parallels. Thus the meridians do not cut the parallels at right angles.

The projection is best suited for countries having a great extent in latitude and little in longitude. The meridian curves are sometimes for convenience replaced by the rectilinear inscribed polygons.

The great usefulness of this projection and of the Rectangular Polyconical Projection consists in the fact that it is easy to construct universal tables since each particular parallel has always the same radius (ν cot lat)

Single sheets not exceeding in latitude four degrees when plotted on either system do not materially differ in any of the principal qualities from sheets plotted on specially devised conical projections such as 3, 11, or 24 (*b*) (*c*) or (*d*). But this only applies to small areas and independently plotted sheets. The best tables to use are those published by the United States Coast and Geodetic Survey—"Tables for a Polyconic Projection of Maps" 1900

Polyconical projections should not be used for large areas and are especially unsuited for maps having a great extent in longitude

26 **Rectangular polyconical projection** sometimes called the **rectangular tangential**. The parallels are formed as in the last projection but the meridians in this case are curves which cut the parallels at right angles. The lengths along the parallels are consequently not true

This projection is described in the War Office pamphlet "On the Projection of Maps" by Major L Darwin R E 1890. A geometrical construction (due to the late Mr O Farrell of the Ordnance Survey) determines the intersection of the meridians and parallels. (See War Office pamphlet which contains the necessary tables or Encyclopaedia Britannica.)

For small areas it does not differ appreciably from the simple polyconical. It is best suited for countries having a great extent in latitude and not much in longitude. It will also do well for a topographical series each sheet being plotted independently

The conical orthomorphic, Lambert's (No 3)
The modified conical equal area Bonne's (No 7)
The simple conical equal area Lambert's (No 10)
The conical equal area Albers (No 11) have been described

Cylindrical Projections

27 **Simple Cylindrical Projection**.—The parallels and meridians are straight lines at right angles to each other. The lengths along the meridians are correct and those along some selected parallel. A projection of no value

Mercator's (1),
Transverse Mercator's (2)
Cylindrical equal area Lambert's (5)
Transverse ditto (6)
have been described

Conventional Projections

It is rather difficult to draw the line between regular and conventional projections, but the distinguishing mark of the latter is that they are devised for simplicity of drawing and not for special geometrical or analytical properties. The globular projection is certainly conventional and it is probably fair to call polyconical projections conventional

Ancient conventional projections are often more arbitrary than these, though sometimes of historical interest they are of no practical importance. An account of them will be found in German

28 **Globular Projection**.—A conventional projection of a hemisphere in which the equator and the central meridian are two straight lines at right angles their intersection is the centre of the circular boundary. The meridians cut the equator at equal distances and are

arcs of circles passing through these points and the poles The parallels are arcs of circles which cut the central and extreme meridians at equal distance

This name is sometimes incorrectly given to La Hire's perspective projection (20) and also equally incorrectly to the polar equidistant zenithal projection (21)

It differs however, but little from La Hire's It is used in popular atlases but has no claim to be treated seriously

29 **Projection by Rectangular Co ordinates**—Sometimes called Cassini's or Cassini Soldner's Any point on the earth is fixed if we know the length of a perpendicular from it on to some fixed meridian and the distance of the foot of this perpendicular from some fixed point on the meridian On this projection these spheroidal co ordinates are plotted as ordinary plane rectangular co-ordinates

The perpendicular spoken of is really a plane section of the surface through the given point at right angles to the chosen meridian and may be briefly called a great circle Such a great circle clearly diverges from the parallel The divergence of latitude in seconds is approximately

$\frac{x}{2\rho}$ tan λ cosec 1 where x is its length ρ that of the radius of curvature to the meridian ... the normal terminated by the minor axis λ the latitude of the foot of the perpendicular

The difference of longitude from the meridian in seconds is sec λ cosec 1

The resulting error consists principally of an exaggeration of scale north and south, and is approximately equal to sec ... it is practically independent of the extent in latitude

This projection should not be used when the map extends more than say, 200 miles from the central meridian It is on this projection that Ordnance Survey 1 inch maps of England are drawn and also the 6 inch Ordnance Survey maps of the United Kingdom and for these comparatively small areas the projection is a very suitable one

30 **Graticules and Field Rectilinear Projections**—*These have been described* (see p 93)

CHAPTER XII

THE REPRODUCTION OF MAPS IN THE FIELD

It is important that the Intelligence Division of an army in the field should possess the means of reproducing maps and sketches. The most suitable methods where large numbers of copies are required are lithography and zincography. In the former, the printing surface is, as its name implies, a stone, in the latter zinc.

Every officer employed with a survey and mapping section should possess some knowledge of these methods, of the equipment and personnel which would be available and of the time which would be required for the work, though it may not be essential for him to be an expert lithographer.

In the South African War of 1899-1902 the Survey and Mapping Sections supplied themselves with stones and lithographic presses which worked admirably at Orange River, Bloemfontein and Pretoria.

The experience gained in this war has resulted in a new equipment. In the heavier equipment stone is carried in preference to zinc.

The former is, of course, much heavier than zinc, and is also fragile, but this disadvantage is compensated for by the greater ease with which it is worked, especially under service conditions.

It is considered that any photographic process, except sun printing, is unsuitable for use *in the field*.

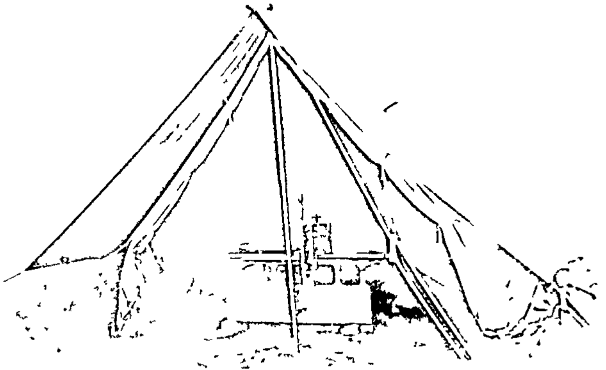
The recently approved Field Lithographic Equipment consists of a portable press, and the necessary stores as well as a tent. Four double-faced stones are carried, and the whole equipment, with the exception of the tent, is carried in six boxes, details of which are given in the following table:—

DETAILS OF FIELD LITHO EQUIPMENT (No. 1)

Number of Box	Contents	Measurements	Weight
			lb. ozs.
1	[illegible] press	4' [illegible] × [illegible] × [illegible]	101 8
[illegible]	[illegible] stones	[illegible] × 1' [illegible] × 10	[illegible] 0
[illegible]	Ditto	[illegible]	[illegible] 0
[illegible]	[illegible]	[illegible] × 1' 8 × 1' [illegible]	197 14
	[illegible] stores	[illegible] × 1' 10 × 1' [illegible]	11 [illegible]
[illegible]	Ditto	[illegible] × [illegible] × 1' [illegible]	[illegible] 14
Total weight of equipment			1,[illegible] [illegible]

This equipment, which can be reduced in emergencies, is intended to be carried in any ordinary cart or wagon, and will reproduce maps up to 18" × 23".

A light field litho press weighing about 190 lbs., capable of reproducing maps up to double foolscap size, i.e., 13" × 16", has also been designed and approved.



Field Lithographic Press (Fd. Cable Transport)

Zinc plates will be used with this press which is intended for mule transport

The personnel required for the lithographic equipment is two lithographic draughtsmen and two lithographic printers

The draughtsmen are specially trained in the art of drawing reversed on stone or zinc but have nothing to do with the actual printing of the plans which is done by the printer

Rates of Work.—A skilful draughtsman should be able to trace in transfer ink, a map of enclosed country with a fair amount of lettering and detail, at a scale of 1 inch to 1 mile at the rate of 10 minutes per square inch If required to draw direct on to the stone or plate, double this length of time would be necessary

A printer should be able to print about 40 copies of the same map in an hour under ordinary circumstances

Lithography and zincography are methods of reproducing maps or plans in greasy ink, in large numbers by mechanical means from stone or zinc Both methods depend on the fact that stone and zinc are to a certain extent porous and capable of absorbing both water and grease Where a grease spot exists water is repelled and *vice versa*

If a drawing in greasy ink, or transfer as it is called, be laid face down on a clean stone and subjected to pressure the grease leaves the paper, and is absorbed by or takes on to the stone or zinc If the stone or zinc plate be then damped and a roller charged with a greasy ink be passed to and fro over its surface the ink will only adhere on the lines of the drawing Should a piece of clean paper be placed on the stone or plate and passed through the press it will lift some of the ink and a reproduction of the original picture will be obtained By damping and rolling up alternately a very large number of copies can be produced

Preparation of Transfers

The first step in the reproduction of a map unless it is to be drawn direct on to the stone or plate is the preparation of a transfer that is to say a copy of the plan or map in greasy ink

The ink used for this purpose is known as litho writing ink and is applied with ordinary pen ruling pen or brush It is obtained in sticks and prepared for use by warming the bottom of a saucer rubbing the ink on a little water and stirring up Its chief constituent is some form of soap *i e* a substance of a greasy nature which has been acted upon by an alkali and which can be subsequently rendered greasy by the application of an acid

The transfer would usually be traced on a specially prepared paper known as **tracing transfer paper**

Where an alteration in scale from that of the original is required the enlargement or reduction would first be drawn on ordinary paper in pencil and then traced in litho writing ink on tracing transfer paper

Drawings in litho writing ink should not be blotted or rubbed

Preparation of Stone

The Stone used in lithography is a kind of lime stone and comes from Bavaria

It is ground with a levigator and silver sand passed through a 40 mesh sieve This gives a level surface, and removes old work

Polish with pumice stone and water

Polish with snake stone and water

Wash with cold water

Transferring Litho Writing Ink Transfer Made on Ordinary Paper to Stone

The stone is placed in front of a fire to warm

The transfer, which should be freshly prepared, is placed face downwards on a clean piece of paper and damped from the back with a sponge wetted with dilute nitric acid (1 in 80) about 4 times at intervals of 2 or 3 minutes The object of the acid is to convert the ink from the soluble to the insoluble form it also renders the paper more pliable The acid must not be strong, or it will rot the paper If the ink were soluble it would wash off the stone The impression would probably roll up but it would not be so strong, and the lines possibly rotten, *i e*, broken

The warmed stone is then placed in the press and the transfer face down on it Grease takes more readily on a warm than on a cold stone

The transfer is pulled through the press half a dozen times with light pressure, transfer damped scraper or stone reversed in case pressure is not equally distributed over the surface of the stone and pulled through another six times

These operations are repeated till transfer is damp throughout Transfer is removed composition washed off with cold water and the stone gummed in

Light pressure has to be used as the composition on the paper is soft and might smash

The transfer is then carefully removed from the stone, which is then gummed in with a gum solution of the consistency of thick treacle The stone is then allowed to dry and cool The gum fills up the pores of the stone where clear of ink, when once dried cannot be removed with water It prevents the lines from spreading, and lessens the chance of scum, that is to say it prevents the printing ink from the roller taking all over the stone The gum is then washed off from the surface of the stone with a sponge and the ink is strengthened by rolling up and damping alternately If a hot stone is rolled up the ink may spread in spite of the gum and as it dries quickly scum might appear A stone should always be damp while being rolled up

The stone is then gummed again cleaned up where necessary with the point of a knife or piece of snake stone and etched

The etching solution is dilute nitric acid (about 1 to 80), such as will give a little roughness on the teeth but no effervescence on the stone It is applied with a sponge followed up with a second sponge with water The stone is then cleaned with a damping cloth gummed again to replace any gum destroyed by the acid and allowed to dry

The object of etching is

1 To prevent the work from spreading
2 To prevent scum appearing on the white portions
3 To leave the work in slight relief and to prevent fine work from filling up with ink

Printing

The ink used for printing is composed of burnt linseed oil lamp black and a pale varnish (as a drier)

It is bought ready ground but for use is thinned down with lithographic varnish

Chalk printing ink is a superior kind of litho printing ink used for fine work

The ink is applied with a roller This is made of calf skin flesh side outwards over two layers of flannel fastened on to a wooden cylinder

When using black ink the roller need not be washed but it should be scraped about three times a day

The stone is washed to remove surface gum, and if the ink has had time to get hard (8 or 9 days with average ink) it is removed with turps. The stone is then damped with a cloth, and rolled up with the roller charged with ink. For fine work the ink should be thick.

The paper is put down to "**lays**," *i.e.*, marks or lines drawn as a guide on the stone with printers' lead, a mixture of lead and antimony. These lays do not wash out, and, not being greasy, do not take the printing ink. The stone should be damp when the lays are drawn.

Only paper of inferior quality requires damping to soften it, and allow it to press better into the work.

The right side (*i.e.*, smooth side) should be used.

The paper having been laid on the inked stone is pulled through press once, with moderate pressure, and removed.

The stone is damped and rolled up again after every impression is taken.

If work is stopped, although only for a short time, the stone should always be gummed in.

Drawing on Stone Direct

To draw on stone direct needs special training, as the drawing has to be reversed.

The outline at least would be usually laid down from a tracing by means of red chalk paper in a similar manner to that employed for duplicating bills, &c., with carbon paper. The wrong side of the tracing would, however, be uppermost. This outline is called an offset or faint.

The stone should be clean and free from scratches.

The outline is inked in with litho writing ink; all, except the actual portion being inked in, should be covered over to prevent the chalk dust from being rubbed off by the hand or arm.

When the drawing has been completed the stone is flooded with a mixture of gum and dilute nitric acid (1 in 60).

Solution washed off, stone gummed and dried. Then oiled, washed with turps, damped, rolled up, cleaned, etched, washed, gummed and dried. It is then ready for printing from.

Alterations on Stone

Wash off the gum with water. Take out any part to be removed as lightly as possible with snake stone or knife, and wash with water. Blot. Make the correction in litho writing ink. Allow ink to dry. Gum. Wash off gum. Roll up. Etch. Gum in again and dry.

If an addition has to be made on a part where nothing has yet been drawn the process is:—wash off gum with water. Wash with citric acid (1 in 80) to eat gum out of stone, wash with water, and draw in new work as before.

Printing in more than one Colour

When a map has to be printed in more than one colour, a **keystone** has first to be prepared for printing, and on it is laid down all the details of the complete map.

Two lines or **lays** are drawn across the plate at right angles to each other, to mark where two adjacent sides of the paper are to rest.

These lays are marked as follows:—

Where the lays will come, wash off gum with water. Apply citric acid (1 in 80), dry with blotting paper. Draw lays with litho writing ink in the manner just described for additions on stone.

In printing the offset the keystone is allowed to dry after being rolled up so as to lessen the chance of the paper stretching or contracting. For the same reason glazed board or enamelled paper should be used. If these are not procurable use a bit of stout paper which has just been pulled through the press.

The **offset** is then dusted over with red chalk powder and the surplus shaken off.

The offset is then laid on a stone prepared in the usual way to receive a transfer, and pulled through the press. This leaves the chalk on the stone. The portions required in one colour are then drawn in in litho writing ink, as described in drawing on stone.

The marks of the lays are cut with a knife and filled in with common ink.

A colour stone is prepared for each separate colour, a fresh offset being required for each stone.

When this is done the keystone is prepared for work by removing all the detail which is not required in black, in the manner described for alteration.

Before printing from the colour stones the black ink must first be washed off with turpentine and the stone rolled up with the coloured ink.

The roller used is similar to the black litho roller except the smooth side of the skin is outwards, as this makes it easier to clean.

The lightest colour is printed first and the print is placed accurately on the next stone, and so on until all the colours have been printed.

Register. Great care is necessary that the paper is layed accurately to the lays on each stone or the colours will overlap.

Zincography

The preparation and putting down of transfers on to zinc is done in the same way as on to stone.

The plates are usually obtained with a smooth surface (termed planished). Before use they are grained with fine sand, the plate being kept wet and the graining done with a muller.

Printing. On removal of the transfer the plate is etched with a solution of gum, galls and phosphoric acid. The solution is allowed to act for $\frac{1}{2}$ to $1\frac{1}{2}$ minutes, producing a fine layer of phosphate of zinc. It is then washed off and a gum solution, to which $\frac{1}{2}$ oz of etching solution to 1 pint has been added. This solution is slightly thinner than that used for stone. The plate is fanned dry, damped, sprinkled with oil followed by turpentine and rubbed with a piece of flannel. It is then rubbed over with a turpentine cloth and rolled up with ordinary litho ink which takes all over the plate owing to the presence of the turpentine. Water is then sprinkled on the plate which is rubbed with a cloth. This causes the ink to leave the plate except on the lines. The plate is again rolled up, gummed in and fanned dry. Gum washed off and plate rolled up. It is then ready to take the first impression. Plates should always be gummed in and fanned dry whenever it is necessary to stop work even for a short time.

The number of impressions which can be obtained from a zinc plate depends on a variety of circumstances but usually from 500 to 1,000 should be obtained. From stone the number would run into thousands.

Sun Printing

When the lithographic equipment is not available for the multiplication of maps in the field recourse must be had to sun printing.

There are several papers now on the market which are suitable for this purpose and although this method is not nearly so satisfactory or expeditious as lithography, there is a very great advantage in the gain in the lightness of the equipment required.

The most suitable paper for this purpose is **Bemrose No 5, Brown Process**, to be had from Bemrose and Sons, 4, Snow Hill, London It is made in three qualities, viz —

E 101 thick
E 100 thin tough
E 102 linen

The Prints produced by this method are either white on a deep chocolate ground, or chocolate on a white ground The former would be produced by printing from a tracing, the latter by printing from a negative print made on the thin paper (E 100)

A tracing on tracing cloth or paper should be made from the original It should be drawn with Indian ink, made as thick and opaque as possible Creases and folds should be avoided It is possible to print direct from an original, unless made on very thick paper, but the printing is of course slower than with a tracing

The tracing is placed in a printing frame, face to the glass and the sensitized paper laid over it, prepared side next to the tracing, the felt pad and back of frame next in succession, care being taken that the paper is in perfect contact with the tracing

The frame should be placed in the sun if possible for ½ to 8 minutes The print should be examined at intervals and removed from the frame as soon as the detail is completely visible

The print is next placed in water for 3 to 5 minutes, and then in a bath of hyposulphite of soda, 1 oz to 3½ pints of water, for ½ to 1 minute or, if preferred, the hyposulphite can be applied with a brush or sponge

The print is then well washed and dried

The paper is supplied in rolls of 11 yards, width 26, 30 and 40 inches, sufficient developer is supplied with each roll

For quick work two men would be required, one to expose the prints, and the other to develop and wash them

The following apparatus would be required (cost about £3) —

1 printing frame 26 × 21
2 zinc baths, 28 × 21
1 developing brush
1 tin case for sensitive paper 26 long,
1 bucket or can for water,
1 pot for developing solution,
1 roll of ordinary tracing paper

The price of the paper per roll of 11 yards is as under —

Thick	7 07 30 wide,	9 , 40 wide
Thin	5 37	7 ,,
Linen		21s ,,

CHAPTER XIII

BRITISH AND FOREIGN GOVERNMENT MAPS AND SURVEYS

National Surveys.—A good topographical map is such a vital military necessity, that the initiative in the matter of national surveys has in nearly every country, been taken by the army This is the case in the United Kingdom India, France, Germany, Italy, Spain, Austria Russia and other countries

Mapping v Sketching.—The necessity for a topographical map having been realised, the method of its formation would become the next point for consideration It would occur to any officer accustomed to sketching (as are the officers of most armies) that a workable map might be formed by combining military sketches As an instance of the effect of such a system we may quote the state of the maps of India before the year 1802 These maps were compiled from sketches and were subsequently tested by the results of the accurate survey In the short distance across India in the latitude of Mysore the maps so compiled were found to be 40 miles in error

As another instance may be quoted the compilations made in Burma after the annexation in 1885 A vast amount of sketching was carried out by officers during the operations, these sketches were compiled the result though better than nothing at all, was highly unsatisfactory

Akin to these were the compilations made from farm surveys during the South African War of 1899 1902 These were reported by many generals to be worse than useless They gave however an abundance of names and served to show the location of farms and the lines of railways The hill features shown on them were quite unreliable, which will not surprise those who have seen the original farm surveys from which they were compiled

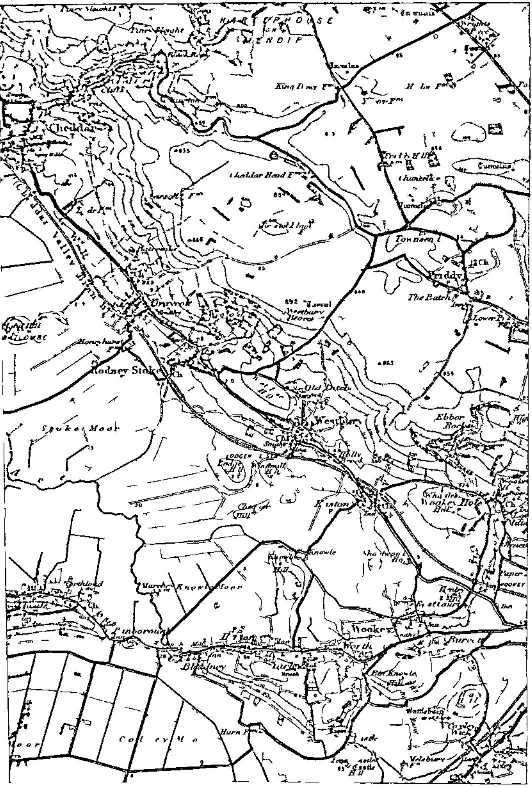
Another instance of the futility of combining sketches to make maps is the present condition of the maps of most British possessions in Tropical Africa Indeed all experience demonstrates the fact that *no system of sketching, no combination of sketches, and no compilation of previously unconnected and unrelated sketches, isolated surveys or farm plans, will result in a satisfactory topographical map*

The Ordnance Survey.—*In the United Kingdom* the first step towards the production of a topographical map was taken in 1747, two years after the rebellion in the Highlands General Watson D Q M G to the Duke of Cumberland's forces, conceived the idea of making a map of the Highlands this was subsequently extended to the rest of Scotland Various schemes were now advocated for the accurate mapping of the whole kingdom, but it was not until 1783 that the Ordnance Survey may be said to have come into existence by the measurement by General Roy of a base line on Hounslow Heath, and it is to be noticed that the whole work was undertaken by soldiers the working party being furnished by the 12th Regiment the reason given by Roy being that, 'not only was this obviously the most frugal method but also because soldiers would be more attentive to orders than country labourers

The subsequent history of the Ordnance Survey is in brief —Until 1824 work was almost entirely confined to the 1 inch scale At that date the 6 inch map of Ireland was commenced, in 1840 the same scale was commenced for Great Britain The map of Great Britain on a scale of [illegible] or approximately 25 inches to the mile, was commenced in 1854 This is periodically revised so that no sheet is more than 15 years out of date The second revision is now in progress in England and Scotland The same scale was commenced in Ireland in 1890 about half Ireland is completed The interest of the 25 inch map for a soldier lies in the fact that

ORDNANCE SURVEY

Scale – 1 Inch to 1 Mile *(Coloured)*



Ordnance Survey Southampton.

SHOWING THE COMBINATION OF CONTOURS AND VERTICAL HACHURES

ORDNANCE SURVEY

Scale. - 2 Mile to 1 Inch 126,720



Ordnance Survey Southampton

[illegible] local roads are [illegible] and [illegible]

[illegible] of the 1 inch, ½ inch, ¼ inch, [illegible] inch [illegible]

[illegible] interesting feature of the Ordnance Survey [illegible]

[illegible] most [illegible]

[illegible]

1 [illegible]
2 [illegible]
3 [illegible]
4 [illegible]

The Survey of India. [illegible] the Ordnance Survey, the Survey of India owes its origin [illegible] 33rd Regiment. After the fall of Tippoo, Major Lambton sub[illegible] Trigonometrical Survey of the Peninsula. Actual work was com[illegible]

[illegible] in 1823 was succeeded by Sir George Everest; the whole [illegible] survey as it now exists was the creation of Everest's brain,' [illegible] The first attempt at a topographical survey [illegible] Mackenzie, who surveyed some 40,000 square miles in the Deccan between 179[illegible] and 1809. Everest was succeeded by Sir Andrew Waugh in 1843, and from this date [illegible] extension of topographical surveys took place. The names of Capt. Robinson and Capt. M[illegible] are specially to be remembered as topographers.

It is to Capt. Robinson that we owe the present accurate method of using the plane table.

It is impossible even to mention the vast numbers of topographical maps of the Survey of India. They cover an area of about 2,000,000 square miles, and vary in scale from ¼ inch to 4 inches to the mile. Generally speaking, a new province is surveyed first on the ¼ inch scale, and subsequently, if required, on the 1 inch. The system of surveying on service has been brought to a high degree of excellence. Perhaps the most interesting of recent surveys is the ½ inch work carried out during the operations in China. A preliminary edition of this was printed in the field. The special merit of the Indian topography lies in its frontier work; no [illegible] rapid and accurate maps under such difficult conditions.

South Africa.—*In South Africa the only topographical map which existed on the outbreak of war in 1899 was a 1 inch map of a small area north of Ladysmith.* This map had been rapidly made some years before; both the time and means allowed for its construction were inadequate.

Foreign Maps.*—The principal topographical maps of other countries are:—

Austria $\frac{1}{75000}$. Hills are shown by contours and vertical hachures, the later are coarse and heavy. Not a good example of a topographical map.

Belgium, $\frac{1}{40000}$, in colours; hill features shown by contours. $\frac{1}{100000}$, military map of Belgium. The hill features are badly defined by feeble contours in brown, which are difficult to follow.

France $\frac{1}{80000}$. 'Carte de l'Etat Major,' the staff map of France, engraved on copper; hills shown by vertical hachures. Similar to, but inferior to, the black edition of the Ordnance Survey 1 inch (Plate XXIII).

The coloured $\frac{1}{50000}$ map of Algeria and Tunis, prepared by the Service Geographique de l'Armée. Hill features shown by contours at 10 metre interval and by shading with an oblique light. These are excellent maps (Plate XX).

* See Pamphlet by Col. Sir John Farquharson, K.C.B., on Foreign Topographical Maps.

The $\frac{1}{100,000}$, "Carte Vicinale" Hill features shown only by stump, difficult to read except in hilly country

Germany, $\frac{1}{100,000}$, engraved on copper, printed in black, somewhat inferior to the Ordnance Survey 1 inch

$\frac{1}{25,000}$, "Positions Karte," the maps of Prussia, Baden, Saxony and Wurtemberg on this scale are good clear maps

Italy, $\frac{1}{100,000}$ contours at 50 metres with vertical hachures (Plate XXV)

Russia, $\frac{1}{126,000}$, or "3 verst" map, engraved on copper, printed in black Hills shown by vertical hachures when there are any $\frac{1}{126,000}$ or 3 verst maps of the Balkan Peninsula Hills shown by contours in brown, and the contours are somewhat close, otherwise a good clear map

Spain, $\frac{1}{50,000}$, in colours, contours at 20 metre intervals

Switzerland, $\frac{1}{25,000}$, and $\frac{1}{50,000}$, contours brown, detail black, beautiful maps

United States, $\frac{1}{62,500}$, $\frac{1}{125,000}$, and $\frac{1}{250,000}$, according to population of country In colour, contours in brown, water, blue Very excellent maps (Plate XXI)

Analysis of Scales of Foreign Maps —We can now analyse the scales used Where two maps of a country are of equal importance they are both put down in the accompanying list —

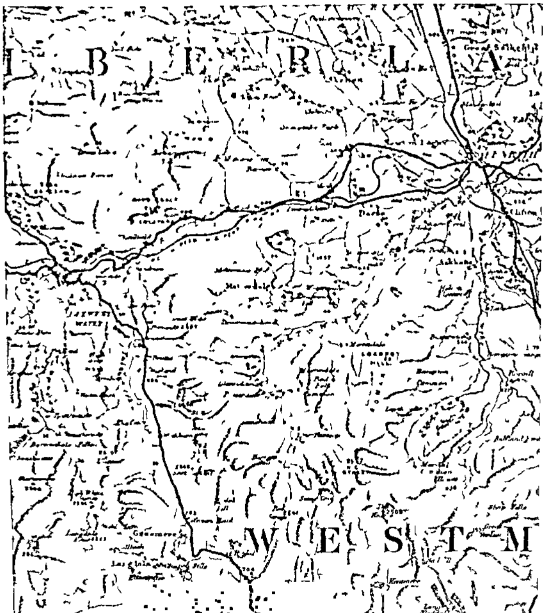
SCALES

The United Kingdom	1 by	63,360
"		126,720
India		63,360
"		253,440
Austria		75,000
Belgium		40,000
"	,	160,000
France		80,000
"		50,000
Germany	,	100,000
"		25,000
Italy	,	100,000
Russia		126,000
Spain		50 000
Switzerland		25,000
"		50 000
United States	,	125 000
	Mean, about 1 by	90,000

The list of the scales of the principal topographical maps of the world, most of which are very good maps, is sufficient to show how important it is that officers should accustom themselves to the use of small scales The same opinion would have been arrived at on the merits of the case by considering those instances in which maps are used in war, and the purposes which they have to serve Even in the case of a small country like England it is simply inconceivable that an army could make any general use of 6 inch maps—the 6 inch map of England is over 15,000 square feet in area

ORDNANCE SURVEY

Scale 4 Miles to 1 Inch (Coloured)



Uses of Maps in War.—The uses to which maps are put in war are:—

(1) For strategical purposes;

(2) For finding the way;

(3) To give the Commander of a force topographical information concerning roads, villages, woods, hills, streams, bridges, ferries, railways, and water supplies, &c.;

(4) To enable a general outpost, offensive or defensive position to be taken up;

(5) And generally to convey information which cannot be given at the moment by the eye alone.

It is clear that, as a general rule, large scales are not required.

Analysis of methods of showing hill features.—It is worth noting that most government maps represent hill features by a combination of contours and shade, contours and vertical hachures, or by contours alone. On very small scales the contours become generalized form lines which may be replaced by vertical hachures. As regards the intervals of the contours, the following list gives the number found by multiplying the contour interval in feet by the scale in inches to the mile:*

France,	1/[illegible]0,000	41
"	1/80,000	52
Prussia,	1/25,000	41
Baden, Bavaria, Wurtemberg,	1/25,000	81
Italy,	1/[illegible]0,000	20
"	1/100,000	102
Spain,	1/[illegible]	81
U.S.A.,	1/[illegible]	50
	"	20
O.S. ½ inch		50
" Colonial		50
" 1 inch		100
Mean		57

We may therefore say that the weight of authority is in favour of a contour interval of about 50 feet for a 1 inch scale and for other scales in proportion, thus 100 feet for a ½ inch map, &c.

These same results might have been arrived at on the merits of the case. All practical topographers know that in ordinary country the contours on the Ordnance Survey unshaded 1 inch map are not sufficiently close to show the ground. It should be added that the rule of 50 feet for a 1 inch scale is a tacit rule on which the form lines of the small scale Survey of India maps are made, e.g., on the transfrontier ¼ inch maps the form lines may be considered to be 200 feet apart.

These considerations have led to the decision to adopt the same rule for military sketching, both on its own merits, and because sketches will necessarily be used in conjunction with Government maps. The rule may be stated thus:—The contour interval in feet is found by dividing 50 by the number of inches to a mile in the scale.

To sum up the analysis of this list of maps:—

* Switzerland (an exceptional case on account of its mountainous character) has the following intervals: for the [illegible] map 10 metres; for the [illegible] map 30 metres.

The modern topographical map will be on a scale of about $\frac{1}{100\,000}$ or from ½ inch to 1 inch to 1 mile

The hills will be represented by a combination of contours and shade

The intervals of the contours will be given by the rule*

$$\frac{50}{\text{No of inches to the mile}}$$

The map will be printed in colours

The Layer System —Another means of showing the relief or hill features of a tract of country is by "layers" or "tints" In this system the height above sea level between any two selected contours, is indicated by a special tint or colour Thus a light tint say of yellow, will represent all the ground between sea level and the 200 ft contour, a somewhat darker tint will show the ground between the 200 and 400 ft contours and so on Mr Bartholomew's excellent tinted half inch maps of the United Kingdom (based on the Ordnance Survey) are well known

A few official maps on this system have been published Amongst them may be mentioned

Austria, [illegible] 11 tints 9 of brown 2 of green
Sweden [illegible] 13 tints, light yellow to dark brown

No system can rival this as a means of showing broadly on small scales the absolute and relative heights of the main features Its disadvantages are, that it gives a stepped appearance to the map that the darker shades obscure the detail, and that it gives very little idea of the steepness of the hill slopes

Medium country on a scale of 2 miles to 1 inch may perhaps be considered to be equally well represented by contours and shade or by layers and the effectiveness of the latter method increases as the scale decreases

"**Metric Scales** —In the British Empire the question sometimes arises as to whether the scale shall be in inches to the mile or shall be a "metric" scale in which the denominator of the fraction is some definite number of thousands For instance shall we choose

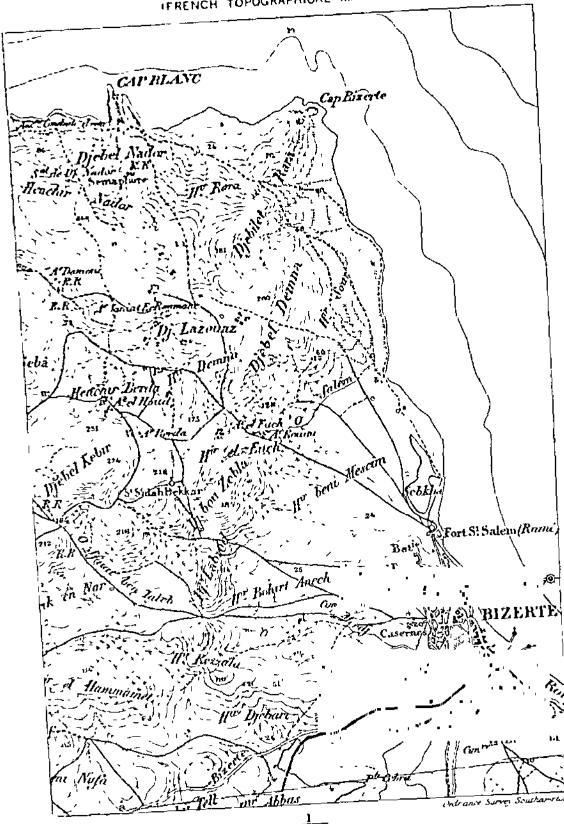
½ inch to 1 mile ($\frac{1}{126\,720}$) or [illegible]?

The answer may as a rule be given in favour of ½ inch to the mile This statement at once brings up a definite mental picture of the relationship between the length on the map and that on the ground in terms of two most familiar units *The other involves a consider*able effort to realize and is indeed only justifiable where the metric system is in use When we adopt a metric system and have got into the habit of thinking in centimetres and kilometres then and not till then should our maps be drawn on a metric scale

Plates XVIII to XXV are examples of some of the more important small scale maps of the United Kingdom, France Germany, the United States, Switzerland and Italy

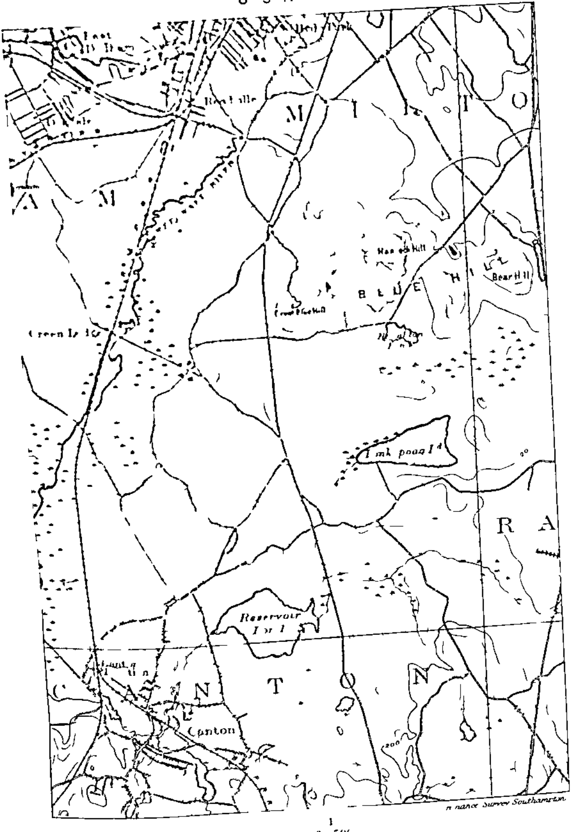
* Or a 25 metre interval for a scale of [illegible]
a 20 ,, [illegible]

BIZERTA
(FRENCH TOPOGRAPHICAL MAP)



Scale $\frac{1}{50,000}$

U S A



1
S al 6_ CIK
CONTOURS AT 20 FEET INTEPVALS

GERMANY

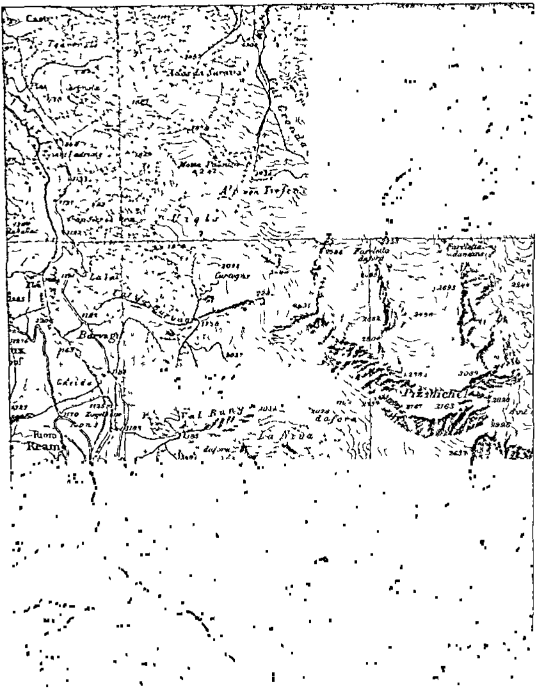


Scale $\frac{1}{100\,000}$



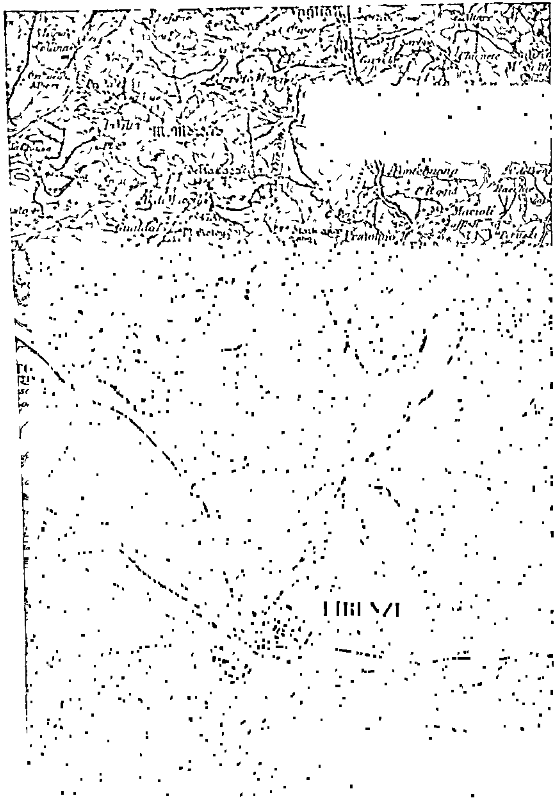
Scale $\frac{1}{80\,000}$

SWITZERLAND



Ordnance Survey Southampton

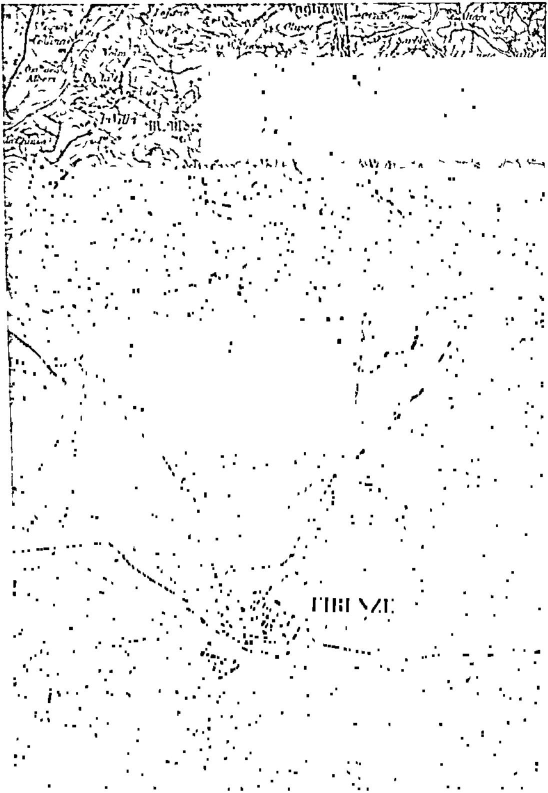
Scale $\frac{1}{50\,000}$



Scale $\frac{1}{100\,000}$

CONTOURS AT 50 METRE INTERVAL

ITALY



Scale $\frac{1}{100\,000}$

CONTOURS AT 50 METRE INTER

ST. HELENA

Scale – 2½ Inches to 1 Statute Mile



CHAPTER XIV

FIELD ASTRONOMY

SECTION I. GENERAL CONSIDERATIONS

The Heavens. [illegible] of the heavens on a clear night is that of a vast concave [illegible] planets and stars. This great concave sphere is called [illegible]

The [illegible] bodies are produced by the revolution of the *earth* on an axis, the North and South [illegible] of the heavens are on the prolongation of this axis.

[illegible] of the heavens is a great concave sphere imagine at its [illegible] small sphere of appreciable dimensions, and while the celestial sphere is [illegible] the earth revolves on its polar axis. Now although appearing to be about the [illegible] the distances of the celestial bodies vary enormously. Thus, the [illegible] earth of the moon is 238 833 miles, of the sun is 92 900 000 miles, and [illegible] 5 000 000 000 000 miles, while the distances to the planets [illegible] depending upon our mutual positions in our several orbits.

Reduction of Observation to a Common Datum point.—A second in arc is equivalent [illegible] either from it any point on the moon, the angle subtended by two points a mile apart on the earth's surface may be as much as one second of arc.

Hence the necessity in observations of the sun, moon, and planets—stars are too far off to be affected—of reducing the observations taken to what they would be at the centre of the earth, so that astronomical tables may be of universal use while they are at the same time kept within reasonable bounds in size, and this involves a knowledge of the shape and size of the earth.

Besides the *diurnal motion* there is the *annual motion* of the earth to be taken into account, and this leads us to the consideration of the Solar System.

THE SOLAR SYSTEM

The Solar System is composed of the sun, the planets (of which there are about 600 very small ones and 8 large ones) and their accompanying satellites.

Some of the planets are accompanied by secondary planets or bodies called *satellites* which revolve round their particular planets just as the system of planets revolves round the sun. The planets and their satellites are opaque non-luminous bodies and only shine by the light of the sun being reflected from them, and thus they can only be seen when this reflected light can be seen, or when they come between an observer and some luminous body like the sun.

The Solar System may be more particularly described as consisting of:—

1. The Sun, the central body.
2. The four inner planets—Mercury, Venus, Earth, and Mars.
3. The small planets or asteroids, of which nearly 600 are known to be revolving between the orbits of Mars and Jupiter.
4. The four outer planets—Jupiter, Saturn, Uranus, and Neptune.
5. The satellites of the planets; the Earth has one called the *Moon*, Mars has two, Jupiter has six, Saturn, nine, Uranus, four, and Neptune, one.

Kepler's Laws.—The laws of planetary motion are three in number, as follows:—

1. The orbit of each planet is an ellipse having the sun in one focus.
2. As the planet moves round the sun its radius vector (*i.e.*, the line joining it to the sun) passes over equal areas in equal times.
3. The square of the time of revolution of each planet is proportional to the cube of its mean distance from the sun.

When a planet is furthest from the sun it is said to be in *aphelion*; when nearest, in *perihelion*.

The major axis of a planet's orbit is called the *apse line*, or *line of apsides*, and that of the

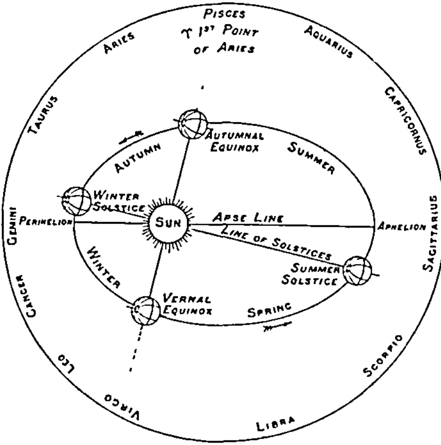


Fig. 46.

earth's orbit is known to be revolving in space 11".78 annually (as measured at the sun's centre) in the direction of the earth's orbital motion.

Motion of Planets in the Solar System.—All the planets revolve round the sun in the same direction, viz., contrary to the motion of the hands of a watch when viewed from the north side of the plane of the earth's orbit. Nothing certain is known of the axial rotation of Mercury, Venus, Uranus, Neptune, or the minor planets. The other planets rotate on their axes in the same direction as they revolve round the sun. The satellites move round their respective planets in the same direction, except the satellites of Uranus, the satellite of Neptune, and the newly discovered ninth satellite of Saturn; the motions of these are retrograde.

The axis of each planet is inclined to the plane of its orbit at a nearly constant angle, it has a proper secular motion, but practically remains for a considerable interval of time parallel to itself during the motion of the body through space

The orbits of all the planets are not in the same plane, but those of the principal planets lie in planes within 9 on either side of the plane of the earth's orbit, which plane is called *the ecliptic*

The Moon

Motion of the Moon—The moon moves round the earth in a path inclined at 5° 8 40′ to the ecliptic, and takes 27d 7h 40m to make a complete revolution with reference to a star The movement of the moon in the heavens is from west to east

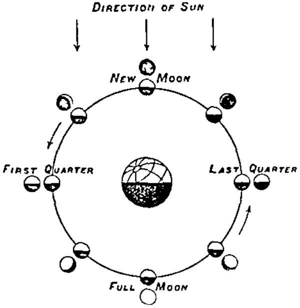


Fig 47

The Phases of the Moon are shown in fig 47, where the unshadowed portion of the outer row of moons show its successive appearance as seen from the earth

The Stars

The great number of stars visible even to the naked eye renders it impossible to distinguish each by a particular name, and thus the stars have been arranged in groups, called *constellations*, for the purpose of more readily distinguishing them, and each group or constellation is given a name

An idea of **the position of the various constellations** can be best obtained by reference to a celestial globe, map or "star chart"

To distinguish the stars of a constellation from one another, Bayer, in 1603, assigned to each of the stars in every constellation a letter of the Greek alphabet, the brightest being α, the next β, and so on The name of the constellation was put after each letter When the Greek letters were exhausted recourse was had to the Roman alphabet and these two alphabets fully answered the purpose at the time The order of the letters *was* intended to indicate the relative brilliancy of the stars in the same constellation, without reference to those in other parts of the heavens, thus α *Aquarii* is a star of the same magnitude as γ *Virginis* Since this

system was introduced some of the stars have changed in brightness Sometimes when there are two near stars of nearly equal magnitude, the numerals 1 and 2 are affixed to them in their order of right ascension as x^1 and x^2 *Tauri*

Flamsteed gave numbers to the stars observed by him in each constellation, the numbers were given according to the order of their right ascension, and have nothing to do with their brilliancy This method of naming stars is often used

Proper Names of Principal Stars —Some of the *principal* stars of the first two magnitudes have also received particular names, thus —

α Canis Minoris	is called	Procyon,
α Lyræ	,,	Vega,
α Boötis	,,	Arcturus,
α Canis Majoris	,,	Sirius,
α Aurigæ	,,	Capella,
α Tauri		Aldebaran

etc

The Pole Star —The star nearest the north pole of the heavens is known as the north polar star The north polar star at present is α *Ursæ Minoris* and is about 1¼° from the pole, which distance will diminish in about 150 years to about ½°, and then it will increase again until the present pole star will be too far from the pole to serve as such Thus the pole star varies 4 600 years ago α *Draconis* fulfilled this office and in 12 000 years α *Lyræ* will become the pole star This change of the pole star is due to *precession*

Distance of Stars —We can best form an idea of the immense distances of the stars by considering the time which light takes to travel from them to the earth Thus, the time the *light of the sun takes to travel* to the earth is 8⅓ minutes whilst Vega is distant 30 light years and Polaris 50 light years

The **annual parallax** of a star is the angle subtended at the star by the mean radius of the earth's orbit

For the nearest star the annual parallax is less than 1″

Beside all known displacements in the apparent position of a star due to the earth's motion in its orbit each star appears to have a small motion of its own which is called **proper motion**

Stars have been classified into *orders* or *magnitudes* of brightness and these "**magnitudes**" are given in the Nautical Almanac for all stars there included, under the heads of Mean or Apparent Places of Stars Those of the sixth magnitude are about the smallest visible to the naked eye The largest telescopes show stars down to about the 17th magnitude

Notation

In practical astronomy the following notation is usually adopted —

A	Azimuth	L A N	Local apparent noon
E	Equation of time	G A N	Greenwich apparent noon
L	Longitude	L M N	Local mean noon
R A or R	Right ascension	G M N	Greenwich mean noon
R O	Right ascension of object	h	Altitude
T	Chronometer (or watch) time of observation	p	Polar distance
L S T	Local sidereal time	t	Hour angle
G S T	Greenwich sidereal time	♈	First point of Aries
L A T	Local apparent time		Declination
G A T	Greenwich apparent time	ζ	Zenith distance
L M T	Local mean time	λ	Latitude
G M T	Greenwich mean time	φ	Latitude

SECTION II.—DEFINITIONS, ETC.

Celestial Sphere.—In the same way as circles are made use of as a means of measuring angles, so in spherical trigonometry a sphere is employed. Therefore we may conceive the concave bowl of the heavens as a measuring sphere with the earth at the centre. This sphere is called the *celestial sphere*, and its diameter may be any size (see HZLN, fig. 48).

The Axis of the Earth is that diameter round which it rotates daily. Its extremities are called N. and S. pole. This axis produced cuts the celestial sphere in the N. and S. poles of the heavens (see PN, fig. 49).

The Terrestrial Equator is a great circle of the earth whose plane passing through the centre is perpendicular to the axis. This plane produced cuts the celestial sphere in the *celestial equator*. The terrestrial and celestial equators are in the same plane, referred to generally as the plane of the equator (see EKQ, fig. 49).

The Zenith is the point in the celestial sphere vertically above the observer's position (see Z, fig. 48).

The Horizon is a plane cutting the celestial sphere at 90° from the zenith (see HLR, fig. 48).

The Meridian plane of a place is the plane passing through the poles and that place.

The Celestial Meridian is a great circle formed in the celestial sphere by the extension of the meridian plane. It clearly passes through the poles of the heavens and the zenith (see HZPLN, fig. 48), L and H are the N. and S. points of the horizon.

The Prime Vertical is the plane through the zenith at right angles to the meridian, and cuts the horizon in E. and W. points.

The Elevated Pole is that pole which is above the horizon.

The Longitude (L) of a place is the arc of the equator intercepted between the meridian of a place and any standard meridian such as Greenwich, from which all longitudes are measured E. or W. up to 180° of arc or 12 hours of time.

The Latitude (ϕ or λ) of a place is the inclination of the normal (or plumb bob) to the plane of the equator, and is N. or S. (+ or −) according as the place is N. or S. of the equator.

The Ecliptic is the great circle in which the celestial sphere is cut by the plane of the earth's orbit. It is therefore the apparent path of the sun among the stars.

The Obliquity of the Ecliptic is the angle which the plane of the ecliptic makes with the plane of the equator, and amounts to 23° 27′ at present.

The points where the ecliptic and equator intersect are called **the equinoxes**, and the intersection of their planes is called **the line of equinoxes**.

The Vernal Equinox is the point at which the sun crosses the celestial equator from south to north (about March 21st).

The Autumnal Equinox is the point at which the sun descends—about September 23rd—from the northern to the southern side of the equator.

The Solstices are the points of the ecliptic 90° from the equinoxes, and are distinguished as summer and winter (*vide* fig. 46).

Astronomical Coordinates

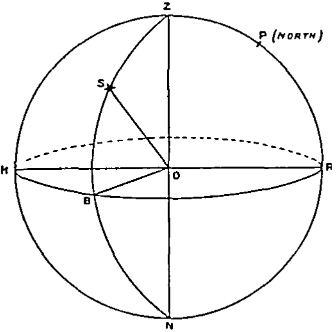
In order to fix the position of any heavenly body at a given instant, it is necessary to employ some system of coordinates. For field astronomy two such systems are employed —

(1) { Altitude
      Azimuth

(2) { Declination,
      Hour Angle, } or { Declination
                         Right Ascension

(1) Altitude and Azimuth

The Altitude (h) of a heavenly body is the arc of a great circle through the zenith and the heavenly body intercepted between that body and the horizon (see SB in fig 48)



The plane of the paper is supposed to coincide with the plane of Meridian

Fig 48

It is clear that the angle SOB would also represent the altitude and this angle can, whenever the heavenly body is visible, be observed with the theodolite

Great circles passing through the zenith are called **vertical circles**

The Zenith Distance (ζ) is the complement of the altitude, *i.e.*, $90 - h$ (see ZS in fig 48)

In working out certain computations which involve *zenith* or zenith distance, care must be taken with the algebraical sign of the zenith distance. It is usual to call

Meridional zenith distance (ζ) + when the heavenly body is S / N of zenith

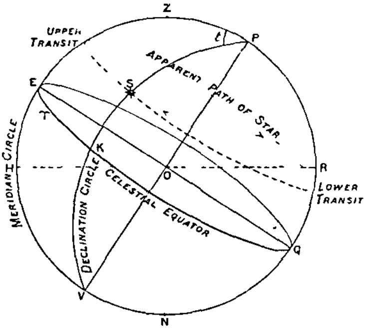
Culmination —When a heavenly body attains its highest altitude it is said to be at its upper culmination. At that its lowest altitude it is at its lower culmination. The latter in many cases, would not be visible as it would be below the horizon

A star is said to be **at elongation** when it is in such a position that the angle at S in the astronomical triangle PSZ becomes a right angle, *i e* when the plane of the vertical circle ZS becomes tangential to the diurnal path or circle of the star

The Azimuth (A) of a heavenly body is the angle between the meridian plane of the observer and the vertical plane passing through the body For astronomical purposes azimuth is measured the nearest way from the meridian plane Thus in fig 48 the azimuth of S = angle KOB = angle PZS

(2) DECLINATION AND HOUR ANGLE

The Declination (δ) of a heavenly body is the arc of a great circle through the poles and the body intercepted between the plane of the equator and the heavenly body It is



The plane of the paper is supposed to coincide with the plane of Meridian

Fig 49

measured from the equator to the poles in degrees 0° to 90° and termed N or S (+ or −) according as the body lies north or south of the equator (see SK, fig 49)

The declination of the sun is given for every day of the year on pages I and II of each month in the 'Nautical Almanac' and that of the principal stars (pp 324 to 452) for every 10th day of the year

An inspection of these tables will show that the sun's declination varies from approximately 23½° N to 23½° S but that the stars vary very slightly during the year

The Polar Distance (p) of a heavenly body is the complement of its declination *i e* (90 − δ) it is styled N or S polar distance (N P D) or (S P D) according as it is measured from the N or S pole (see P S in fig 49)

The Hour Angle (t) of a heavenly body is the angle at the pole between the *meridian* of a place and the declination circle through the heavenly body (see angle Z PS, fig 49) It is usually measured westward from the meridian from 0° to 360°, or 0h to 24h, one hour being equivalent to 15° of arc From fig 49 it is clear that this angle is the same as the

arc of the equator intercepted between the meridian and declination circle, also measured westward

In this system in place of the hour angle the right ascension of the heavenly body may be used

Right Ascension

Right Ascension (R A or Æ) of a heavenly body is the arc of the equator intercepted between the first point of Aries and a declination circle through the body (see ♈ K, fig 49) It is reckoned in time *eastwards* from 0 to 24 hours

The R A and declination (δ) of all the principal heavenly bodies are tabulated in the 'Nautical Almanac,' which gives their values at particular instants of time for the Greenwich meridian *as if they were seen from the centre of the earth*

Astronomical Triangle

A heavenly body at any moment forms a triangle with the zenith and elevated pole (see fig 50) the sides being arcs of the meridian ZP, the vertical circle ZS, and the declination circle PS

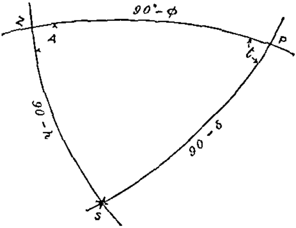


Fig 50

Now in fig 49 the arc FZ is the latitude ϕ of the observer, hence PZ is equal to $90 - \phi$, or co-latitude The latitude is always known approximately, or can be found from one of the methods described later

The side PS, or polar distance, is equal to 90 − declination, and its value can be found from the 'Nautical Almanac'

The side ZS or zenith distance, is equal to 90° − altitude, and h the altitude can always be observed, whenever the heavenly body is visible, with the theodolite

The angle at Z is the azimuth

The angle at P is the hour angle (t) *

The angle at S is the parallactic angle

Given any three of these, the others can be deduced For formulæ, &c , see Section III

Note.—It will be easily seen from fig 49 that **the latitude of a place is equal to the declination of the zenith or the altitude of the pole**

* More correctly as drawn 360° − hour angle

Hour Angle and Time (t)

When any point of the celestial sphere is on the meridian of an observer, that instant is designated the *transit* of that point over the meridian also the *meridian passage* and *culmination* In one complete revolution of the sphere about its axis every celestial body crosses the meridian twice It is therefore necessary to distinguish between the two transits The meridian is bisected by the poles of the heaven The transit over that portion which contains the observer's zenith is the UPPER TRANSIT, and the transit over the other portion is the lower transit At the upper transit the hour angle (expressed in time) of a heavenly body is 0 hours, at the lower transit its hour angle is 12 hours (see fig 49) A knowledge of the hour angle of a heavenly body at any moment will on account of the uniform rate at which it varies, at once give the time that it will take to reach the meridian if it be on the east side of it or the time elapsed since it crossed the meridian if it be on the west side In other words, if a clock be so regulated as to go 24 hours between two successive transits of any heavenly body and to read 0h 0m 0s at its upper transit, *the hour angle of the heavenly body (t) would be the time shown by the clock*

A sidereal day is the interval of time between two successive (upper) transits of the first point of Aries, *i e*, the true vernal equinox over the same meridian It is divided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds When ♈ is on the meridian the sidereal time is 0h 0m 0s and it is *sidereal noon* at that place, and the sidereal time at any instant is the hour angle of ♈ at that instant reckoned from the meridian westward, 0 to 24 hours

From the definition of right ascension it is clear that all stars will succeed each other across the meridian in the order of their right ascensions

An apparent or solar day *is the interval between two successive transits of the sun's* centre across the same meridian and **apparent time** (AT) at any instant is *the hour angle of the sun* measured westwards throughout the circle expressed in time (15° – 1 hour)

Now if the orbit of the earth were a circle with the sun in the plane of the equator and in the centre, this apparent day would give us a constant interval of time, but since the orbit is an ellipse and since it is inclined to the equator these intervals are continually varying day by day, and to obtain a measure of the day that is uniform and at the same time corresponds fairly well with the recurrence of day and night the average length of all the solar days of the year is taken and used as the *mean solar day* This is the day to which ordinary clocks and watches are regulated, and it is adjusted so as to bring 12 noon as near to the time when the sun is on the meridian as possible for every day of the year

Mean Time —The *mean solar day* is thus regulated by a fictitious or *mean* sun and is the interval between two successive transits of the *mean sun's centre* across the same meridian, the interval being divided into 24h 0m 0s *Mean time* (MT) *at any instant is the hour angle of the mean sun's centre at that instant*

Astronomical Time —Since the ordinary division of the civil day into two periods of 12 hours each is somewhat ambiguous and inconvenient for astronomical purposes, the day (apparent mean, or sidereal) is conceived by astronomers to commence at noon, and is divided into 24 hours the hours being subdivided as usual into minutes and seconds

Equation of Time —As the mean sun is an entirely fictitious assumption, we have no phenomenon by which we can directly observe its progress We are therefore reduced to observing the motion of the real sun, and by knowing the interval between the real and fictitious sun at any instant, we are enabled to arrive at the mean time at that instant This interval is called the *equation of time* (E), and is defined as the difference between apparent and mean time, or the angle at the pole between the hour circles passing through the centres of the true and mean sun, expressed in time

The variation in the equation of time is due to the two following causes —

(1) The obliquity of the ecliptic,
(2) The eccentricity of the earth's orbit

Firstly —Neglecting the effect of the eccentricity of the earth's orbit (*i e*, supposing the orbit a circle, and therefore by Kepler's Laws, that the earth moves uniformly along it) and for convenience imagining the sun to move round the earth with a uniform velocity, then (see fig 51) let a mean sun, M be made to move along the equator with the same velocity as the true sun T moves in the ecliptic, the two passing through ♈ (the vernal equinox) at the same instant Now when the true sun is at T the mean sun will be at M, where ♈T = ♈M and if T^1 be the point where the hour circle through T cuts the plane of the equator, then T^1M will measure the angle between the hour circles of T and M, and therefore the equation of time

A little consideration will show that T and M will coincide (*i e*, will lie in the same hour circle) only at the equinoxes and solstices and that from equinox to solstice M will be ahead of T, and behind it from solstice to equinox

The maximum difference between the true and mean suns, on account of the obliquity of

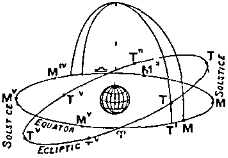


Fig 51

the ecliptic is about 10 minutes and occurs about the 5th February, 7th May, 8th August, and 8th November

Secondly —Neglecting the obliquity of the ecliptic, Kepler's second law tells us that the sun would appear to move fastest when the earth is at perihelion (see fig 46) and slowest when at aphelion. Hence a true sun would be ahead of a mean sun from perihelion to aphelion, and *vice versa* on the return journey

Any point of the orbit may be taken as the starting point of the two suns, but one has been chosen which will cause the equation of time at any period of the year to be nearly a minimum With the point so chosen, the greatest difference due to the elliptical form of the earth s orbit is about 7 minutes

A combination of the above two causes of variation gives the following data —The equation of time vanishes (*i e*, the hour circles passing through the mean and true suns, coincide) four times in the year, and one of them having been fixed on the 24th December, the others fall necessarily on the 15th April 15th June, and 31st August, its maximum positive and negative values as applied to AT, and their respective amounts are as shown below, and occur about the dates given against them

	minutes
10th February	+14 26
14th May	− 3 51
26th July	+ 6 16
2nd November	−16 21

The value of F is given in the 'Nautical Almanac' for every day of the year at apparent and mean noon at Greenwich

We have seen (p 131) that the sun's hour angle (t) at any particular instant and place is the apparent time at that place

Now the difference of the hour angles of any celestial body at two places at the same instant is equal to the difference of longitude between those places It follows therefore that the difference of the solar times of two places at one instant is equivalent to their difference of longitude

Thus if

T_0 = Greenwich time
T = local time
L = longitude of place,

then $T_0 = T \pm L$ (+ or − according as longitude is west or east) this equation being true *whatever time is used* whether solar or sidereal as long as T_0 and T are *both of one kind* of time

Mean and Apparent Times

In converting apparent into mean time or *vice versa* we use the formula —

$M T = A T \pm E$,

where M T = the mean time
A T = the apparent time
E = the equation of time (+ or − according as it is additive to or subtractive from the apparent time)

Example 1 —In longitude 30° 45' W on December 7th civil date the L M T chronometer marks 3h 14m 13 5s (A M) and is fast 25m 18 7s what is the G M T ?

	h	m	s
L M T chronometer =	3	14	13 5 A M
Chronometer correction = −		25	18 7
Local mean time Dec 7th =	2h	48m	54 8s A M

	h	m	s
L M T, Dec 6th =	14	48	54 8
Longitude (in time $\frac{30\ 45}{15}$) +	2	3	0
the G M T = Dec 6th	16h	51m	54 8s

And the civil time is Dec 7th 4h 51m 54 8s A M *

Example 2 —In longitude 60 W, local apparent time May 24th 1903 3h 12m 10s what is the local mean time ?

	h	m	s
L A T = May 24th	3	12	10
Longitude in time = +	4	0	0
G A T May 24th	7h	12m	10s

The value of E here required is that of the Greenwich instant corresponding to the local time instant 3h 12m 10s

* In the further examples unless A M or P M is inserted astronomical time will always be understood

From the 'Nautical Almanac' of 1903 we get E at G A N, May 24th, = 3m 27 64s, and the hourly variation = + 0 2 seconds

Hence at the given G A T instant

E 3m 27 64s + (0 2 × 7 21) – 3m 29 08s and is subtractive

	h	m	s
Thus since L A T – May 24th	3	12	10
and E =		3	29 08
L M T – May 24th	3h	8m	40 92s

The Year

The unit of measurement for large intervals of time is the *year*

The *solar year* (sometimes called Tropical or Equinoctial) is measured between two successive transits of the sun through the mean vernal equinox, that is to say, the interval in time taken by the earth in describing the whole of its orbit around the sun

There are not, however, an exact number of *mean solar days* in the year, the following being given by Bessel, the accepted authority —

One solar year – 365 2422 mean solar days

Mean and Sidereal Times

Since in the time the earth has described the whole of its orbit round the sun the latter has made one transit less over any given meridian than the first point of Aries—

366 2422 sid days = 365 2422 mean solar days
1 „ – 0 99726957 „
or 24 hours S T = 23h 56m 4 091s M T
1 hour S T – 1h M T – 9 8296s M T (i)
and 1 hour M T = 1h S T + 9 8565s S T (ii)

Equation (i) shows the *retardation* of M T on S T and (ii) the *acceleration* of S T on M T

Tables to facilitate the conversion of the *intervals* of one species of time into *intervals* of the other are given in the 'Nautical Almanac

Example—Convert 18h 4m 51s S T units into M T units

	h	m	s
By 'Nautical Almanac p 576, 18h S T =	17	57	3 07 M T
4m „ =		3	59 34 „
51s „ =			50 86 „
M T equivalent required =	18h	1m	53 27s

Conversion of Mean into Sidereal Time and *Vice Versa*

In practical astronomy we are constantly requiring to compute the S T corresponding to a given M T, or *vice versa*

In such a conversion, it is necessary to refer to some *epoch* which is given in terms both of mean time and of sidereal time

Two such epochs are given in the 'Nautical Almanac,' viz —The S T of mean noon (p II) and the M T of sidereal noon (p III), *i e*, the M T of transit of the first point of Aries both at Greenwich

The *interval* of the known time, which has elapsed since the epoch, is then to be converted into an *interval* of the other time by means of the tables (see example), and the result added to the time of the epoch (given in 'Nautical Almanac') will complete the computation

Now, as the epochs in the 'Nautical Almanac' are given for Greenwich time and date, if the instant we wish to convert is given in local time, it is necessary to reduce the given local time to its equivalent Greenwich instant

This process thus depends on the value of L S T of L M N, and this can always be obtained from the G S T of G M N (given on page II, 'Nautical Almanac') by correcting the latter for the divergence in time between the M T clock and the S T clock, which has occurred in the interval elapsed between the M N at Greenwich and the M N at the locality

The difference in *rates* between the two times (M T and S T) may be taken at 9·86 secs per hour (see equation (ii) p 134), and the elapsed interval in time is the difference in longitude, therefore the above correction = 9·86 secs × longitude in hours and is additive in W longitude and subtractive in E longitude

Example I —CONVERSION OF SIDEREAL TIME INTO MEAN TIME

On the 1st of May, 1898, in longitude 15 E the L S T is 20h 41m 12·65s What is the L M T ?

		h	m	s			h	m	s
Sidereal time to be converted						=	20	41	12·65
G S T of G M N	=	2	37	44·58	L S T of L M N	=	2	37	34·72
Correction for acceleration at 9·86s per hour of long + for W − for E	=		−	9·86					
					Sidereal interval from L M N	=	18	3	37·93
M T equivalents					hours	=	17	57	3·07
					mins	=		2	59·51
					secs	=			36·90
					·93	=			·93
					M T interval from L M N, or local M time		18	0	40·41

Example II —CONVERSION OF MEAN TIME INTO SIDEREAL TIME

On the 1st June, 1898, in longitude 60 W the L M T is 10h 15m 20·73s What is the L S T ?

		h	m	s			h	m	s
Mean time to be converted						=	10	15	20·73
S T equivalents					10 hours		10	1	38·56
					15 mins	=		15	2·46
					20 secs	=			20·05
					·73	=			·73
					Sidereal interval from L M N	=	10	17	1·80
G S T of G M N, 1st June	=	4	39	57·81	L S T of L M N		4	40	37·25
Correction for acceleration at 9·86s per hour of long + for W − for E	=		+	39·44					
					Local sidereal time	=	14	57	39·05

Sidereal and Apparent Times

In computing sidereal time from apparent time, the A T may be first converted into M T and from this the S T computed, or a more direct way may be used by means of the sun's apparent R A

Thus

$$\textit{for the sun} \quad \mathrm{ST} = \mathrm{RA} + t = \mathrm{RA} + \mathrm{AT},$$

where t is the hour angle

SECTION III —THE APPLICATION OF SPHERICAL TRIGONOMETRY TO PRACTICAL ASTRONOMY

It will be seen that at any moment the sun or star in question will form with the zenith and elevated pole a spherical triangle of which the sides will be portions of great circles of the celestial sphere

The sides of the triangle have already been shown to have values as follows —PZ the

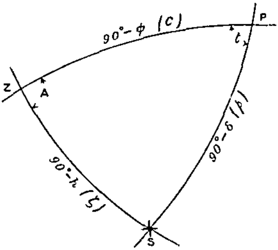


Fig 52

co-latitude $(c) = 90 - \phi$, ZS, the zenith distance $(\zeta) = 90 - h$, and PS the polar distance $(p) = 90 - \delta$

Also the angle at P = hour angle (t) required for time and the angle at Z = azimuth

Knowing any three of the above elements the remainder can be deduced

Then by the general formula for a spherical triangle we get

$$\tan \frac{t}{2} = \sqrt{\frac{\sin (S - p) \sin (S - c)}{\sin S \sin (S - \zeta)}},$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (S - c) \sin (S - \zeta)}{\sin S \sin (S - p)}},$$

where

$$S = \frac{p + c + \zeta}{2}$$

Put for c its value $(90 - \phi)$ and for ζ, $(90 - l)$, then if $S = \frac{h + \phi + p}{2}$, we obtain the following —

(i) For time $\tan \frac{t}{2} = \sqrt{\sin (S - h) \sec (S - p) \operatorname{cosec} (S - \phi) \cos S}$,

(ii) For azimuth $\tan \frac{A}{2} = \sqrt{\sin (S - l) \sin (S - \phi) \sec (S - p) \sec S}$

Solution of Right angled Spherical Triangles—The formulæ for the solution of right-angled spherical triangles are very easily written down by 'Napier's Rules of Circular Parts'

In a right-angled spherical triangle there are 5 so-called 'circular parts' viz, 2 sides, the complements of the two angles, and the complement of the hypotenuse, the right angle being left out of consideration

These are written down in the 5 divisions of a circle in the order in which they occur in *the triangle, any one may be taken as the middle part*, then the two on either side are called the adjacent parts, and the other two the opposite parts, then the rules are—

Sine middle part = product of tangents of the adjacent parts,

" " " = " cosines of the opposite parts

Thus in a triangle ABC where C is the right angle, we have the following figure —

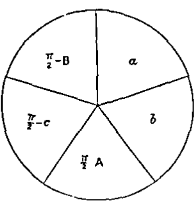


Fig 53

Star at Elongation—Another method of obtaining azimuth is to observe a star at "Elongation" *i e*, when the angle S of the triangle PZS is a right angle

In this case we have

$$\cos t = \tan p \cot c,$$

or (i)

$$\cos t = \tan p \tan \phi$$

Again

$$\sin p = \sin c \sin A,$$

$$= \cos \phi \sin A,$$

whence (ii)

$$\sin A = \sec \phi \sin p$$

From (i) the time of elongation can be computed and (ii) gives the azimuth of the star

Sun or Star on the Prime Vertical.—It is sometimes desirable to observe a heavenly body when it is on or near the Prime Vertical. In this case the angle at Z will be a right angle.

$$\cos t = \cot p \tan c,$$

or (i)

$$\cos t = \tan \delta \cot \phi,$$

also

$$\cos p = \cos \zeta \cos c,$$

or

$$\sin \delta = \sin h \sin \phi,$$

whence (ii)

$$\sin h = \sin \delta \operatorname{cosec} \phi$$

Here (i) gives the time of crossing the Prime Vertical and (ii) gives the altitude of the body at that instant. See plates XXXIII, XXXIV.

SECTION IV.—ELEMENTS FROM THE 'NAUTICAL ALMANAC.'

The 'Nautical Almanac'* supplies us with the data necessary for the solution of problems in field Astronomy.

When the time of observation, for which the quantities are required, is exactly one of the instants for which the required quantity is put down in the 'Nautical Almanac' then we have only to take out this quantity; but as a rule this time falls between two of the instants in the 'Nautical Almanac' and, as a mere inspection of this book shows that the quantities in it are continually changing, we see that a reduction is necessary when the data are required for any instant of time differing from that for which the quantities are registered. This reduction is done by *interpolation* and to facilitate this interpolation, the 'Nautical Almanac' contains the rate of change or alteration of the quantities.

When dealing with with the sun and stars, interpolation by first differences, called *simple interpolation*, is generally sufficient.

Example.—Let it be required to find the declination of the sun at 6 o'clock p.m. on a certain day, when at noon $\delta = + 12° 10' 0''$, and is increasing at the rate of 9 seconds an hour.

The result will be $12° 10' 0'' + (9'' \times 6) = 12° 10' 54''$.

We have seen that difference of longitude corresponds with difference of time; therefore as the data given in the 'Nautical Almanac' are for Greenwich noon they have to be corrected for any other place according to its longitude.

Generally for obtaining a quantity for a certain *local time* we reduce that time to its corresponding *Greenwich* time and take out the quantity accordingly.

The following should be specially noted :—

For each month page I, 'Nautical Almanac,' gives data for G.A.N.
" " page II, " " " " G.M.N.

The R.A. of a star does not practically alter in 24 hours, therefore the R.A. of a star given in the 'Nautical Almanac' for any date is accepted as its R.A. at any other locality on the same date, or within 24 hours of it.

* The 'Nautical Almanac' is sometimes called the 'Astronomical Ephemeris' (ἐφημερος daily); an ephemeris is properly a table of daily positions of the heavenly bodies.

The data commonly required in field astronomy are as follows —

(1) **Sun's R A and δ at time of observation** To find either of these assume your longitude as nearly as possible, turn it into time and add it to your L M T if you are W of Greenwich, subtract it if E

This result is the G M T of observation for which R A or δ can be found in the 'Nautical Almanac,' page II of each month

Example—In longitude 15° E required the R A of sun at 10 hours on May 1st, 1903

L M T	=	10 hours
Correction for longitude	=	− 1 "
G M T	=	9 "

		h	m	s
R A of sun at G M N May 1st	=	2	30	6 23
Correction due to G M T *i e* 9 hours after noon − 9 × 9 5[?]s	=		1	25 68
R A at time of observation	=	2	31	31 91

The variations in 1 hour for R A δ and E are only given on page I 'Nautical Almanac' for apparent time quantities These figures however are equally applicable to the mean time quantities on page II

(2) **Equation of Time** Find G T of observation as above and take out E, by simple interpolation, from the column on page I or II of the month in 'Nautical Almanac' according as it is required for A T or M T Particular attention must be paid to the heading of the columns of the 'Nautical Almanac' from which the sign + or − will be obtained

(3) On page II in the last column is G S T of G M N its hourly variation is 9 86 seconds (see p 134)

On page III of the month in 'Nautical Almanac' is a column headed *Transit of the First Point of Aries* This is identical with G M T of G S N its hourly variation is 9 83 seconds (see p 134)

(4) **The R A and δ of the stars** These are given for all the more important stars under the head *Apparent Places of Stars* They are arranged in the order of their right ascensions for every ten days of the year, and eight stars near to each pole are further detailed for every day of the year Care must be taken to notice the sign N or S against the head of the column for declination It signifies north or south of the equator

It sometimes happens that there are two transits of a star in one day since the sidereal day is shorter than the solar day When this occurs on any day on which the R A and δ are given in the 'Nautical Almanac' both are given in small figures and when the double transit occurs *on an intermediate day* the date alone on which it takes place is given in small figures In the latter case the procedure for interpolation is fully described in the *Explanations* at the end of the 'Nautical Almanac'

The quantities given for R A and δ in the 'Nautical Almanac' under the heading *Mean Places of Stars should not be used in computations*

The Tables I II, and III (near the end of 'Nautical Almanac') for determining the *latitude by observation of the Pole Star out of the meridian* may be used to derive quantities from by simple interpolation

The computation is as follows —

From the observed altitude of *Polaris* corrected for refraction &c is subtracted 0 1 0

To this reduced altitude the further corrections are applied (1) Table I with argument LST of observation, (2) Table II with argument LST and altitude (3) Table III LST and date

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also

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or

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Approximate Details of Certain Theodolites

Instrument	Weight of with Box	Weight of Stand	Price	Telescope Powers	Readings of Verniers	Average Bubble Value	Details
	lbs	lbs	£ s d				
3 inch Transit Theodolite (Vernier)	11	5	22 10 0	about 12	1	1 div 1	
5 inch Transit Theodolite (Vernier)	20	10	31 0 0	12 to 18	30	1 div 30	
* 5 inch Transit Theodolite with Micrometer Microscopes to horizontal and vertical circles	30½	12	42 0 0	16 to 25	10 (estimate to 1)	Vert bubble 1 div 5 plate bubble 1 div = 30	With transit axis level and aluminium star sights
6 inch Transit Theodolite (Vernier)	35	10	36 10 0	12 to 18	20	1 div = 20	With transit axis level

A transit theodolite has already been described and instructions given for its adjustment and setting up (p 15)

Notes on the Theodolite when used in Field Astronomy —

1 All altitudes should be taken on both faces

2 Note whether the eye piece used reverses horizontally or vertically or both

3 When observing the sun the dark glass should be used

4 For observations to stars it is necessary to light up the field of the telescope This may be done in two ways (1) By using the axis lamp which fits on to a small bracket, and reflecting through the axis of the theodolite lights up the field The result generally is far from satisfactory A more certain method is as follows (2) A piece of white paper about 1 inch wide is fitted to the object end of the telescope by means of an elastic band and bent slightly over the object glass On this paper is directed the light from any small lamp held at the side of the theodolite The reflection from the paper then lights up the interior of the telescope A bicycle lamp answers very well for the purpose

Care must be taken to throw just enough light on the field to show up the cross wires without obscuring the star

The 3 inch transit theodolite given in the list is a very good little instrument but is recommended more for rapid triangulation than for astronomical work However for rapid exploration azimuths and latitudes may with care be obtained to within 30

TIME KEEPERS

The time keepers used in field astronomy are either **box chronometers** or **chronometer watches**

A box chronometer is invaluable for stationary work or work on ships, but is not suited for land journeys

* Where weight is not a consideration of the first importance a 6 inch Transit Micrometer Theodolite costing about £50 and weighing about 70lbs is to be preferred especially for astronomical work

A keyless half chronometer watch is recommended for work in the field These watches are less expensive than pocket chronometers are less liable to get out of order, and more are easily repaired They may be carried in the pocket under conditions of rough usage short of actual violence The watches used by the R G S have backs and faces to screw off, and caps to fit on the winding button Such a watch costs from £20 to £30

Both box chronometers and chronometer watches may be regulated to keep either sidereal or mean time

STAR ATLAS MAPS AND CHARTS

A set of star charts is given at the end of this book

From these or similar charts, the *relative* positions of the various stars are seen at once, and then their positions in the heavens can be found by referring these positions to imaginary lines joining well defined stars or groups of stars and following these lines in the heavens

In using a star chart the observer may be supposed to be standing (as he is in reality) at the centre of the celestial sphere, in which case the chart approximately represents the stars as seen when looking upwards When the chart is therefore placed on the table the stars will appear in their proper relative positions but the E and W points will be reversed with regard to the *true* E and W

All observations should be worked out on set forms a sufficiency of which is an important aid to the topographer The following books are required for working out observations —

An Angle Book (for booking observations in the field)
The 'Nautical Almanac or other Ephemeris for current and *next* years
Book of logarithms *
Astronomical Tables (refraction &c —see Tables XIV -XVI)
Computation forms (see Forms K P)
Star Charts (see Plates XXX -XXXIII)

SECTION VI —OBSERVATIONS

Observations in field astronomy consist of determinations of the altitude of the sun or star for latitude and time or for azimuth the horizontal angle between some ' referring object and the sun or star, in addition, at the same instant

A Star through the theodolite appears as a bright point It is only necessary therefore to bisect the star with the centre horizontal cross wire of the theodolite when taking altitudes and with the vertical wire when taking horizontal angles Use the same part of the wire on each face

The Sun however is a large body, and hence, in order to make all observations possible of comparison the centre must be observed As however we cannot tell the precise instant the wire of the theodolite crosses the centre we are reduced to observing one edge or ' limb as it is called For altitudes, the time of contact of the horizontal wire with the upper or lower limb as seen through the theodolite is noted In the same way, when taking the horizontal angle between some ' referring object ' and the sun the right or left limb is observed by making a contact with the vertical wire

* It saves an enormous amount of time and trouble to use a logarithm book which gives the logarithms of the trigonometrical functions for each second of arc *e g* Shortrede's log tables

In order to do away with the necessity for applying a correction for the sun's semi diameter, an equal number of observations on opposite limbs are made, that is, "upper" and "lower" for altitudes, and "right" and "left" for azimuth angles

It is advisable to record the limbs in the Angle Book as they *appear* through the telescope, and state which way the eye piece reverses This last, however, is not always necessary, if the "pairing" of observations is strictly adhered to

As already explained, *collimation errors are eliminated by taking an equal number of* observations on each "face" of the theodolite No altitude obtained from a theodolite on one "face" only is of any value

DETERMINATION OF TIME

Time by Single Altitudes

We have seen that time, either sidereal or solar, is dependent on the hour angle of the 1st point of Aries or of the sun Since we are able by means of astronomical observations of the sun, to compute its hour angle, the solar time is readily found, and so with a star

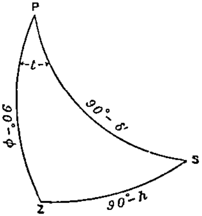


Fig 54

Time is known when we have determined the hour angle of the sun or a star If we adjust a theodolite so that the centre wire describes the plane of the meridian, the determination of the instant of passage of sun or star (with a knowledge of the R A) gives the simplest way of computing the time But this is rarely possible in the field However since we are able by an observation of altitude of sun or star, when some distance from the meridian to deduce its hour angle, we have a ready means of determining the time at the instant of observation

An examination of the astronomical triangle PSZ (see fig 54) shows us that the value of the hour angle ZPS depends to a great extent upon the side ZS for of the other sides PZ is constant, and PS varies only slightly with the change of declination during the time taken in making an observation Now ZS is the complement of the altitude and thus in a position of the heavenly body such as is expressed in the above figure the hour angle changes in accordance with the changes of the altitude

Obviously, so far as concerns accuracy of observation the most favourable period to observe altitudes for time will be when they are changing as rapidly as possible *i e*, *when for*

a given change of altitude the corresponding alteration in time is as small as possible, for as in all observations certain imperfections and inaccuracies are to be expected, we must choose a period for our observations when these errors will have the least injurious effect on our results

Selection of Stars—*In selecting a star for the determination of time by single altitudes choose one on or near the Prime Vertical (*i e*, nearly E or W) It is easy to show that small errors in the observed altitude, or in the assumed latitude produce the least effect upon the computed time when the star is on the Prime Vertical *See Table on p* 160

For convenience of observation, and to avoid excessive refraction, the star should have more than 15° and less than 45° altitude

When a star is on the Prime Vertical, it necessarily follows that its declination must be of the same name (*i e* additive for N and subtractive for S lat), and less than the latitude of the place of observation

The diagrams on Plates XXXIV and XXXV show at a glance the Hour Angle of a heavenly body when on the Prime Vertical and its Altitude at that instant The position of a star at any instant can be found from a knowledge of its R A (*i e*, L S T, of transit) and declination The Nautical Almanac and star charts should therefore always be consulted

Example —It is required to take observations at about 8 P M in lat 40° N Now on looking up the G S T of G M N (p II, 'N A') and correcting it at the rate of 9 86 secs per hour of longitude (W additive E subtractive), we find the L S T of L M N to be 10h 25m 30s and therefore the sidereal time of 8 P M is 18h 25m 30s The star chart shows a large star with declination 25° N and R A 14h 55m 30s

Referring to Plate XXXIV it will be seen that with this declination and in latitude 40° a star will cross the P V 3h 45m before and after meridian transit The star will therefore be on the P V in the west at 18h 40m 30s L S T that is at 8 15 P M local watch time

The time at which the observation should be made is thus easily found and it is seen from Plate XXXIII that the star has a suitable altitude at that instant, in this case 41°

At or near the equator, the above rules cannot be applied, but any star whose meridional zenith distance $(\pm\phi) - (\pm\delta)$ does not exceed 15° may be used

Pairing Observations—When taking observations for time (and also for azimuth), one set should be taken on an east star, and one on a west star Stars should therefore be selected beforehand to enable the observer to make the observations consecutively

Time should be found by either of the methods hereafter described to an accuracy of less than one second of time

Chronometer Error—The object of the observation is to obtain and record *the error of the chronometer*, which for any instant is the difference between the time it shows and the time it should show, this error being styled *fast* when it is ahead of proper time, and *slow* when it is behind

The error of the chronometer thus obtained is that *at the mean of the times of observation* This moment should be noted to the nearest minute

Data Required.—For this purpose, besides our observed altitude, we must know two other elements one of these must be PZ, the co-latitude, the other PS the polar distance, which can be derived from the 'Nautical Almanac'

A small error in the assumed latitude, and even a considerable error in the assumed longitude would produce no practical error in the result

* The selection of a star near the Prime Vertical is a counsel of perfection Practically all that is required is to choose a star well to the East or West and between 15° and 45° in altitude

The following equation is used in the computation for time (see p 136) —

$$\tan \frac{t}{2} = \sqrt{\cos S \sin (S - h) \operatorname{cosec} (S - \phi) \sec (S - p)}$$

where

$$S = \frac{h + \phi + p}{2}$$

Let ϕ always positive and p reckoned from the *elevated* pole

NOTE —If a number of observations of the same star at the same place are to be individually computed it will be most readily done by the fundamental equation

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

here ϕ and δ are constant for all the observations and it only remains to compute $\sin h$

Instructions for Observing (*General*) —For precision several altitudes will be taken and the mean of these with the corresponding mean of the times noted will be taken for computation If the time assumed is much out the computation must be repeated with new functions obtained by means of the corrected time

The most favourable position for the observer for taking this observation is on the equator, the worst at either of the poles

When the latitude is undetermined a small error in its assumed value has no appreciable effect on the results of the observation if the stars taken are near the prime vertical (see p 160)

To eliminate instrumental errors errors of refraction and assumed functions, observations must be taken in pairs on either side of the zenith, in the case of the sun by morning and afternoon observations in that of the stars by two stars one east and one west of the meridian

With Theodolite —When observing the sun adjust the instrument turn the telescope on the sun and make contact with the upper or lower limb on the centre horizontal wire noting the time of observation and the readings of both ends of the level and of the vertical circle, next change face and make contact with the other limb, noting readings as before

The procedure for observing a star is the same except that the observations are taken by allowing the star to make its own passage across the centre horizontal wire near to the vertical wire Four* observations complete a set half on each face of instrument, thus F R F L, F L F R

Time by Equal Altitudes

If a star, the declination of which is constant during the observation be observed at the same altitude before and after transit over the meridian it is obvious that the hour angles of the star at both instants of observation must be equal to each other though on opposite sides of the meridian, and that the mean of these instants will give the actual time that the star is on the meridian

As a star rises in altitude we may record an instant (say t) which by our timekeeper corresponds to the time that the star reaches a certain altitude then by waiting till the star has crossed the meridian and descends to the same altitude and again recording the instant (say t') we can find the moment of transit which should have been $\frac{t + t'}{2}$ A comparison of this time with the R A of star would determine for us the error of our timekeeper

This method of equal altitudes is independent of an accurate knowledge of latitude or declination or of errors in the graduation of the instrument It is (with a star) very simple

* Clearly all that is absolutely necessary is F R, F L

in its procedure but requires a clear sky for a considerable interval of time, and for altitudes less than about 45° it is liable to variations on account of changes in temperature, and, therefore, of refraction

When the sun is used with this method, a correction has to be made for the change of declination that occurs between the times of observation before and after the passage over the meridian

Having obtained the mean of the two times of observation, it may be corrected to the time of apparent noon *i e*, time of transit by the following formula —

$$\frac{\pm t \times d\delta}{15}(\tan\phi \operatorname{cosec} t - \tan\delta \cot t),$$

where

t — half the interval between the two observations,

$d\delta$ = the change of declination in one hour,

δ — the declination of the sun when on the meridian

This correction is positive when the sun is leaving the elevated pole, negative when it is approaching it

In observation by equal altitudes of both stars and sun, instead of one observation only being made each side of the meridian it is necessary, for precision to obtain a set as in observing for absolute altitudes The instrument must be set forward and readings taken at intervals of 10, 20, or 30 minutes of arc, on the upward journey of the heavenly body, and to these readings the vernier can be easily adjusted on the downward journey of the sun or star The mean time of each set of observations will then be used as if they were the instants corresponding to one pair of altitudes only

Comparison of Watches

Suppose it is intended to compare a chronometer watch (a) and a chronometer watch (b) The observer A watches the seconds hand of (a), while B watches (b) When the seconds hand of (a) appears to cross a definite division, A makes a rap on the table, B notes the time of this by (b) to the nearest $\frac{1}{10}$th second

B then signals in a similar manner to A, who notes the time of the signal

They then change watches and go through the same procedure

The mean of these four determinations will give a very fair comparison

Comparison of Box Chronometers

In order to get a good comparison, it is desirable that at least one of the chronometers should be sidereal, if the others are mean, and *vice versa*

Box chronometers always tick half seconds

As the difference of the two times, sidereal and mean, is approximately 10 seconds an hour and the clocks tick half seconds, a coincidence will occur every 3 minutes

Place the two chronometers on the same table, and take up the time on one counting one half two half three, &c When the coincidence appears exact, while still counting, note the time shown by the other chronometer

Comparison of Watch and Chronometer

Owing to the fact that a watch beats five times in 2 seconds, it is impossible to get a satisfactory coincidence It is therefore best to compare them in the same way as two watches

DETERMINATION OF AZIMUTH

To determine the direction of the meridian line or the azimuths of any number of points from a station, it is usual to select some well defined terrestrial object suitably situated to

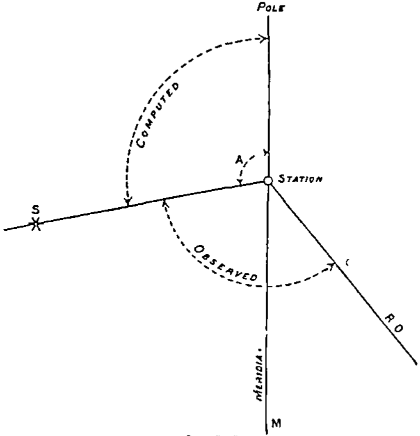


Fig 55 *

which all azimuth angles can be referred Such an object is called **"The Referring Object"** (R O)

In all **astronomical observations for azimuth** the procedure adopted is, by means of an observed altitude, to *compute* the azimuth (A) of the sun or star (S) (see fig 55) and to *observe*, at the same time, the azimuthal angle between the R O and the sun or star From these two angles (one *computed* and the other *observed*) it is simple to deduce the azimuth of the R O

The computed azimuth is always measured from the elevated pole, the shortest way

If we can determine the precise instant a heavenly body is on the meridian the determination of azimuth is simple, and can be done by measuring the horizontal angle at that instant between the heavenly body and the R O

But, when on the meridian, a heavenly body is moving at the greatest possible rate in azimuth, so this method is impossible and it is necessary to select a time for observing when a small error in altitude or time will have the least possible effect on the result

Stars should be observed for azimuth at or near elongation, and sun or stars on or near the prime vertical As regards the latter practically all that is required is to choose a star well to the east or west and between 15° and 45° in altitude

* Azimuths used for the purposes of triangulation as described in Chapter III are measured from the South by the West Thus if the deduced azimuth of the R O in fig 55 were 150 the azimuth used for triangulation would be 330 (the North being the elevated pole in this case)

(5455)

AZIMUTH BY OBSERVATION OF SUN OR STAR EAST OR WEST

As regards the selection of stars for observation by this method the condition that they are to be well to the E or W necessitates their selection being made in the same way as those required for the determination of time (*q v*)

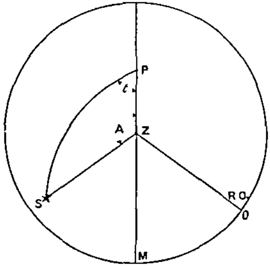


Fig 56

For the purposes of the determination of azimuth the theodolite having two circles a horizontal (or azimuthal) and a vertical (or altitude) circle has particular advantages

In fig 56 S represents the sun or star O the referring object and P the pole

We can *observe* the angle SZO with the theodolite and there only remains for us to *compute* the angle SZP *i e* A or the azimuth of the sun or star

In the astronomical triangle *we have the following quantities* —

$$PS = 90 - \delta,$$
$$PZ = 90 - \phi$$
$$ZS = 90 - h$$
$$\text{Angle } SPZ = t$$

It is required to compute the value of the angle SZP, this can be done with either of the two following sets of data —

(*a*) The three sides as above

(*b*) *The two sides (90 – δ) (90 – φ) and included angle "t"*

ASTRONOMICAL FORM 3 utilises the data (*a*) in the following equation and can be employed with either sun or star observations —

$$\tan \frac{A}{2} = \sqrt{\sec S \sec (S - p) \sin (S - \phi) \sin (S - h)}$$

where

$$S = \frac{h + \phi + p}{2}, \text{ and A is the angle SZP}$$

Instructions for Observing for Azimuth —The R.O selected should be suitable for intersection with the cross wires and as far off as possible, so that solar focus may be used At night the R.O will be a light not less than 400 yards off

In all cases care must be taken that the lower plate remains clamped throughout the set of observations

It is not advisable or necessary that the R O should read exactly zero

In all azimuth observations an endeavour should be made to take as many observations on one "swing" as on the other "swing"

The observed altitudes and azimuths are simultaneously recorded

The times are not absolutely necessary except for sun observations (to correct δ) but are usually recorded, as then a fair value for time (which is always useful) can be computed on Astronomical Forms 1 or 2 from the observed altitudes

The procedure in observing **a star** for azimuth is as follows —

Note the reading of the R O on the horizontal arc and turn the telescope on the star, take an observation with the intersection of the centre cross wires and note the time, the readings of both ends of the bubble, and of the vertical and horizontal arcs Now reverse the telescope, and take another observation of the same star, noting time bubble readings and vertical and horizontal arcs, as before, then read the R O This completes one set of observations on a single star

The procedure for obtaining *simultaneous* azimuth and altitude readings **with the sun** is as follows —

Place the *horizontal* wire in *advance* of the limb to be observed and allow it to make its own contact just before the moment of contact keep the *side* limb to be observed in contact with the *vertical* wire by means of the slow motion screw, just as the contact with the horizontal wire is completed remove the hand from the vertical wire slow motion screw note the time and then read level error, vertical and horizontal angles

The following order of observations should be followed —

> Read R O
> Observe (as above) the right upper limb
> Reverse the telescope
> Observe (as above) the left lower limb
> Read R O

This completes one set of observations

A rough diagram should always be drawn in the angle book, showing the relative positions of the observer sun or star and referring object, and also the meridian line

In rapid triangulation it is frequently of the greatest importance to determine the azimuth of some side of the triangulation In such cases it is often convenient to determine this by observations of the sun in the afternoon Three sets should be taken, and each set worked out independently, so as to obtain a check

Azimuth by Circum Polar Star at Elongation

A Circum polar Star useful for this observation is one with a polar distance of less than 10° As already explained a star is at elongation when the angle at S in the triangle PZS is a right angle

The time of elongation is computed from the formula $\cos t = \tan \phi \tan p$ Then the angle between the R O and the star is observed at this time

The azimuth of the star can be computed for the same moment from the formula $\sin A = \sec \phi \sin p$

It is sufficient to take an observation on each face within a few minutes of the computed time of elongation Only the times and horizontal angles need be recorded Thus the observations would be —Face right, R O, star Face left, star, R O With a quick observer these might be repeated

The means of the angles between the R O and the star can then be taken, provided that in ordinary latitudes no time differs from that of elongation by 5 minutes either way This gives a range of 10 minutes for observing

The angle A is then added to or subtracted from this observed angle to obtain the azimuth of the R O A rough diagram should be drawn

The altitude of the star at elongation is given by, $\sin h = \sin \phi \sec p$

The method is an easy one, both as regards observing and computing and gives good results provided the time limits are not exceeded

The computation is so simple that no example is thought necessary

Rough Methods

The foregoing observations occupy considerable time in both observing and computing, and a shorter and simpler method of obtaining azimuth is sometimes necessary

There are two such methods of obtaining an approximate azimuth by circum polar stars, at culmination and at elongation these are as follows —

Circum-polar Star at Culmination (rough)

Compute the chronometer time of upper or lower transit of Polaris, or some other known circum polar star As this time approaches, bisect, with the central wire of the telescope, the R O and read the horizontal arc Then at the computed instant of the star's culmination bisect the star with the central wire and read the arc Observe the R O again The difference between the reading of the star and the mean of readings of R O will give the azimuth of R O E or W

Repeat the process with other circum polar stars if required

Circum polar Star at Elongation (rough)—Assume the elongation to be *about* 6 hours from the time of transit easily found from the R A given in the 'Nautical Almanac'

As nearly as possible *at this time* take a set of observations to obtain the horizontal angle between the R O and the star

Now the azimuth of a star at elongation is obtained from the relation $\sin A = \sin p \sec \phi$

A fair value for the azimuth of the R O is obtained, therefore, by adding or subtracting the angle A from the *greatest* of the observed angles, according as the elongation is E or W

In the northern hemisphere there are two stars, α and δ, *Ursæ Minoris*, which come to elongation at intervals of approximately 6 hours from each other One of these can therefore always be found at elongation in the first half of the night In the southern hemisphere there are two suitable stars *β Hydri* and *β Chamæleontis* each about 12° from the pole, and of the third and fourth magnitude respectively

A plan of the constellation *Ursa Minor* is given in fig 57 which may assist in the identification of the several stars



Fig 57

Ursa Minor

The elements of all these seven stars, except η which is a double star, are given in the 'Nautical Almanac'

DETERMINATION OF LATITUDE.

General Remarks

In fig 58, in which the plane of the paper is supposed to coincide with the plane of the meridian —

It is evident that the *latitude of a place* is equal to the *declination of the zenith* EOZ

It is also the *altitude of the pole* NOP, for in fig 58, ZE = latitude of the place = ϕ, and

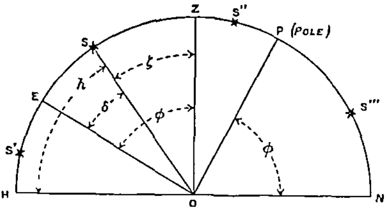


Fig 58

therefore PZ = 90 − ϕ, but PN = 90 − PZ Hence PN or PON, which is clearly the altitude of the pole, is equal to ϕ

Let h = altitude of a heavenly body

ζ = meridian zenith distance This is $\pm$ when the heavenly body is $\frac{S}{N}$ of zenith

δ = declination, ϕ = latitude of place } These are $\pm$ when $\frac{N}{S}$ of equator

Then it is clear from the figure that $\phi = \zeta + \delta$

If the heavenly body be—

at S ζ is +ve, δ is +ve
If at S ζ is +ve, δ is −ve
,, S ζ is −ve, δ is +ve
,, S ζ is −ve,* and δ is 180—true δ

In the first two cases the altitude (h) is observed while facing south
,, last ,, ,, ,, ,, ,, north

Now suppose the observations are made in *south* latitude (see fig 59) Then if the heavenly body be at S we have $\phi = \zeta + \delta$, whence giving ζ and δ *their proper signs*, ϕ *if* south must come out negative

From the above results it will be seen we get the following general formula giving ϕ, ζ, and δ, their proper signs —

$$\phi = \zeta + \delta \qquad \ldots\ldots\ldots\ldots \quad (1)$$

* δ is here measured from E through the zenith and elevated pole, and is equivalent to the angle S'' OE, that is to say when a star is taken at its *lower* transit the δ is obtained by taking the supplement of the value given in the 'Nautical Almanac'

We have thus an easy method of determining the latitude, viz —by measuring the altitude of any celestial body at the exact moment when it is on the meridian, δ being obtainable from the 'Nautical Almanac,' if we know the approximate Greenwich time of observation

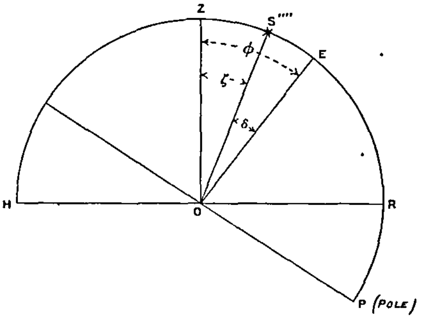


Fig 59

Latitude by Meridian Altitudes

This is the simplest way of determining latitude though not the most accurate

First calculate the *time of transit* of the sun or star (see p 153)

Then, having adjusted the theodolite, commence observing a minute or so before the time of transit and take a series of altitudes F R, F L, F L, F R, F R, F L, &c, noting the times The mean of the pair of observations (F R and F L) nearest to the time of transit is to be taken as the observed meridian altitude Or, if the error of the watch is not accurately known the pair which gives the *greatest* altitude must be taken (In the case of the sun, the correction for semi-diameter must first be applied in order to ascertain this)

Correct the meridian observed altitude so obtained, for level error, refraction (and parallax in case of the sun) and from this obtain the true meridian zenith distance (ζ) Then, as before —

$$\phi = \zeta + \delta \qquad \text{(taking care of the signs)}$$

To get good results from meridian altitudes, the following rules should be observed —

(1) No altitude should be less than 40°

(2) Equal numbers of North stars and South stars should be observed

(3) The North and South stars should be balanced, *e g*, if a North star of altitude 40 is observed, try and find a South star of the same altitude

It is clear that a sun altitude will be unbalanced and that a latitude by meridian altitude of the sun can only be an approximation

Latitude by Circum Meridian Altitudes

When a number of altitudes are observed very near the meridian, they are called *circum meridian* altitudes

This method is an accurate and satisfactory one

Each altitude reduced to the meridian gives nearly as accurate a result as if the observation were taken on the meridian

It can be shown by investigating the formula that near the meridian, the correction varies as the *square* of the hour angle and not simply in proportion to the time. Hence it is that near the meridian we cannot reduce a number of altitudes by taking their mean to correspond to the mean of the times as is done (in most cases without sensible error) when the observations are remote from the meridian. We are obliged therefore to reduce *each* altitude separately and take the mean of all the results.

(*a*) The zenith distance should not be greater than 50° and under no circumstances less than 10°.

(*b*) The hour angles should not, as a rule, exceed 10 minutes.

(*c*) The closer the meridian (and therefore the more rapidly the observations are taken) the better will be the results.

(*d*) Equal numbers of North stars and of South stars should be observed.

(*e*) The North and South stars should be balanced.

Selection of Stars.—When selecting stars care must be taken to choose those which transit at a convenient time. The stars chosen should have zenith distances less than 50°, and greater than 10°. This can be found from the equation

$$\zeta = \phi - \delta$$

by giving ϕ an approximate value and carefully looking to the signs of ζ, δ and ϕ. The altitude (h) of the star at transit will be 90 – ζ.

Time of Transit.—The time of meridian transit is obtained in the case of a star, from the R.A. given in the 'Nautical Almanac' which is the same thing as the L.S.T. of transit. This last can be reduced to L.M.T. from a knowledge of the G.S.T. of G.M.N. p. II 'Nautical Almanac' (see Astronomical Form 4).

With the sun the L.M.T. of transit is obtained from the *equation of time* (p. I, 'Nautical Almanac') corrected for longitude. If a sidereal clock is used the L.S.T. of transit will be the R.A. of the sun as given in the 'Nautical Almanac' corrected for longitude (see Astronomical Form 5).

PRACTICAL HINTS ON OBSERVING FOR LATITUDE

Programme of Work.—Before commencing observations for latitude an observer should make out a programme of the observations he intends to take, determining the approximate time of each by his watch and selecting such stars as will enable him to carry out his programme at convenient hours.

As the travelling rate of the chronometer can rarely be depended on, observations for time should always form part of the programme; these can be taken at any convenient time while waiting for the culmination of latitude stars.

For example in latitude 40° N. on the 1st January observations are required for latitude. The sidereal time at mean noon on that day is 18 h. 50 m. and supposing we wish to begin at dark and complete the observations before 10.30 P.M. we must choose stars for latitude which culminate between 0 h. and 5 h. 30 m. sidereal time.

The zenith distance of a star at culmination is found from the equation $\zeta = \phi - \delta$ where the signs of the several quantities must be carefully attended to.

Then knowing that the latitude is approximately 40° 0' and looking out the declination of α Cassiopeia from the 'Nautical Almanac' we obtain the approximate zenith distance of this star thus:—

$$\begin{array}{rl} \phi & + 40° \quad 0' \\ \delta & + 55 \quad 58 \\ \hline \phi - \delta = \zeta = & - 15 \quad 58 \text{ north of the zenith} \end{array}$$

Similarly we shall find for—

α Arietis	ζ = − 17	3 south
ε Cassiopeiæ	ζ = − 23	9 north
α Tauri	ζ = + 23	42 south
Polaris	ζ = − 4[illegible]	45 north
β Orionis	ζ = + 4[illegible]	19 south

forming three pairs of approximately equal north and south zenith distances.

As regards the times of transit —If the watch is M.T. look out in the "Nautical Almanac" the G.S.T. of G.M.N. and correct this for longitude at the rate of 9·86 seconds per hour (W + E −). This will give the L.S.T. of L.M.N. Now the R.A. of the star is its L.S.T. of transit, hence the difference of the R.A. and the L.S.T. of L.M.N. gives the sidereal interval from L.M.N. which can then be reduced to a M.T. interval and used with a M.T. watch (corrected for any known error). If the watch shows S.T., no computation is necessary except to apply any known error.

The programme can then be drawn up as below entering altitudes instead of zenith distances.—

Time	Star	Mag.	Aspect	Altitude	Remarks
h. m.					
[illegible] 41	α Cassiopeiæ	Var.	[illegible]	74 [illegible]	For latitude circum-meridians.
6 31	Polaris	2·2	[illegible]	41 15	
6 56	ε Cassiopeiæ	3·6	[illegible]	66 51	
7 11	α Arietis	[illegible]·0	[illegible]	[illegible] 58	
9 39	α Tauri	1·0	[illegible]	67 18	
10 19	β Orionis	0·3	S	41 41	

E. and W. stars suitably placed for time observations can be observed between the observations to α Arietis and α Tauri.

Instructions for Observing Latitude.—The observer should commence observing altitudes about 15 minutes before the transit, reversing the telescope after the *first* observation and then after *each pair* of observations.

In the case of the sun, altitudes are taken to the upper and lower limbs alternately.

The observations should be continued till it is quite evident that the sun or star is falling and that some 10 minutes has elapsed since transit.

As the altitudes are changing very slowly contacts have to be *made* by the slow motion screw.

At the time of observing then set the vertical circle to the proper altitude (h) and turn the telescope a little to the east of the meridian, the star should then be in the field of the telescope. The altitudes of the star on both faces are recorded in the *field book*, and the star is easily found by setting the vertical circle to these altitudes approximately.

The most favourable series of observations can afterwards be selected to compute from, as no greater accuracy can practically be obtained by computing from a large number of observations in preference to six or eight well situated, *i.e.*, near the meridian.

It is not necessary to have an *equal* number of [illegible] on either side of the meridian, but the better they are *balanced* the better will be the results. [illegible] example, three observations on one side of the transit and five on the other may be said to be well balanced.

The Chronometer Error or correct local time at the mean of the observed times must be known and is usually obtained by an independent set of observations (see p. 143).

The chronometer error has to be applied to the *true time* of transit to obtain the *chronometer time of transit*

The *Rate* of chronometer is practically only required when the chronometer or watch used is unreliable, or when the *interval* between the time of transit and the time at which the chronometer error was determined exceeds one or two hours

Example —*Error* of watch determined by set of observations for time at 20h 5m 0s was found to be 2m 20s *slow*

Rate of watch ascertained to average about 1 3s per hour, *gaining*

Mean of times of transit 24h 10m 0s on the same day

We have therefore —

Watch gained in elapsed interval 4 1h × 1 3s = 5 3s

Watch *error at time of transit* = 2m 20s less 5 3s

= 2m 14 7s *slow*

In observation with a star the chronometer error is generally obtained by an independent set of observations, within one hour or less of the circum meridian altitude observations, and any good ordinary watch may be relied on to keep sufficiently accurate time for such a small interval without any correction for *rate*, but it is advisable to ascertain the rate in case it should be unusually high No good ordinary watch should gain or lose more than 12 seconds per diem

The notation and formulæ employed in the computations for latitude by circum meridian altitudes are as follows —

ϕ = latitude N or S ζ = mean zen dist δ = dec at mean of times of obsn

ϕ_1 = approx lat ζ_1 = approx mer zen dist

$m = \dfrac{*2 \sin^2 \frac{t}{2}}{\sin 1}$ Z_1 = correct do do do

m_1 = mean of the values of m (computed separately for each hour angle)

$$\phi_1 = \delta + \zeta_1 \text{ and } \phi = \delta + Z_0$$

$$Z_0 - \zeta_0 \quad Am_0 \text{ and } A = \frac{\cos\phi_1 \cos\delta}{\sin\zeta_1}$$

As regards signs, treat A and m_0 as +ve Z_0 must be less than ζ_0

In this method of reducing circum meridian altitudes the computation of the correction Am_0 can be repeated a second time with the computed values for the latitude and zenith distance, instead of the *approximate* ones ζ_1 and ϕ_1, and a closer result obtained

Astronomical Forms 4 and 5 provide for the computation of latitude from circum meridian altitudes of the star and sun respectively

Latitude by Pole Star

When the correct local time and the approximate longitude are known the latitude may be found from observations of the altitude of the *Pole Star* at any time

To obtain the correct local time it is usual to take an independent set of observations for *time*, either immediately before or after the observations for latitude

The following formula is employed —

$$\phi = h - p\cos t + \tfrac{1}{2} p \sin 1 \sin^2 t \tan h$$

The hour angle t is to be deduced from the sidereal time, and the star's right ascension by the formula—

$$t = \text{S T} - \text{R A}$$

Calculation Form 8 provides for the reduction of observations by these formulæ, and also by Tables I, II, and III at the end of the 'Nautical Almanac' The latter tables are useful when great accuracy is not required, but when precision is necessary it is best to work out the

* See Table XVI.

formula given above, as interpolation in the tables can only be accurately done by means of second differences—a somewhat lengthy calculation

Latitude by Rough Methods

A useful method of finding latitude which does not entail a knowledge of the declination of any celestial body is the following —

The latitude of a place is equal to the altitude of the pole at that place Hence if we take a circum polar star and observe its altitude at both upper and lower transit the mean of these altitudes (corrected for refraction, &c) will give at once a value for the latitude of the place Since however two such observations at upper and lower transit must necessarily be separated by a period of 12 hours, it is not always practicable to make them

A single observation of the altitude of a star at time of its transit gives the latitude readily by the formula—

$$\phi = \zeta + \delta,$$

but it must be recollected that when the star is observed at its lower transit, δ then is taken as the supplement of the declination given in the 'Nautical Almanac

Another rough method has already been described on p 152

SECTION VIII —GENERAL PRACTICAL HINTS IN FIELD ASTRONOMY

(*a*) Before taking observations for *time* or *azimuth* suitable stars one E , and one W , may be selected from the star chart or else stars may be picked up east and west of the observer in the sky It is necessary of course to know the names of the stars used In every case where observations for *time* are taken, comparison of watches should be made, either before or after observation

(*b*) For *latitudes* the altitude and chron time of transit of the star must be worked out previous to observation (see p 153 and Ex Form 6)

(*c*) Before commencing work an observer should make out a programme of the observations he intends to take determining the approximate time of each by his watch, and selecting such stars as will enable him to carry out his programme at convenient hours

(*d*) Observations should always be made in pairs, with face right and left alternately, so as to eliminate collimation and level errors

The same principle should be adopted in selecting stars for observation, thus for *latitude*, pairs of stars of opposite and approximate equal zenith distance should be observed and for *time* and *azimuth* pairs of stars east and west

(*e*) A booker should always be employed if possible

(*f*) Observations should be made in the following sequence —

1st The observer carefully sets up and adjusts the theodolite

2nd The observer informs the booker what the observation is for and the name of the star used The booker enters these in the angle book and also the following data the approximate watch correction, the thermometer and barometer readings the value of one division of the vertical circle level the place date and referring object when required

3rd The observer gets the star into the field of the telescope telling the booker to stand by ' On making a contact the observer says ' up *

4th The booker takes the time and records it

5th The observer reads out the level readings always giving that at the eye end of the theodolite first Thus ' E, 7 4 , O 5 2 '

6th The observer reads out the vertical angles, C and D verniers

7th The observer reads out the horizontal angles A and B verniers

* Some observers consider it better to use a word beginning with a dental consonant such as ' tip '

(*g*) The booker must always repeat the readings after he has written them down to show that he has got them correctly

The booker should remind the observer each time he is about to observe what face should be used and for sun observations what limb (upper, lower right or left) he is to take To simplify this the limbs and face to be used should be written down in the angle book in their proper order for the particular observation before observing

(*h*) The only occasion where both vertical and horizontal angles are simultaneously recorded is for azimuth observations Computation Form 4 The method of making the contacts has already been explained p 149

(*i*) After observing everything required by the angle book form should be inked in and the book signed This is to ensure that no necessary data are omitted

IDENTIFICATION OF STARS

When making observations of the stars for time latitude or azimuth it is very helpful to be able by mere inspection of the heavens to pick out the star previously determined on for observation or at any rate the constellation in which that star exists

This involves a knowledge of the relative positions of the larger stars in the different constellations

Star charts are as a rule on stereographic or globular projections and do not represent satisfactorily the relative positions of stars

For this reason a short description of the positions of some of the principal stars is given

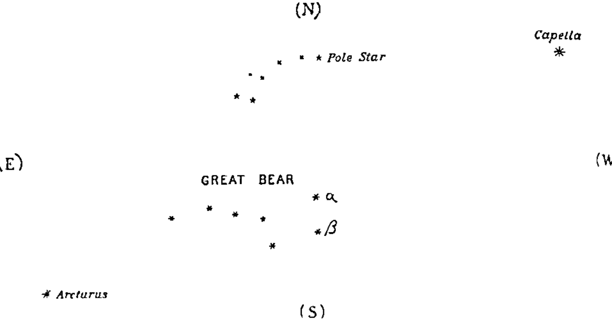


Fig 60

It must be understood that when speaking of a straight line between two stars what is meant is the portion of a great circle of the celestial sphere intercepted between and passing through those stars A little consideration will show that owing to the position of the observer being at the centre of the sphere a diagram of what is seen will have the E and W points interchanged

The angular distance between two stars can be very fairly accurately estimated, if one compares it with half the distance from the point in the heavens above the head to the horizon which of course represents 45°

(I.) Referring first to the constellation "Ursa Major" or "Great Bear," the two stars α and β point nearly to *Polaris* or the POLE STAR The Pole Star itself forms the end of the tail of the "Little Bear" or "Ursa Minor"

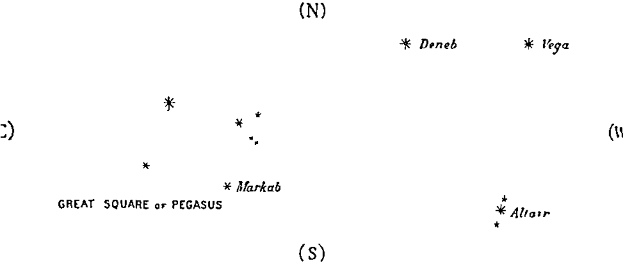


Fig 61

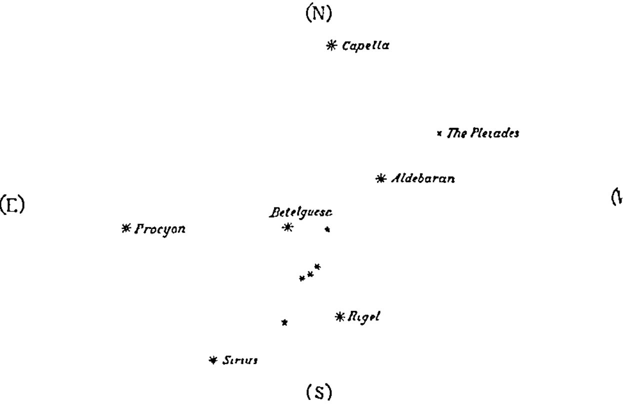


Fig 62

The stars α and β of the "Great Bear" are generally known as the pointers

(2) If the curve formed by the tail of the "Great Bear" be continued away from the pointers, the bright star ARCTURUS or α *Bootis* will be seen at about 30° from the end of the tail

(3) A line drawn from the Pole Star perpendicular to the line from the Pointers passes through CAPELLA or α *Aurigæ*, which is distant about 50° from the Pole Star

(4) A line from the Pole Star through the last star but one of the tail of the Great Bear will, if produced, pass through SPICA or α *Virginis* This star will be found to be about 30° beyond Arcturus

(5) In a line with the pointers, on the opposite side of the Pole Star, and distant about 60° from it, will be seen the "Great Square of Pegasus" formed by three stars of that constellation, and one, nearest the Pole, belonging to "Andromeda" The star at the opposite corner is called MARKAB

(6) Midway between the "square" and the Pole Star will be noticed the constellation "Cassiopeia," formed by five stars in the shape of a W

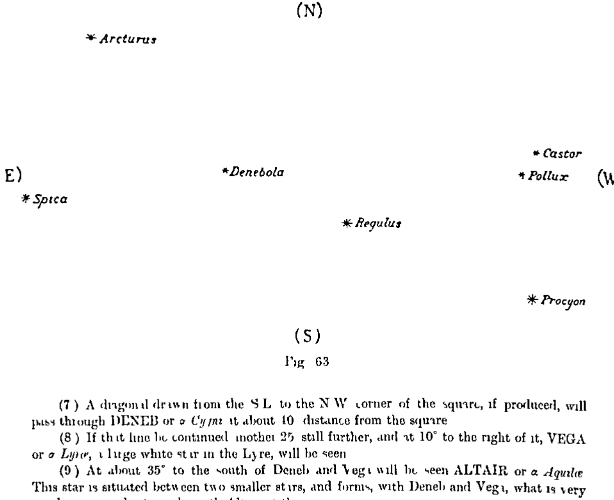


Fig 63

(7) A diagonal drawn from the S E to the N W corner of the square, if produced, will pass through DENEB or α *Cygni* at about 40° distance from the square

(8) If that line be continued another 25° still further, and at 10° to the right of it, VEGA or α *Lyræ*, a large white star in the Lyre, will be seen

(9) At about 35° to the south of Deneb and Vega will be seen ALTAIR or α *Aquilæ* This star is situated between two smaller stars, and forms, with Deneb and Vega, what is very nearly an isosceles triangle, with Altair at the apex

(10) A line from the Pole Star through Capella will touch RIGEL, a star in "Orion," distant about 65° from Capella This line also passes between two other bright stars, ALDEBARAN or α *Tauri*, 10° to the west, at about 30° from Capella, and BETELGUESE in Orion, 10° to the east, at about 40° from Capella

(11) A line from Aldebaran through "Orion's Belt" passes close to SIRIUS or α *Canis Majoris* the brightest star in the heavens, and at about an equal distance on the other side of the belt This star, with Rigel, Betelguese, and Aldebaran, forms a trapezium, having "Orion's Belt" in the centre of it

(12) At about 50 to the east of Betelguese will be seen PROCYON or α *Canis Minoris* This star, with Betelguese and Sirius, forms a very nearly equilateral triangle

(13) About 30° due north of Procyon will be seen CASTOR, which, together with POLLUX, 5 to the S E of Castor, are the two well known second magnitude stars in "Gemini"

(14) At a distance of 45° eastwards is seen REGULUS or α *Leonis* This star forms the apex of an isosceles triangle with Castor and Procyon

(15) Continuing the line from Procyon to Regulus, at about 30° beyond the latter will be seen DENEBOLA, a second magnitude star also in Leo

(16) It will be noticed that Denebola forms an equilateral triangle with Arcturus and Spica The positions of these last two have already been referred to

(17) 30° from the Southern celestial pole is the well known constellation the Southern Cross, the cross being formed of 4 stars (2 first mag, 1 second, 1 third) The star nearest to the pole is α, that furthest away is γ When the line α, γ is in a vertical plane these stars are approximately on the meridian

(18) 40° south of Sirius is α Argûs (CANOPUS), a first magnitude star

(19) Canopus, the Southern Cross, and ACHERNAR (α Eridani), form a right angled triangle with the right angle at Canopus Achernar is about 40° distant from, and transits about 5 hours before, Canopus

(20) CANOPUS, ACHERNAR, and FOMALHAUT (α Piscis Australis) are in a line and nearly equi distant, being about 40° apart

---

Effect on determination of Time (by single altitudes) of error of 10″ in observed Altitude See p 144

Azimuth from N	90°	75° or 105°	60° or 120°
	seconds	seconds	seconds
Latitude 60°	1 33	1·38	1 54
40°	0 87	0 90	1 00
20°	0 71	0 73	0 82
0°	0 67	0 69	0 77

Effect on determination of Time (by single altitudes) of error of 10′ in assumed Latitude.

Azimuth from N	90°	75° or 105°	60° or 120°
	seconds	seconds	seconds
Latitude 60	0 00	0 [illegible]	0 77
40	0·00	0 21	0 [illegible]
20	0·00	0 19	0 41
0°	0·00	0 18	0 38

EXAMPLES

OF

ASTRONOMICAL OBSERVATIONS.

Specimen Page of Angle Book

Instrument "
Observat ons for T me
To S n ~~w~~ B 1 99 ~~F~~ (or M T) C r n No 1 Ra e Apj x Lo g l ~~w~~ h m s 0 " 9
Place R F In t tu e The 50 Appr x Chron Err r n ~~F~~ (or M T) = h m s 0 8 1 s w I f rr ng (ject (R O)
Date 2 4 01 (L v Va e d I D v f Le e 0 M gn t o l aring

Apparent I n b or Object	Face	VERTICAL ANGLES: C Vernier °	′	″	D ′	″	Means of C and D °	′	″	Level: E	O	Chron Times T: h	m	s	HORIZONTAL ANGLES (f r Az mut only): A V n r	B	M ans f A and B	A gle Diff (R O and Star	Means of Pairs (with P O)
O	L	3	6	0	6	30	3	26	15		7	0	11	45					
O	R	6	3	0	3	0	24	27	30	0	10	0	1	0					
O	R	6	5	0	51	0	27	5	30	4		0	1	1					
O	I		0	0	0	0	25	0	0	0	1	0	18	34					

WITH SUN
Eye p1 ec re ers —
Horizo ta ly No
Vert al y Y

NOTE The d figures are tho e wh h would be en ered at t) e t me of observ ng The blue fig res as soon after observ ng as po s ble

Ob r A B C
Re o ler A A 7

Specimen Page of Angle Book

Page 2

Instrument ...
Correction for ...
Year, Day ...
Place ...
Date ...

Bar. ...
Ther. ...
Value of 1 Div. of Level

True Meridian ...
Approx. Corrn. From ... 0 40 slow

Ref.
Referring Object (R.O.)
Magnetic Bearing

Apparent Limb of Object	Face	C Vernier			D		Mean of C and D			Level E	Level O	Chron. Time h	m	s
		°		″			°		″			h	m	s
●	L	11	43	0	48	33	1	11	14	20	10	10	46	1
	R.	1	43	4	41	0	15	43	20	1	1	10	49	
	R	1	3		1	4	15	3	5	10	1	10	40	[illegible]
	L	42	4	4	4	40	16	1	17	10	0	10	51	43

HORIZONTAL ANGLES (for Azimuth only)

A Vernier	B	Means of A and B	Angle Diff. of R.O. and Star	Means of Diffs. (with R.O.)
°		°	°	°

Wire few ...
Eye-piece reverses —
Horizontally ...
Vertically ...

NOTE.—The red figures are those which would be entered at the time of observing. The blue figures as soon after observing as possible.

Observer A. B. C.

Recorder X. Y. Z.

Specimen Page of Angle Book

In ment
Ob er at o f Az t
To Sun
Face R E I Inr p t
D te 0)

B JJ o
I 6
Va f I I f I I 10

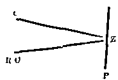
T (or M T) (N 1
Appr x C r I r r n S T (or M T)

Lat
h m s
0 4 s ow

Approx Lo g
h m s
0 - p
R ferring (I t R) (I g) n t
C re T g S at n
Magnetic bearing

Appa nt Limb or (t	Face	VERTICAL ANGLES (C) °	′	″	D			M s f C nd D °		L	O	Chron Times h	m	s	HORIZONTAL ANGLES (for Azimuth only) A °	′	″	B °	′	Means f A and B °	′	″	Angle Diff (RO and Star) °	′	″	Means f Pairs (with RO) °	′	″
RO	1															3	1	3o	5	2	35							
O	1	11	41	11	41	7				½	½	9	1	1	7			6	17			5 5	1	50	47			
O	2		19	6	18	46			1	⅓	4	19		½	47			1	1	4	4	1				1		
RO	R														45	3	4	3	3	4	3	3	1	49	47			

W n S
T e p r —
Hor z ta ly Y s
V r lly No



NOTE Th red t n e th e wh h w u d e r d at th t f r t The th g n s a fter b ervi g n p tle

Observer X Y C
Recorder X Y Z

Specimen Page of Angle Book

Instrument [illegible]
Observations for [illegible]
To Star [illegible]
Place [illegible]
Date [illegible]

Lat. [illegible]
Ther. [illegible]
Value of 1 Div. of Level [illegible]

M.T. Chron. No. [illegible]
Approx. Chron. Error on M.T.

Rate
h m s
9 1 21 [illegible]

Approx. Long. [illegible] h m s [illegible]
Referring Object (R.O.)
Magnetic Bearing

Apparent Limb or Object	Face	VERTICAL ANGLES C Vernier °	′	″	D ′	″	Means of C and D °	′	″	Level E	O	Chron. Times T h	m	s	HORIZONTAL ANGLES (for Azimuth only) A Vernier °	′	″	B	Means of A and B °	′	″	Angle Diff. of R.O. and Star °	′	″	Means of Pairs (with R.O.) °	′	″
☉															Rejected												
	R	11	4	3	4	1	48	1	33	6																	
	1	4	0	[illegible]	0	[illegible]	48	0	31	9	1		1	58													
	L	4	2	[illegible]	[illegible]	4	48	2	14	8	4			9													
	1	11	7	9	6	4	48	[illegible]	51	3																	
	R	11		18	6		48	[illegible]	51				5														
	L	4			0	6	48	0	46	9	4			40													
	1	[illegible]	0	10	0	[illegible]	48	0	6	9	1			9													
	R	11	3		3	2	48	0	30	6			4	10													
															Rejected												

* Warm Sun

Eye-piece reverses —
Horizontally [illegible]
Vertically [illegible]

NOTE.—The red figures are those which would be entered at the time of observing. The blue figures as soon after observing as possible.

Observer A. B. C.

Recorder X. Y. Z.

Specimen Page of Angle Book.

Instrument

Observations for Latitude

To Sun ~~or Star~~

Place R.E. Institute

Date 15.7.14 (C.A.)

Bar. 30.400 "

Ther.

Value of 1 Div. of Level 15

S.T. (or M.T.) Chron. S.

Approx. Chron. Error on ~~S.T.~~ (or M.T.) ... 0 0 ... fast

" Rate

h m s

Approx. Long. ~~East~~ W. h m s 0 ... ...

Referring Object (R.O.)

Magnetic Bearing

Apparent Limb or Object	Face	VERTICAL ANGLES: C Vernier °	′	″	D ′	″	Means of C and D °	′	″	Level E	O	HORIZONTAL ANGLES (for Azimuth only): Chron. Time T h	m	s	A Vernier °	′	″	B	Means of A and B	Angle Diff. of R.O. and Star	Means of Pairs (with R.O.)
O	I	230	46	8	46	13	59	46	1	[illegible]	[illegible]	1	5	14.5							
O	R	209	30	5	30	0	60	9	4	4											
O	R	99	9	40	9	8	6	0	1	5	6			11							
O	I	40	20	0	0	8	60	20	19												
O	I	33	48	40	18	40	59	48	4	6	5		58	48							
O	R	1	13	10	40	10	60	10	5		5		1								
O	R	60	1	20	11	47	59	4	54	5	5			14							
O	I	40	17	30	18	10	[illegible]	1	5	7			4	1							

Specimen Page of Angle Book

Page 6

Instrument ... ...
Observation of ... [illegible]
Time Star [illegible] ... Bar. 3[illegible] ... Sidereal M.T. Chron. No. 17 ... Rate ... Approx. Long. E. or W. 0 h – m [illegible] s
Place [illegible] ... Ther. 4[illegible]° ... Approx. Chron. Error on ~~Local~~ M.T.) 0 h 3 m 24 s fast ... Referring Object (R.O.) ...
Date [illegible] ... Value of 1 Div. of Level 15″ ... Magnetic Bearing ...

Apparent Limb or Object	Face	VERTICAL ANGLES C Vernier			D		Means of C and D			Level E	O	Chron. Times, T			HORIZONTAL ANGLES (for Azimuth only) A Vernier			B		Means of A and B			Angle Diff. of R.O. and Star			Means of Pairs (with R.O.)		
		°	′	″	′	″	°	′	″			h	m	s	°	′	″	′	″	°	′	″	°	′	″	°	′	″
☉	L	2[illegible]1	[illegible]	15	[illegible]	10	[illegible]1	8	14	7	5	7	3	4														
	R	2[illegible]4	[illegible]	[illegible]	0	12	[illegible]1	33	[illegible]	4	8		8	1														
	R	2[illegible]4	12	9	19	[illegible]	[illegible]1	4[illegible]	[illegible]	4	8		11	0														
	L	2[illegible]1	42	0	43	[illegible]	[illegible]1	40	[illegible]	[illegible]	5		16	33														

* Wire Sew ...
Free bubble reverses —
Horizontally ... ...
Vertically ... ...

NOTE.—The red figures are those which would be entered at the time of observing. The blue figures as soon after observing as possible.

Observer A. B. C.

Recorder X. Y. Z.

EXAMPLES

OF

ASTRONOMICAL COMPUTATIONS.

TIME by Altitudes of Sun

From Page 1 Angle Book

	°	′	″		h	m	s
Vert Angles	23	26	15	Times	20	13	45
	24	27	30		20	15	20
	27	5	30		20	17	15
	25	0	0		20	18	34
Totals	99	59	15		81	4	54
Means	24	59	49		20	16	13½

Level Correction *

	E	O
	5	7
	0	10
	4	7
	0	12
Totals	9	36

Correction

$$\frac{(36 - 9)}{8} \times 20$$

$$= + \frac{27 \times 20}{8}$$

$$= + 1' \ 7 \ 5$$

	°	′	″	
	24	59	49	
+	0	1	8	
	25	0	57	Correct mean observed altitude

* With micrometer theodolites the level should be read to one place of decimals

Form K.—(S.M.E. Astronomical Form 1)

Time by Altitudes of the Sun

Those parts in Italics are only necessary when the Chronometer keeps Sidereal Time

Place	Lat. N	M.T. / S.T. Chron. No	Bar. 29.9
Date			
Page in Angle Book	Approx. Long. E or W	Approx. Chron. Corr. on L.M.T. + 0 8 1	Ther

ELEMENTS

Time Ell. N.A.

L.S.T. of L.M.N.

Time in Angle Book 1 p

† Mean of Obsd. Times 10 14 5

Approx. Chron. Corr. ± + 0 8 15 (Slow + Fast −)

L.S.T. (Approx.)

* *L.S.T. of L.M.N.*

S.I.I. of L.M.N.

M.T.E. H / M / S

L.M.T. of Observation (Approx.) 4 3

Long. W (+) or E (−) ± 0 9

G.M.T. of Mean of Observed Times 1

‡ Interval of hours from G.M.N. 3

POLAR DISTANCE

This is reckoned from the Elevated Pole 90 ± δ

For Dec. δ at time of observation see below

Dec. δ N or S — 90 0 0

Polar Distance p

If the Lat. (φ) and Dec. (δ) are of the same name δ is subtracted from; if of different name added to 90°

FORMULA AND COMPUTATION

$$\tan\frac{t}{2} = \sqrt{\cos s \sin(s-l)\ \mathrm{cosec}(s-\phi)\ \sec(s-p)}$$

where $s = \frac{l+\phi+p}{2}$

The Lat. (φ) is always positive and p is reckoned from the Elevated Pole

From Angle Book

Obsd. Mean Altitude	5 0 07		
Refraction	= − 0 2 41		
Semi-diam.	±		
Parallax	− + 0 8 1		
True Alt. (l)	= 1 1		
Lat. (φ)			
Polar Dist. (p)	= 8 1 1		
	2		
s	= 5 10 3	Log cos	9 0 600
s − p	= 4	Log sec	10 001 1 6
s − φ	= 9 0 8	Log cosec	10 031 11
s − l	= 5 0 5	Log sin	9 91 30 1
			2 / 19 4
		Log tan t/2	9 1 111
		t/2 =	21 00 4
		t in arc =	51 40 5 8
		t in time =	3 1 (h m s)

If before noon subtract from 24 0 0

L.A.T. of Observation = 20 0 40

Equation of Time ± = + 0 3 51 3

L.M.T. of Observation = 20 4 31 8

Not required with M.T. Chron.: *S.T. Equivalents* Hrs / M / S; *S.I. Int. for L.M.N.* −; * *B.L.S.T. of L.M.N.*; *L.S.T. of Obs.*

† Chron. T. of Obs. = 16 13

Chron. Slow or Fast ± on L.S.T. or L.M.T. = 9 13 (h m s)

DECLINATION

Dec. (δ)

At Greenwich Noon (Page II N.A.) N or S 4 45 7 4

Hourly Variation (Page I N.A.) × ‡ Interval in hours from G.M.N. ± − 0 3 1

At mean of Obsd. Times N or S 4 4 35 7

EQUATION OF TIME

(Page I N.A.) m s ± + 3 48

± + 0 1

± + 3 51 3

REFRACTION

Refn. due to Alt. 4 1 (″)

Corr. for Bar. ± 0 0

Corr. for Ther. ± 0 0

Refraction 2 41

PARALLAX

Hor. Par. (Page I N.A.) ×

cos l

Par. in Alt. 0 8 1

TIME by Altitudes of an E Star

From Page 2, Angle Book

	°	′	″		h	m	s
Vert. Angles	15	11	14	Times	10	46	12
	15	40	50		10	48	32
	15	53	55		10	50	2
	16	1	17		10	51	43
Totals	62	47	16		43	16	29
Means	15	41	49		10	49	7 25

Level Correction

	E.	O.
	20	10
	15	15
	19	12
	10	20
Totals	64	57

Correction

$$\frac{(57 - 64) \times 5''}{8}$$

$$= -\frac{7 \times 5''}{8}$$

$$= -4''$$

	°	′	″	
	15	41	49	
−	0	0	4	
	15	41	45	Correct mean observed altitude

FORM I.—(S.M.E. Astronomical Form 2.)

Time by East and West Stars

STAR | E or W | Lat. (φ) N or S | Bar.
Ther.
Date | Approx. Long. E or W | Date | Approx. Corr.
Page in Angle Book | M.T. Chron. No.

$$\tan \frac{t}{2} = \sqrt{\cos s \sin (s-h) \operatorname{cosec} (s-\phi) \sec (s-p)}$$

where $s = \frac{h+\phi+p}{2}$, with φ always taken positive and p reckoned from the Elevated Pole.

h = Altitude. p = Polar Distance.

COMPUTATION

FROM ANGLE BOOK

		° ′ ″
Mean Obs. Alt.	−	1 11 45
Corr. for Refn.	−	3
True Alt. h		15 3 8
Lat. φ	−	51 3 0
Polar Dist. p	=	5 3 1
	2	1 25 1

REFRACTION

		″
Refn. due to Alt.		3 1
Corr. for Bar.	± = −	1 4
Corr. for Ther.	± = +	0 0
Refraction		3 1

POLAR DISTANCE

		° ′ ″
Decl. (δ) N or S	± =	1
		90 0 0
Polar dist. (p) or 90 ± δ	−	3 3 1

NOTE.—p is reckoned from the Elevated Pole. If the Lat. and Dec. are of the same names subtract from, and if of different names add to 90°.

s		6 1 4	Log cos	=	9 1 9
s − p	−	1 1	Log sec	=	
s − φ	−	1 4 1	Log cosec	=	1
s − h	−	1	Log sin	=	9 9 9 9 1
				2	19 6 9 5
			Log tan $\frac{t}{2}$		9 4 9
			$\frac{t}{2}$	=	° 5 1 9
			t in arc 5		0 4 40 1
				3	11 6 57 3

		h m s
G.S.T. of G.M.N. (Page II N.A.)	=	1 59 7 5
Corr. for Long. at 9.86 s per hour (− E + W)	± −	0 0 0 35
*L.S.T. of L.M.T.	=	1 59 3

		h m s
t in time ± (− E + W)	−	1 4 19 1
R.A. of Star		1 9 9
L.S.T. of Obsn.		1 18 3
(*L.S.T. of L.M.N.)		1
S.T. int. from L.M.N.		10 5 1
M.T. Equivalents: H s		9 8 1 011
M.T. Equivalents: M ns		0 5 5 8 57
M.T. Equivalents: S s		0 0 1 1
		0 0 0
L.M.T. of Obsn.		10 5 49
FROM ANGLE BOOK — Chron. Time of Obsn.		10 49

		m s
Chron. Fast or Slow on L.M.T. or L.S.T.	± +	6 0

Azimuth by the Sun.

From Page 3 Angle Book

	°	′	″		h	m	s
Vert Angles	31	41	17	Times	19	51	21
	32	41	14		19	54	9
Means	32	11	15 5		19	52	45

	h	m	s	
Chron watch is approx	0	3	29	slow
L M T of observation is	19	56	14	

Level Correction

	E	O
	3 5	7·5
	7 5	4 0
Totals	11	11 5

Correction

$$\frac{(11\ 5 - 11) \times 10''}{4}$$

$$= +\ 1''\ 25$$

	°	′	″	
	32	11	15 5	
+	0	0	1 3	
	32	11	16 8	Correct mean observed altitude

FORM M.—(S.M.E. Astronomical Form 3.)

Azimuth by Sun or Star with the Theodolite

SUN or STAR	Lat. φ N or S	° ′ ″	R.O. [illegible]
Page in Angle Book	Approx. Long. E or W		T [illegible]
Place	Date		
[illegible]	[illegible]		Mag. Bearing

Tan $\frac{A}{2}$ = [illegible] sin s − φ sin (s − h) ... $s = \frac{h+\phi+p}{2}$

A = Horizontal Angle between the Elevated Pole and the Sun or Star.

In this formula the latitude φ should be taken with the positive sign whether N or S and Polar Distance (p) is then to be reckoned from the Elevated Pole.

ELEMENTS

[illegible] Induced Alt.	—	1 [illegible]
Corr. for Par.	±	0 1 4
Corr. for Te[illegible]	±	0 4 5
Refraction		1 1
Hor. Par. (Page [illegible] N.A.) × Cos. Alt.		
Parallax in Alt.	—	0 [illegible]

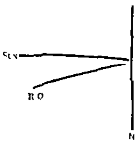
WITH SUN OBSNS. ONLY

		h m s
G.S.T. of G.M.N. (Page II N.A.)	—	
Corr. for Long. at 9.86 sec. per hour (+ W − E)	=	
L.S.T. of L.M.N.	=	
L.S.T. of Obsn.	=	
Sid. Int. from L.M.N.	=	
M.T. Equivalents: Hrs / Mins / Secs		
L.M.T. of Obsn. (If M.T. Chron. used this is taken direct from Angle Book)	—	10 [illegible] 1[illegible]
Long. E (−) or W (+)	± —	0 [illegible] 9
G.M.T. of Obsn.	=	10 [illegible] [illegible]
Int. from G.M.N. in Hrs.	—	4 10
Decl. (δ) at G.M.N. (N or S)	±	+ 19 45 [illegible]1
Hr. Var. (Log [illegible] N.A.) × Int. from G.M.N.	±	— + 0 [illegible] 0
Dec. (δ) of Sun or ✱ at time of Obsn.	±	— + 1[illegible] 4[illegible] 11
		90 0 0
Polar Dist. (p) or 90 − (± δ)	—	0 11 49

NOTE.—p is reckoned from the elevated pole. If φ and δ are of different names δ will be negative and p = 90 + δ.

COMPUTATION

FROM ANGLE BOOK Mean Obsd. Alt.		1[illegible] 1[illegible] 9	
Corr. for Refraction	—	0 1 1	
Corr. for Parallax	+	0 0 7 4	
True Alt. h		0 9	
Latitude φ		[illegible]	
Polar dist. p	—	0 11 4	
		153 45 17	
s	—	[illegible]	Log sec = 10.6139005
s − h		[illegible]	Log sec = 0.00[illegible] 8
s − φ		[illegible]	Log sin = [illegible] 7
s − p	—	41 4 10	Log sin = [illegible]
			[illegible] 6[illegible]
			Log tan $\frac{A}{2}$ = 10.0 [illegible] 0
			$\frac{A}{2}$ = 49 1[illegible] 11
			A = 98 [illegible]
Angle between R.O. and ✱ (or Sun) (from Angle Book)			= 1 49 41
† AZIMUTH of R.O. from Elevated Pole			= [illegible] 4



† NOTE.—Azimuth for trigonometrical purposes should be measured from the South (by West). It is best ascertained above data by consideration of a figure showing the relative positions of Star, Pole and R.O.

LATITUDE by Circum meridian Altitude of Star

		h	m
October 7th, 1897, "ε Pegasi,"	R A =	21	39
G S T of G M N	=	13	6
		8	33
Chron Correction	+	0	4
Approx Chron Time of Transit		8	37

NOTE —This computation is made before observation and guarded in the angle book

From Form 4 Chron Time of Transit (T_0) = 8 h 36 m 37 5 s

From Angle Book Page 4 Chron Times (T)	Hour Angles ($T - T_0$)	Values of m (Vide Table XVI)	Face	From Angle Book, Page 4, Vert Angles
h m s	m s	′		° ′ ″
8 29 12	− 6 55 5	94 1	R	48 1 33
8 31 58	4 39 5	42 6	L	48 0 31 5
33 39	2 58 5	17 4	L	48 1 14 5
36 26	0 11 5	0 1	R	48 2 51
38 5	+ 1 27 5	4 1	R	48 2 51 5
40 49	4 11 5	34 5	L	48 0 46
42 39	6 1 5	71 3	L	48 0 6
46 10	9 32 5	178 7	R	48 0 30 5
	Totals	442 8		10 24
	Means	55 3		48 1 18

Level Correction

	F	O
	6	6
	8	4
	8	4
	5	7
	5	7
	8	4
	8	4
	6	6
Totals	54	42

Correction

$$= \frac{(42 - 54) \times 15''}{16} = -11'2$$

	°	′	″	
	48	1	18	
−	0	0	11·2	
	48	1	6 8	Correct mean observed altitude

Form N —(S M E Astronomical Form 4)

Latitude by Circum Meridian Altitudes of Star

STAR | Approx L n F | h m | Page in Angle Book

Place | Bar | M T | No

Date | Ther | Approx Chron | Error ± | Rate ±

NOTE —Chron T of Transit and mean value of

COMPUTED CHRON TIME OF TRANSIT (T_0)

L S T of L M N — 13 37 0

M T F on a

L M T of Transit

Chron Error

Chron Time of Transit T_0

MEAN ZEN DIST ζ

Refraction due to Obsd Mean Alt	=	0
Corr for Par		
Corr for Ther	= +	5
Refraction Corrected	=	
Obsd Mean Alt From Angle Book)	=	° ′ ″
Refract n (see above)	= −	
True Mean Alt (h)	=	4 0 1
		90 0 0
ζ_0 Mean Zen Dist (90 − h)	= +	41 59 47

NOTATION AND FORMULÆ

ϕ — Lat N or S ζ_0 Mean Zen Dist

ϕ Approx Lat ζ_1 = Approx Mer Zen Dist

δ Dec at Mean of Times of Obsn Z_0 Correct Mer Zen Dist

$2\sin^2\frac{t}{2}$ / sin 1″ (Values found in Table XVI See above note)

m_0 Mean of Values for m

$\phi_1 = \delta + \zeta$ and $\phi = \delta + Z_0$

These Eqns hold good in all cases if the signs of the functions ϕ δ and ζ are carefully attended to as follows —

Lat and Dec are $\frac{+}{-}$ when $\frac{N}{S}$ of Equator

Zenith Dist is $\frac{+}{-}$ when the Star is $\frac{S}{N}$ of Zenith

$Z_0 = \zeta_0 - A\, m_0$ and $A = \frac{\cos\phi_1 \cos\delta}{\sin\zeta}$

APPROX LATITUDE (ϕ)

From Angle Book Greatest Obsd Alt	=	48 °
Refraction (use that for ζ)	=	− 0 1
Approx Mer Alt	=	48 2
		90 0
ζ (Approx Mer Zen Dist)	=	+ 41 58
Dec (δ)	=	+ 9 4 5
Appx Lat $\phi = \delta + \zeta$	=	+ 51 23 5

COMPUTATION FOR TRUE LATITUDE (ϕ)

Log cos ϕ	=	
Log cos δ	=	
Log cosec ζ	=	1 1 1
Log A	=	9 84 1
Log m_0 (n_0 =)	=	1 74
Log A m_0		1 706
A m_0	=	50 93
A m_0	=	0 0 50 93
ζ_0 or Mean Zen Dist	=	41 59 4 0
Z_0 or (ζ_0 − A m_0)	= +	41 58 50 0
Dec (δ) N ~~or S~~	= +	9 4 33
Lat $\phi = \delta + Z_0$	=	1 N ~~or S~~

LATITUDE by Circum meridian Altitudes of Sun

		h	m	
Equ of Time, 15th July, 1897	= +	0	6	(Page I, N A)
Chron Correction	= +	0	1	
Approx Chron Time of Transit	=	0	7	

NOTE —This computation is made before observation and guarded in the angle book

From Form 5 Chron Time of Transit (T_0) = 0h 6m 40 7s

From Angle Book Page 5 Chron Times (T)	Hour Angles ($T - T_0$)	Values of *m* (Vide Table XVI)	From Angle Book, Page 5, Vert Angles
h m s	m s	′	° ′ ″
23 59 14 5	− 7 26 2	108 6	59 46 10 5
23 62 35	4 5 7	33 0	60 9 47 5
65 11	1 29 7	4 4	60 0 36
67 5	+ 0 24 3	0 3	60 20 19
68 48	2 7 3	8 8	59 48 40
71 2	4 21 3	37 2	60 19 50
72 14	5 33 3	60 6	59 47 54
74 23	7 42 3	116 5	60 17 50
Totals 193 0 32 5		369 4	480 31 7
Means 0 7 34		46 2	60 3 53 4

Level Correction

	E	O
	7	2
	4	5
	3	6
	7	2
	6	3
	5	5
	5	5
	7	2
Totals	44	30

Correction

$$= \frac{(30 - 44) \times 15'}{16} = -13' 1$$

	°	′	″	
	60	3	53 4	
−	0	0	13 1	
	60	3	40 3	Correct mean observed altitude

Form O (S M F Astronomical Form 5)

Latitude by Circum Meridian Altitudes of Sun

Place

Appx Long E or W h m s

Date

Hour Angle Book
MT or ST: No ?, Error ± 0 0 56 8 Fast, Rate ±

COMPUTED CHRON TIME OF TRANSIT T_0

	h m s
L A T of L A N	0 0
Eqn of T (Page I N A)	+ 0 5 43 51
Corr fl	− 0 0 0 009
L M T of L A N (Tr) M T of Tran it)	0 5 4 9
M T Chron Error ± Fast + Slow	+ 0 0 56 8
Chron T of Tran t T_0	0 6 40 7

† NOTE — Mean Values computed on a separate sheet of paper. Values for ... are found in Table XVI. If a S T ... and some of the Hour Angle Ints are great (say over 10 mins) all of them should first be converted into Mean Time Ints before taking out value of m from the tables.

* This Corr = Long in Hours × Hourly Var of Eqn of Time Page I N A. In E Longs if Eqn of T is increasing numerically the Corr should be given a contrary sign to that of the Eqn of T and vice versa. In W Longs if Eqn of T is increasing numerically the Corr is given the same sign as the Eqn of T and vice versa.

MEAN OF HOUR ANGLES $(T-T_0)$

	h m
From Angle Book Mean Chron Time T	
Chron T of Tran T_0	
Mean of Hour Angles $T-T_0$ ±	+ 0 0 3

NOTE —

TIME INT OF OBSN FROM G M N

	h m
G M T of G A N Eqn of T p I N A) − Long in Hrs × Hourly Var of Eqn of T (see Note *) ±	0 0 0 0
L M T of L A N or Corr Eqn of T) Long Add if W sub if E)	
G M T of L A N; Mean of Hour Angles (see above) ±	+ 0
Int of Mean of Obsn Times from G M N	0 4 25

DECLINATION

Dec at G M N (Page II N A) N	1 7 69
Hourly Var (Page I N A) 24 03 × 07; Int in Hrs from G M N ±	− 0 0 1 8
Dec (δ) at Mean of Times	= 1 27 5 1

COMP FOR APPX LATITUDE (ϕ_1)

	°
From Angle Book Greatest Obsd Alt	60 0
Refraction (use Refr for ζ_0)	− 0 0
Semi Diam ±	0 1 5
Approx Mer Altitude	60 4
	90 0 0
ζ Approx Mer Zen Dist	= + 9 5
Dec (δ)	= + 21 7 0
Appx Lat $\phi = \delta + \zeta$	+ 51 23

NOTATION AND FORMULÆ

ϕ Latitude N or S; ζ_0 Mean Zen Dist
ϕ_1 − Approx Lat; ζ − Appx Mer Zen Dist

(see Note †) Z_0 Correct Mer Zen Dist

δ — Dec at Mean of Times of Obsn

— Mean of Values of (for each hour angle)

$\phi = \delta + \zeta_1$ and $\phi = \delta + Z_0$

These Eqns hold good in all cases if the signs of the functions ϕ, δ and ζ are carefully attended to as follows —

Lat and Dec are $\pm$ when $\frac{N}{S}$ of Equator

ζ ... is $\pm$ when the Sun is $\frac{S}{N}$ of the Zenith

$Z_0 = \zeta_0 - A m_0$ and $A = \frac{\cos\phi \cos\delta}{\sin\zeta}$

COMP FOR MEAN ZEN DIST (ζ_0)

	°
From Angle Book Obsd Mean Altitude	60 3 40 3
Refraction	− 0 0 3 0
Parallax	+ 0 0 4 35
True Mean Altitude	60 3 1 7
	90 0 0
ζ_0 Mean Zen Dist ±	= + 21 6 47 7

REFRACTION & PARALLAX: Refraction due to Alt; Corr for Bar; Corr for Ther; Refraction; Hor Par × cos ?; Parallax in Alt = 4 35

COMP FOR TRUE LATITUDE (ϕ)

Log cos ϕ	= 9
Log cos δ	9 9 983
Log cosec ζ	= 10 30193
Log A	10 06 99
Log m_0 (m_0 46)	1 66 64
Log A m_0	1 7 63
A m_0	53 Secs
A m_0	0 0 53 3
ζ_0	+ 3 56 4 7
Z_0 or $\zeta_0 - A m_0$)	+ 3 55 3 9
Dec δ	+ 1 27 35 1
Lat $\phi = \delta + Z_0$	51 3 29 0 N

LATITUDE by Altitudes of Polaris

From Page 6, Angle Book

	°	′	″		h	m	s
Vert Angles	51	37	12·5	Times	7	3	54
	51	39	39·5		7	8	12
	51	40	29·5		7	11	0
	51	40	55·5		7	16	33
Totals	206	38	17		28	39	39
Means	51	39	34·2		7	9	54·8

Level Correction

	E	O
	7	5
	4	8
	4	8
	7	5
Totals	22	26

Correction

$$+\frac{(26-22)\times 15}{8}$$

$$= +\frac{4\times 15''}{8}$$

$$= +7''\,5$$

	°	′	″	
	51	39	34·2	
+	0	0	7·5	
	51	39	41·7	Correct mean observed altitude

FORM P—(S M I Astronomical Form 6)

Latitude by Altitudes of "Polaris"

P ace | Approx Long | h m s | Bar 0 2
Date | L.M.T. Chronometer No 1 | Ther 4
Page in Angle Book | Error m 1 1 1 |
| Rate |

ELEMENTS.

	h m s
S T at M N page II	1 1
Corr	± 0 0 0 3
L S T L M N	3 3

	h m s
From Angle Book Mean of Obsd Times	
Corr to LMT or LST	± 1
L M T	—
S T Hrs	= 1 0
Mins	= 1
Secs	=
Sid In from L M N	= 1 4 5
L S T of L M N	1 3
* L S T of Obsn	= 13 1

REFRACTION

Refn due to Obsd Mean Alt (Angle Book)	=	1
Corr for Bar	± = +	0 4
Corr for Ther	± = +	
Refraction Corrected	—	4 1

TRUE ALTITUDE (h)

		° ′ ″
Obsd Mean Alt (Angle Book)	=	1 3 41
Corr Refraction	= —	4 1
True Altitude (h)	=	51 39 4 0

COMPUTATION BY TABLES IN NAUTICAL ALMANAC

		°
True Alt (h) (see above)	=	1 3
Subtract	=	0 1 0
Reduced Alt	=	1
With argt LST* h 13m 15s 1st Corr ±	—	0 13 4
Approx Latitude	=	1 1 13
With argt LST* and True Alt 2nd Corr	+	0 0
With argt LST* and date 3rd Corr	= +	0 1 1
Latitude N	=	1 1

COMPUTATION BY FORMULA

$\phi = h - p \cos t + \frac{1}{2} p^2 \sin^2 t \sin 1'' \tan h$

This computation gives more accurate results than those obtained by use of N A Tables

POLAR DISTANCE (p)

		° ′ ″
Dec (δ)	=	89 45 0 8
		90 0 0
p (= 90 − δ)	=	1 14 0
p (in seconds)	—	41

HOUR ANGLE (t)

		h m s
* L S T of Obsn	=	13 11
R A of Star	=	1
t in time (S T − R A)	± — +	18 0 48 0
If t is negative subtract from 24 hrs	= +	
t in arc	=	° 4 8

Log p (p in seconds)	=	3 18 1
Log cos t	=	9 31 37
Log 1st Corr	=	7001 11
1st Corr (in seconds)	=	0 3
Log p	—	01 6 1
Log sin t		0
Log p sin t	=	3 010
Log $p^2 \sin^2 t$ = 2 × (Log p sin t)	=	60 0
Log tan h	=	1 1 1
Log sin 1″ †	=	1 8
Ar comp Log 2 †	=	1 0
Log 2nd Corr	=	1 1
2nd Corr (in seconds)	= +	
True altitude	= +	° 1 3 1 1
1st Corr (opposite sign to cos t) ± (If t is in 2nd or 3rd Quadrant cos t is −)	=	0 1 18 3
		1 1
2nd Corr (always add it)	— +	1 0
Latitude N or S	—	1 24 51

† log sin 1″ = 6·685 49

Ar comp Log 2 = $\bar{1}$ 0989 00

CHAPTER XV

LATITUDES AND AZIMUTHS

ON expeditions and boundary commissions it is frequently impossible to keep up a regular triangulation. Recourse may then be had to the device known as 'latitudes and azimuths.'

Suppose A and B to be the two mutually visible points. If a latitude is observed at A and the azimuth AB is determined and then subsequently the latitude of B is determined it is clear that provided that there is a considerable amount of northing and southing in the line AB values can be obtained for the length AB and the difference of longitude between A and B either graphically by drawing on a plane table or better, by means of a calculation.

Degree of Accuracy.—Obviously no very great degree of refinement can be obtained by this method. We will analyse the error which will result in a fairly favourable case. Let the azimuth of AB be about 45°, let latitudes be observed at A and B with a probable error of 3″. Then the probable error in the range of latitude is $3\sqrt{2}''$ and the probable error in the length of $AB = 3\sqrt{2} \times \sqrt{2} = 6''$ or 600 feet. Now suppose AB to be about 40 miles, the

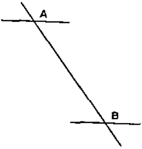


Fig. 64.

ratio of error in the length of $AB = \dfrac{600}{40 \times 5280}$, or $\frac{1}{352}$. Now, although this is a large error as compared with trigonometrical errors, it is smaller than would usually result from any rapid system of traversing.

The most favourable case is that of a journey nearly north and south with long rays. Latitudes and azimuths should rarely be attempted with rays of a greater azimuth than 45° from N. or S.

Supposing the explorer on a rapid march is fixing his framework by this means, when he observes a latitude and azimuth he should take a round of angles to all likely hills anywhere near to his probable line of march. Very often he cannot predict the hill which he is likely to visit next. Even with this precaution he may find either that he cannot visit or cannot identify with sufficient accuracy any hill included in his round of angles from his first station. He must then give up the idea of identifying the hills previously observed to and must observe a latitude and take an azimuth from any convenient hill *back* to his first station, on which he should if possible have erected some cairn or pole or mark.

Latitudes and azimuths were freely used on the march of the Afghan Boundary Commission, as is shown on Plate VIII.

They were also much employed in the exploration of the southern frontier of Abyssinia in 1902-03.

Computation.—Let A and B be two stations at which latitudes have been observed and at A the azimuth of AB has been determined.

Let the difference of the two latitudes in seconds $= \Delta\lambda$.

Let the azimuth of AB at A $= A$.

It is required to find

(i.) The length of the line AB $= c$.
(ii.) The difference of longitude of A and B in seconds of arc $= \Delta L$.

Let the azimuth at the middle point of AB $= A + \frac{\Delta A}{2} = \alpha$.

Let the mean latitude of A and B $= \lambda$.

Let the difference of longitude determined approximately (from a plane table, or rough method) $= \Delta L$.

Then

$$\frac{\Delta A}{2} = \frac{\Delta L}{2} \sin\lambda, \qquad (1)$$

Now apply this value of $\frac{\Delta A}{2}$ to A (remembering in which way the meridians converge) and thus obtain α.

Then for the required difference of longitude

$$\Delta L = \frac{\rho}{\nu} \tan\alpha \sec\lambda \times \Delta\lambda \qquad (2),$$

where $\log \frac{\rho}{\nu}$ is the value of Q in Table I. (for the *mean* latitude).

Also, for the length AB,

$$c = \Delta\lambda \sec\alpha \times \rho \sin 1'' \qquad (3)$$

where $\log \rho \sin 1''$ is the arithmetical complement of P in Table I. (for the *mean* latitude).

CHAPTER XVI

LONGITUDE

The Longitude of a place is the angle made by its meridian plane with some one fixed meridian plane arbitrarily chosen and is measured by the arc of the equator intercepted between these two meridians.

Longitudes are reckoned east or west of the fixed meridian up to 180°.

The fixed meridian in use in all British territories is that of Greenwich.

Since the earth makes a complete revolution once in 24 hours it is clear that each meridian is moving round at the rate of 15° an hour; we may therefore describe the relative positions of two meridians in arc or in time. We may say, for instance, that the longitude of a place is 30° E or 2 hours E.

Now as has been explained in Chapter XIV, the local time is measured by the angle at the celestial pole between the local meridian and the mean sun, and the local sidereal time is the angle between the local meridian and the First Point of Aries.

It follows that the difference in longitude of two places is the difference at any instant, of their local times mean or sidereal.

The longitude of a place may be determined by any one of the following methods:—

Method	
(1) Triangulation	Relative methods
(2) Exchange of telegraphic signals	
(3) Latitudes and azimuths	
(4) Transport of chronometers	
(5) Occultations of stars by the moon	Absolute methods
(6) Moon photographs	
(7) „ culminations	
(8) „ altitudes	
(9) Lunar distances	
(10) Eclipses of Jupiter's satellites	

TRIANGULATION

(1) **Triangulation** is by far the best of all these methods, not only on account of its accuracy but also because the process is itself necessary for the detail survey. Even in wooded countries such as parts of Burma it is still usually possible to triangulate, and the accumulated error even when it is impossible to erect signals will only amount perhaps to some 5 seconds of arc in 100 miles. The method has been fully explained in Chapter III. If however the longitude of the starting point is not known it must be determined by telegraph, chronometers or by some absolute method.

A good example of the combined use of several methods is afforded by the Nyasa-Tanganyika Boundary Survey of 1898. The head of the transcontinental telegraph line had, at that time reached a point (Nkata) near the shore of Lake Nyasa about 140 miles south of the starting point of the boundary (which was on the lake) and some 1900 miles from Cape Town.

The difference of longitude between Cape Town observatory and Nkata was determined by exchange of telegraphic signals on three nights and a good value obtained.

Fourteen chronometers were now transported backwards and forwards between Nkata and

Kambwe near the starting point of the boundary. From this point a continuous triangulation was carried out along the whole length of the boundary zone (over 200 miles) to Lake Tanganyika.

EXCHANGE OF TELEGRAPHIC SIGNALS

(2) **Exchange of Telegraphic Signals.**—*For a description of the refined determination* of differences of longitude by telegraph for geodetic purposes by means of chronographs, the reader is referred to any standard work on geodetic methods: we shall here describe the simple means used to obtain telegraphic differences of longitude with sufficient accuracy for map making.

Let A and B be two stations which are connected by telegraph, and of which it is required to determine the difference of longitude.

Arrangements must first be made to insure direct communication between A and B; this may involve the use of relays. The line must be free of all traffic during certain hours of the night, and there must be a telegraph operator at each end.

At each end there must be a good box chronometer; one of these should be a mean time chronometer and the other a sidereal time chronometer. The reason for this is, that if both were sidereal or both mean time chronometers, the intervals between the beats of the chronometer would be constant, and there would be no coincidences.

The chronometer should be on the same table (in tent, hut, or house) as the telegraph key, and close to it, so that the person sending signals can easily read the chronometer.

Before commencing the exchange of signals, a programme should have been arranged between A and B.

A convenient programme is as follows:—

Signals to be exchanged on three nights.

Each night's work to consist of at least one set of signals sent in each direction.

Unsatisfactory sets to be repeated.

Each set of signals should be preceded by a rattle on the key about 30 seconds before the set begins. Each set should end with a rattle. A message should be sent at the end of a set to ask if it was received all right.

Each set of signals to last 5 minutes.

Each set to begin at 0 seconds. Succeeding signals to be sent at 10 seconds, 20 seconds, 30 seconds, 40 seconds, 0 seconds, 10 seconds, &c., omitting the 50th second.

When the key is depressed to send a signal it should be kept down for 2 seconds.

Receiving.—The chronometers should beat half seconds, and in receiving signals it is necessary to count mentally the beats of the chronometer, thus: one, half, two, half, three, half, &c. If now the signal from the other end should arrive, say, between half and two, the person receiving must judge whether it was 1.6, 1.7, 1.8, or 1.9; if he is doubtful he should say 1.8. It requires a little practice to judge the intervals.

If the chronometers beat half seconds a coincidence will occur every three minutes; at a coincidence the chronometer beat and the signal will sound together, the receiver will then say "up"; the beat and signal will sensibly coincide two or three times running.

Before and after exchanging signals, observations for time should be taken. These observations should be made either with a portable transit instrument or with a theodolite; if with the former, two N. stars and two S. stars before and the same number after; if with a theodolite, two E. and two W. stars before and the same number after.

The box chronometer should not be moved from the table on which it rests during the operations; any movement will tend to give it an irregular rate.

Its error should be determined by transit instrument or theodolite as laid down, each

moment of transit or intersection being referred to the chronometer in one of the following ways —

By having a recorder at the chronometer, and

(i.) Fitting up a temporary telegraph line a few yards long from instrument to hut and sending signals by key;

(ii.) By shouting "up" to recorder; or

(iii.) The time may be transferred by stop watch; this is only suitable for use with a theodolite which must be near the hut.

The next morning, after an interchange of signals, each station should send a message to the other station —

My local mean (or sidereal) time of first signal sent was .... (to tenths of seconds).

My local mean (or sidereal) time of first signal received was .... "

The list of received signals should now be inspected; there should be a slow progressive increase or decrease in the decimals; any obvious mistake should be corrected.

The difference between the means of the corrected local times of receiving and sending is the difference of longitude in time. This is burdened with two errors in addition to instrumental errors: (1) Rate of transmission of signal along the wire; this is eliminated by sending the same number of times in each direction, and is very small. (2) Personal equation. Every observer records a transit, intersection or signal a little too early or too late. This error is best eliminated by exchange of observers from one end of the line to the other when half the work has been completed. This is not usually practicable in topographical work.

In view of the existence of personal equation it must be expected that a telegraphic difference of longitude determined as above described will be in error 3 to 5 of a second of time, or in equatorial regions 150 to 250 yards.

(3) **Latitudes and Azimuths**.—See Chapter XX.

TRANSPORT OF CHRONOMETERS

(4) **Transport of Chronometers**.—Suppose that it is required to ascertain the difference of longitude between two places A and B. Determine the error of a chronometer at A on local time and also determine the "rate" of the chronometer, *i.e.*, the amount which it gains or loses in a day. Now transport the chronometer to B and determine its error on local time at B. Since we know the error at A and the rate we can also at any instant find the local time at A; compare this with the local time at B at the same instant; the difference of these two local times is the difference of longitude between A and B.*

As an example —

At A at 9 A.M. November 3, 1903, the chronometer was 2 minutes 5.6 seconds fast, gaining 1.4 seconds a day.

At B at 10 45 A.M. November 4, 1903, the same chronometer was 4.2 seconds slow; what is the difference of longitude?

The interval in time between the two determinations is 1 day 1.75 hours = 1.07 day.

The rate = 1.4 seconds; the chronometer has therefore gained 1.4 seconds × 1.07 seconds = 1.5 seconds.

The difference of longitude is therefore —

m	s
2	5.6
+0	4.2
+0	1.5
2	11.3

and B is east of A 0 32 49 5.

* It is clear that the difference of the local times is the same as the difference at the same instant of the errors of the same chronometers on local time.

In practice it is found that for land journeys box chronometers give unsatisfactory results for such work good lever half chronometer watches costing from £24 to £30, are the best

Experience shows that the rate of a watch at rest is not the same as its rate when being transported it is therefore desirable when possible to ascertain the travelling rate of the watch This may be done by sending the watch out for a march every day, the march to last as long as it is expected marches will be on the actual journey After 3 or 4 days a fair rate will be obtained The travelling rate in this sense includes the rate when at rest at night

When every precaution has been taken it will be found that a single watch is very variable in its rate It is thus necessary to carry several five is a good number Each watch should be known by some distinctive letter or number One watch should be used for the time observations and comparisons with all the others should be made before and after the observation

Watch Comparisons—Watches usually beat 5 times in 2 seconds, and the comparison of two watches by ear is difficult the following is a useful method —Let A and B be two observers X and Y two watches Let A take watch X and B watch Y A should make a sharp tap on the table when the seconds hand of X is exactly by eye on some definite second B should record the position of Y to the nearest tenth B now signals to A in the same way and A records A and B now change watches and repeat the operations There are now four values for the difference between the watches and the personal equations of the observers are if constant eliminated

Return Journeys—If a watch be transported from a place A to a place B and then immediately carried back to A and if its errors on local time before starting at A, on arrival at B and on return to A be determined a very good value of the difference of longitude A to B can be found

For if no halt is made at B beyond the ordinary night's rest, the travelling rate can be found by dividing the difference between the errors at A (starting and returning) by the number of days which elapsed

If a halt is made at B the rate at rest should be determined and applied for the period of the halt

Chronometer Journeys by Sea—Sometimes it is necessary to determine the difference of longitude of two places on the sea coast or connected by a navigable river, or on one of the great African lakes In such a case as many box chronometers as can be obtained should be carried and the method of return journeys as above described, should be adopted

Variations in Rate—Watch rates are variable, and the only way to eliminate the effects of such variability is to carry a large number of watches The attached table (p 188) will serve to show the variations which may be expected as judged by comparisons of four watches on a journey in E Africa

LONGITUDE BY OCCULTATION OF A STAR

(5) **Longitude by the Occultation of a Star**—We now come to the determination of longitude by *absolute* methods The rapid movement of the moon amongst the fixed stars gives the means of determining by their relative positions the Greenwich time at the moment of observation, and the difference at the same instant, between the Greenwich time and the local time is the longitude, in time, E or W of Greenwich

These *absolute* determinations of longitude depend only upon observations made on the spot and the best of all absolute methods is undoubtedly the occultation of a star

The *apparent* movement of the moon in the heavens from east to west does not take place at the same rate as that of the stars, but is slower, in other words the *relative*

TABLE of Daily Comparisons of four watches carried from River Sabi to River Limpopo and back, between September 8th and September 28th, 1892

Differences between Watches

1 and 2	Δ	1 and 3	Δ	1 and 4	Δ	Dates
m s		m s		m s		
+ 0 29		+ 0 13		+ 0 34 5		8th Sept
	12		4		11 5	
17		9		23		9th "
	6		8 5		13 5	
9		0 5		9 5		10th "
	8		14		7 5	
0		− 0 14 5		− 0 17		11th "
	12		10 5		16	
− 0 12		25		33		12th "
	11 5		11		19 5	
23 5		36		52 5		13th "
	12		11 5		19 5	
35 5		47 5		1 12		14th "
	13 5		9 5		15	
49		57		27		15th "
	10 5		10 5		18	
59 5		1 7 5		45		16th "
	10		9		17	
1 9 5		18 5		2 2		17th "
	10 5		8		16	
20		26 5		18		18th "
	11		8		12 5	
31		34 5		30 5		19th "
	15		3 5		10 5	
46		38		41		20th "
	7		16 5		30	
53		54 5		3 11		21st "
	9		6 5		13	
2 2		2 1		24		22nd "
	6		10 5		30	
8		11 5		54		23rd "
	1 5		3 5		30	
9 5		25		4 24		24th "
	11 5		4		16 5	
21		29		40 5		25th "
	10 5		7 5		14 5	
31 5		36 5		35		26th "
	3		16 5		28 5	
34 5		53		5 23 5		27th "
	10		8		15	
44 5		3 2		38 5		28th "

Comparisons on this journey were only made to the nearest half second

movement* of the moon among the stars is from west to east at a rate of about one diameter per hour.

It follows therefore that in the course of its monthly revolutions round the earth the moon shuts out for a time from the view of an observer on the earth's surface certain of the fixed stars. The disappearance of a star behind the moon's disc is called its immersion and the re-appearance its emersion; and it follows from what has been said that the immersion will always take place on the eastern limb of the moon and the emersion on the western.

Data for these phenomena are given in the 'Nautical Almanac' under the heading 'Elements of Occultations,' together with the G.M.T. of their occurrence as they would be seen from the earth's centre. The G.M.T. of the phenomena as observed from any station on the earth's surface can be computed having given the latitude and approximate longitude of the station; and thus by observing the L.M.T. at which the phenomena actually occur the difference of time, and therefore of longitude, between the station and Greenwich is determined.

To observe an occultation it is desirable to know beforehand the approximate L.M.T. at which it will take place at the observer's station.

This can be done roughly by applying the approximate longitude of the station (in time) to the G.M.T. of conjunction in R.A. of the moon and the star (which is given in the 'Nautical Almanac'), *adding* the longitude if it is east and *subtracting* if west.

Two hours before the time so found the telescope should be directed at the eastern limb of the moon and search should be made for any star (the magnitude being known is a great assistance) approaching it from a distance equal to one or two diameters of the moon.

There can be little difficulty in identifying the star, owing to the rarity with which these phenomena occur, and a star chart is unnecessary for the purpose.

Parallax in R.A. will have a considerable effect on the time at which the occultations will take place, and with a little practice the observer will be able to save himself a good deal of waiting by studying its probable effects.

Thus he will be able to say whether the hour found from the 'Nautical Almanac' will be accelerated or retarded, and roughly how much, simply by ascertaining whether the moon will be to the east or west of the meridian respectively, and how far.

It is evident, however, that this system of waiting considerable periods of time for an occultation (the actual occurrence of which will in many cases be doubtful up to the last moment) will not commend itself to the skilled observer.

A preferable method is to predict beforehand the circumstances of an occultation. This can be done sufficiently correctly, the accuracy of the prediction depending on the exactness of the position of the observer's station assumed in the calculation.

Thus it can be ascertained—1st, whether the occultation will take place at all; 2nd, the L.M.T. of immersion and emersion; 3rd, the exact position of the moon at these instants; 4th, the points on the moon's disc (measured in degrees of arc from the moon's vertex or from its north point) at which the star will disappear and re-appear; and 5th, whether the immersion and emersion will occur in the moon's bright or dark limb—a most important consideration in actual practice.

A method for predicting the circumstances of an occultation is given in the "explanation" of the 'Nautical Almanac,' and there is also a graphical method devised by Colonel S. C. N. Grant, C.M.G., R.E., which is explained further on.

In 1896 several quantities (q_0, p^1, q^1) were added to the "Elements of Occultations" in the 'Nautical Almanac,' and Mr. A. C. D. Crommelin uses these quantities graphically in an

* Of course the moon has a second relative movement among the stars from N. to S. and *vice versâ* corresponding to its changes of declination. The latter cause is also responsible for the circumstance that the range of fixed stars which are at one time or another occulted is such a wide one.

adaptation of Grant's method (published in pamphlet form by the Royal Geographical Society) and thereby considerably reduces the labour required in applying the latter

One can now, with a little practice predict the whole of the circumstances of an occultation in a quarter of an hour, and the times of immersion and emersion will be found to be correct within 2 or 3 minutes An example of such a prediction is appended

Longitude by the observation of a single occultation cannot ordinarily be depended upon to within less than about 1 second of time (15 seconds of arc or a quarter of a mile on the equator) and therefore whenever possible, more than one phenomenon should be observed at the same place To get full benefit from the accuracy of the method it is necessary to obtain from some fixed observatory the *observed* declination and right ascension of the moon during the night in question so as to correct the co-ordinates given (by prediction) in the 'Nautical Almanac' differences even in the second place of decimals of seconds in these quantities appreciably affect the result of the calculation

Another disadvantage possessed by this method of ascertaining longitudes consists in the scarcity of suitable occultations that are visible at any one place during the year, and a large percentage of those that take place in every climate cannot be observed owing to the state of the weather

Thus in England during the winter months the heavens are generally obscured by clouds, and in the tropics climatic causes such as clouds the dust haze accompanying the Harmattan wind (which makes small stars difficult to see when in close proximity to the full moon), bring about many disappointments

In the 'Nautical Almanac' about 150 stars are given for each month, the occultation of which is visible at some point or other of the earth's surface

In considering any one station however about half of these must be struck out owing to the station being outside the limits* of latitude (last column) within which, only, the occultation will be visible

Even now it by no means follows that because the station is within the stated limits of latitude the occultation will be visible there for in many cases among the stars that still remain the occultations will take place either in daylight or when the moon is below the local horizon to say nothing of the instances in which the occultations will be 'spoiled' by the effects of parallax both in declination and in right ascension more particularly the latter Generally about five satisfactory occultations only will be found to be visible at any one station each month, and in actual practice, with field instruments, dependence can only be placed on an average of two of these being observed

In Northern Nigeria, in 1903 the Anglo French Boundary Commissioners (having two sets of instruments at their disposal) waited two months at one place before a set of three occultations was obtained

The longitudes when worked out had an outside variation from one another of only 9 seconds of time At another place on the other hand, with the second telescope in use during the same period of time, three occultations were observed in a fortnight, the resulting longitudes having a maximum variation of three seconds

It should be stated however, that in all six cases the calculations were worked out without using the *observed* co-ordinates of the moon those predicted in the 'Nautical Almanac' only being available at the time

Occultation of a Planet—This observation can be made with field instruments, but it cannot be expected that the result will be as accurate as in the case of a fixed star owing to the appreciable time taken by the disc of the planet to disappear into or emerge from the moon and the consequent difficulty of judging the moment at which the *centre* of the planet comes in contact with the moon's limb

In making this observation accurately the times should be taken of the *immersion* of the

* Owing to the possible accumulation of neglected quantities these limits should be extended by 1°

first limb of the planet into, and the *emersion* of the second limb from the moon's disc, and a correction can then be applied for the apparent semi-diameter of the planet.

In the calculation also the variations of right ascension and declination of the planet as well as its parallax have to be considered, and further complications arise (especially when considering occultations of Jupiter, Saturn and Saturn's ring) in determining the extent of illumination of the planet's elliptical outline.

Practical Hints.—For field work a telescope with a 3 in. or 3½ in. object glass will be found suitable, but care should be taken that the stand on which it is fixed is firm, and if possible, as an extra precaution against vibration, the object end of the telescope should be secured to the stand by an arm working in conjunction with that attached to the eye end of the instrument for the purposes of elevation.

During observation the telescope should of course be protected as much as possible from the wind by placing it inside a roofless hut, or else within a building having a door or window facing the moon. It will be found, too, that an eye-piece of comparatively low power is preferable for use:—

(*a*) For better definition.

(*b*) To minimise the effects of vibration on the instrument.

A diagonal eye-piece is more convenient than a direct one, as it enables the observer by adjustment of the legs of the telescope to remain seated while waiting, but it should be remembered that the image thus seen, although upright, is inverted in azimuth, and allowance should be made accordingly, when expecting an emersion.

Stars of which the occultations are predicted are nearly all invisible to the naked eye, being of the 5th, 6th or 7th magnitude, for the most part.

These are so small that even when seen through a powerful telescope it may be taken as a general rule that an immersion into or an emersion from the *bright* limb of the moon, even when observed, cannot be depended upon for great accuracy of result. Nor, of course, can observations into or from the dark limb be made until some time after sunset or before sunrise, especially if the moon is in the west or east respectively—though as a matter of interest it may be stated that occultations of large stars are often observed in observatories in broad sunlight. Generally speaking also if the moon is only 15° or less above the horizon, the observation will be impeded by haze.

So much stress has been already laid on the scarcity of suitable occultations at any one place that it need hardly be said, when an opportunity occurs of observing one every precaution should be taken to ensure success; for example, observations for time should be made as shortly as possible before *and after* each phenomenon, so that the error of the chronometer on L.M.T. may be known as nearly as possible.

Also every timekeeper, however experienced, is always liable to make mistakes in booking times; when possible, therefore, there should be two timekeepers, with separate chronometers, which latter should be compared immediately after the observation.

If this cannot be done the observer should use a stop watch as a check on the timekeeper, and the procedure would be as follows:—

At the moment that the signal is given by the observer, he should start the stop-watch; then after the time has been booked by the timekeeper, the observer should go to the chronometer and stop his watch when the seconds hand of the chronometer has exactly reached some minute or half minute; the time indicated by the chronometer should then be written down, and the number of seconds shown by the stop watch subtracted from it.

If the time thus arrived at is the same, within one or two tenths of a second, as that booked by the timekeeper, the latter may be accepted as correct.

It should be borne in mind that an ordinary stop watch cannot be relied upon after it has been going several minutes.

If an accurate stop-watch is available, it would perhaps be simpler to start it from the chronometer at some exact minute shortly before the phenomenon is due to take place, and to then stop it at the moment the signal is given

The hours and minutes can then be booked from the chronometer, and the seconds and fractions of a second from the stop-watch

As regards the calculation of longitudes from occultations there are several ways of doing this, all depending on the same principle

The method described in the 'Connaissance des Temps' for example, is different in form from that given in the 'Hints to Travellers,' which is taken from Raper, and an example of which is appended the latter will be found to give good results—although formidable in appearance it is not so in reality, and depends almost entirely on accurate arithmetic

In using it it should be noted that it is preferable to use five places of decimals in the logs instead of four only, ternary proportional logs to five places being given in Chambers' tables

In conclusion the attention of the beginner might with advantage be drawn to two points in this calculation —

(i) Care should be taken to *reduce to date* the assumed places of stars given in the 'Mean Places of Occultation Stars' in the 'Nautical Almanac,' by adding (arithmetically) the quantities $\Delta\alpha$ $\Delta\delta$ (columns 4 and 5 in 'Elements of Occultations' Nautical Almanac) to the R A and declination respectively given under 'Mean Places of Occultation Stars,' Nautical Almanac'

It will be noticed that $\Delta\alpha$ is given in seconds of time and $\Delta\delta$ in seconds of arc

(ii) Table XVIII for reducing the observed latitude to geocentric latitude is based on Clarke's first figure (1858) the value for compression being $\frac{1}{294.26}$

A compression of $\frac{1}{300}$ has been used in the following example, the value adopted makes an appreciable difference in the resulting longitude

In the calculation of longitude by occultations the value for the moon's semi-diameter given on p III Nautical Almanac, should be diminished in the proportion of 15′ 34″·09 to 15′ 32″·65 or ·93409 : ·93265 The former value is the mean semi-diameter adopted for transits, and the latter that adopted for occultations'

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COMPUTATION

OF THE

LONGITUDE OF BUSA

FROM THE

IMMERSION OF 76 LEONIS, ON MAY 6TH, 1903

Computation of the Longitude of Busa from the Immersion of 76 Leonis, on May 6th, 1903

		h m s
Chron time of Immersion	=	10 1 13 8
Watch slow	= +	0 10 8·0*
M T at Place	=	10 11 21 8
Approx Longitude E	= −	0 24 7 5
Corrected Green Date	=	9 50 14 3

* *Error of Watch*

		m s
By a Leonis (W)	=	10 8 04 slow
By a Coronæ (E)	=	10 7 90 slow
Mean	=	10 7 97 slow

☽'s Semi diameter noon 6th	=	15 54 15 (p III N A)
Do midnight	=	15 50 54
12 hourly diff	=	0 3 61
3 61 × 9 837 ÷ 12	=	0 2 96
☽'s Semi diameter for Green Date	=	15 51 19

		′ ″
☽'s Hor Parall noon 6th May	=	58 15 77 (p III N A)
Do midnight do	=	58 2 53
12 hourly difference	=	0 13 24
Now 9h 50m 14 3s = 9 837h		
9 837 × 13 24 ÷ 12	=	0 10 85
☽'s Hor Parall for Green Date	=	58 4 92
Correction for Latitude	= −	0 0 68 (Table XVII)
☽'s Reduced Hor Parall	=	58 4 24

		° ′ ″
Latitude	=	14 25 24
Reduction	=	0 5 30 9 (Table XVIII)
Geoc Latitude	=	14 19 53 1

		° ′ ″
☽'s Declination at 9h	=	2 25 35 0 (p VI, N A)
Correction	=	**0 8 58 37
☽'s Reduced Declination	=	2 16 36 63 N

		h m s
Star's R A		11 13 56 16 (*a*)
Correction	= +	0 0 1 90 (*b*)
	=	11 13 58 06

** Var in 1′ = 10 716
50 24 × 10 716 = 8 58 37

		h m s
Sid Time of G M N	=	2 52 49 13 (p II 'N A')
Acceleration (Table XXXI 'Hints to Travellers') { 9h	=	0 1 28 71
50m	=	0 0 8 21
14s	=	0 0 0·04
Reduced Sid Time	=	2 54 17·09
M T at Place	=	10 14 21 80
R A of Meridian	=	13 8 38 89
Do Star	=	11 13 58 06
Star's Hour Angle	=	1 54 40 83 W
	=	28° 40′ 12 45 (in arc)

		° ′ ″
Star's Declination	= +	2 10 56 0(N) (*a*)
Correction		0 0 14 9 (*b*)
		2 10 41 1

(*a*) (*a*) Mean Places of Occultation Stars, 'N A'
(*b*) (*b*) '$\Delta\alpha$ $\Delta\delta$ in Elements of Occultation,' 'N A'

		° ′ ″
Star's Declination	=	2 10 41 1
Arcs (A C)	=	0 14 21 6 (+ Latitude and Declination are same name)
		2 25 2 7
Arc B	= −	0 1 55 57 (−, Hour Angle is less than 6 hrs)

) s R l ed Hor P rall *Prol	1913
Co c G oc L t t de	10 00037
See S ar D n t on	10 00011
	1 09800

*Prolog of 14 1 84 Ar A

	=	19139
S	=	10 013 3
C e	=	11 19010
See S ars Ho r A gle	=	10 0 80
		1 98 01

*Prolog of 1 59 57 Arc B

	=	1313
	=	10 01373
(e	=	10 318
		8.10
		2
		1 01804
		1 8 00 (n ta t)
Cot Stars De l n t o	=	11 11981
		4 01089

*I r l g f 0 0 24 = Arc C

Pr p r l De n on	23 10 13
) D l o	- 16 36 3
D ff e	0 6 33 5
S l an ter	0 15 51 19
D ff c	0 9 1 69
S	2 21 6)

Co	9 90962
Co t	1 1 610
½ *Prolog	01359
½ *Pro og	1 910
	2 7161

*P olog of 0 83 I rt I

9 996
1 17010
8910 (u of 3 logs l to f nd Arc C)
1 999 1

*I rol of 1 48 f Part II

St r s R A	11 13 58 06	
	0 0 57 87	(Part I I ers o)
	11 13 0 2	
	0 1 48 0 5	(Part II + H of Mer l)
s R A	11 14 48 34	
D l l r (	11 1 59 8	{ 9 }
D o ( )	11 15 9 38	{ 10 l rs } (l M I M A)
D f b n l (	1 48 5	
D (l ( )	5f	

*P o o	1 9 12
*Ar thu et cal comple nent of Prolog	8 0 9046
Co tant	4 100
	540 8

*Prolog of 50 15 55 x

H r f ( )	l n s
	9 J
x	0 9 15 5
G M T of I r	9 50 15 55
L M T of In r on	10 14 21 80
D fferen e of Lo g t	24 6 25 L
	f 1 33 75 E (n Arc)

* l f l Pr P l L r l) f Tl l l l rot A c l l 3 l oc l Tl s T k l s f ra t t (tl f l T r A lu l t se o d Tl r lt ll t e tl l l u e l

NOTE For gr at r accur cy tl e *obse red* places of tl moon lo ll b obta n l from Gre w h and ub t tute l n the bove n l t e resultin long tu le hou l l e u d at tl beg n ng n lo g tl e calcula t on econ l t m

Computation of the Longitude of Busa from the Immersion of 76 Leonis, on May 6th, 1903

		h m s
Chron time of Immersion	=	10 4 13 8
Watch slow	= +	0 10 8 0*
M T at Place	=	10 14 21 8
Approx Longitude	= −	0 24 7 5
Corrected Green Date	=	9 50 14 3

*** Error of Watch**

	m s
By α Leonis (W)	10 8 04 slow
By α Coronæ (E)	− 10 7 90 slow
Mean	10 7 97 slow

☽'s Semi-diameter noon 6th	=	15 54 15 (p III, 'N A')
Do midnight	=	15 50 54
12 hourly diff	=	0 3 61
3 61 × 9 837 ÷ 12	=	0 2 96
☽'s Semi diameter for Green Date	=	15 51 19

		′
☽'s Hor Parall noon 6th May	=	58 15 77 (p III 'N A')
Do midnight do	=	58 2 53
12 hourly difference	=	0 13 24
Now 9h 50m 14 3s = 9 837h 9 837 × 13 24 ÷ 12	=	0 10 85
☽'s Hor Parall for Green Date	=	58 4 92
Correction for Latitude	= −	0 0 68 (Table XVII)
☽'s Reduced Hor Parall	=	58 4 24

Latitude	=	14° 25 24
Reduction	=	0 5 30 9 (Table XVIII)
Geoc Latitude	=	14 19 53 1

☽'s Declination at 9h	=	2° 25′ 35 0 (p VI, 'N A')
Correction	=	**0 8 58 37
☽'s Reduced Declination	=	2 16 36 63 N

		h m s
Star's R A	=	11 13 56 16 (a)
Correction	= +	0 0 1 90 (b)
	=	11 13 58 06

** Var in 1′ 10 716
50 24 × 10 716 = 8 58 37

Star's Declination	= +	2° 10′ 56 0 (N) (a)
Correction	=	0 0 14 9 (b)
		2 10 41 1

(a) (a) Mean Places of Occultation Stars 'N A'
(b) (b) 'Δα Δδ' in Elements of Occultation 'N A'

		h m s
Sid Time of G M N	=	2 52 40 13 (p II 'N A')
Acceleration (Table XXXI 'Hints to Travellers') 9h	=	0 1 28 71
50m	=	0 0 8 21
14s	=	0 0 0 04
Reduced Sid Time	=	2 54 17 09
M T at Place	=	10 14 21 80
R A of Meridian	=	13 8 38 89
Do Star	=	11 13 58 06
Star's Hour Angle	=	1 54 40 83 W
	=	28° 40 12 45 (in arc)

Star's Declination	=	2° 10′ 41 1
Arc (A C)	=	0 14 21 6 (+, Latitude and Declination are same name)
		2 25 2 7
Arc B	= −	0 1 52 57 (−, Hour Angle is less than 6 hrs)
Prepared Declination	=	2 23 10 13

☽'s Reduced Hor Parall *Prolog = 49132
Co sec Geoc Latitude = 10 00637
Sec Star's Declination = 10 00031
1 09800

*Prolog of 14 21 84 = Arc A

= 49132
Sec = 10 01373
Cosec = 11 42016
Sec Star's Hour Angle = 10 05680
1 98201

= *Prolog of 1 52 57 = Arc B

= 49132
= 10 01373
Cosec = 10 31807
82102
2
1 64804
1 8200 (constant)
Cot Star's Declination = 11 41984
4 64988

= *Prolog of 0 0 24 = Arc C

Prepared Declination	2 23 10 13
☽'s Declination	16 36 43
Difference	0 6 33 5
☽'s Semi diameter	0 15 51 19
Difference	0 9 17 69
Sum	0 22 24 49

Co	9 99962
Constant	1 17610
½ *Prolog	64352
½ *Prolog	45210
	2 27164 = *Prolog of 0 57 783 Part I

= 9 99962
− 1 17610
82102 (sum of 3 logs used to find Arc C)
1 99974

*Prolog of 1 48′ 063 Part II

		h m s	
Star's R A	=	11 13 58 06	
	−	0 0 57 783	(Part I — Immersion)
		11 13 0 277	
	+	0 1 48 063	(Part II +) II of Merid)
☽'s R A		(i) 11 14 48 340	
Do preceding hour	=	(ii) 11 12 9 84	{ 9 hrs / 10 hr } (N A)
Do following hour	=	(iii) 11 15 9 38	
Difference between (i) and (ii)	=	0 1 48 502	
Do (ii) and (iii)	=	0 2 9 56	

*Prolog		1 19 912
*Arithmetical complement of Prolog	−	8 07 016
Constant	=	177100
		54058

= *Prolog of 50 15 55 = x

		h m s
Hour of (ii)	=	9 0 9
x	=	0 50 15 55
G M T of Immersion	=	9 50 15 55
L M T of Immersion	=	10 14 21 80
Difference of Longitude	=	0 21 6 25 I
	=	6° 1 33 75 F (in Arc)

* To find the Prolog (Proportional Logarithm) of TIME less than 3 hours or of ARC less than 3° proceed thus From 4 03342 subtract the logarithm of the TIME or ARC reduced to seconds The result will be the Prolog required

NOTE —For greater accuracy the *observed* places of the moon should be obtained from Greenwich and substituted in the above and the resulting longitude should be used at the beginning in doing the calculation a second time

*Example of Prediction of Occultation by Graphic Method

Let us examine the circumstances of the occultation of 76 Leonis on May 6th, 1903, at Busa (a French military post in the Western Soudan)

Latitude = 14° 25′ 24″ N (by observation)

Approx Longitude = 24m 7·5s E of Greenwich (in time)

Data —

76 Leonis, Magnitude 6·5		‘Nautical Almanac,’ “Elements of Occultations”
T_0 G M T of conjunction in R A = 9h 26m 58s		
Limits of latitude, 44 N to 25 S		
q_0 = + 0·1736		
p' = ·5572		
q = − ·1815		

	h	m	s	
Assumed R A 1903 =	11	13	56·16	‘Nautical Almanac,’ “Mean Places of Occultation Stars”
Assumed Declination, 1903 = +	2	10	56·0 (i e, N)	

Reduction to Date —

$\Delta\alpha$ + 1s·90	‘Nautical Almanac,’ “Elements of Occultations”
$\Delta\delta$ = − 14″·9	

There are two diagrams to be used Plate XXVII is †Grant's, and from it we obtain the parallaxes in declination and R A of the moon, which are transferred to Plate XXVIII (Crommelin's) this second diagram being drawn on such a scale that the parallaxes found on Plate XXVII can be entered on it unaltered (It is in this that the advantage of Crommelin's adaptation of Grant's method is to be found) On the second diagram the remainder of the work is done

Plate XXVII represents an orthographic projection of earth (OO, being the equator) The horizontal lines are parallels of latitude and the up and down lines hour circles

In Plate XXVIII the circle in the centre represents the moon, the radius of which is taken as being constant and 0·2725 of the earth's radius

Latitude —It will be seen that Busa is situated well within the limits of latitude given in the data

Moon From p IV of the ‘Nautical Almanac for May, 1903, it will be seen that the meridian passage of the moon is at 8h 17m 24s G M T

Now G M T of occultation = 9h 26m 58s, from which it will be seen that the moon will be well up above the local horizon —in fact near, and to the W of, the meridian

The effect of the moon being to the W of the meridian will be to *delay* the occultation which may be expected to occur about 10 P M

Sunlight —Evidently at 10 P M the sun will not interfere with the observation

Parallax The effects of parallax in declination and in R A have now to be considered, and we come to the use of the diagram

On Plate XXVII draw a horizontal line DD, to represent the latitude of Busa 14° 25′ 24″ N (If the place had a south latitude, DD would be drawn below OO)

Draw a straight line through C, the centre of the diagram to F, the point on the circumference of the circle representing the moon's declination at 9 or 10 P M on May 6th (F would be below OO if the moon had a S declination)

Parallax in Declination —Now from D, inwards towards D, set off the moon's hour angles at T_0, T_1, T_2, i e, at the G M T of conjunction in R A, one hour later and two hours

* See a paper by Mr A C D Crommelin published in 1902 by the Royal Geographical Society entitled "An Adaptation of Major Grant's Graphical Method of Predicting Occultations to the Elements given in the Nautical Almanac"

† Taken from a diagram by Colonel S C N Grant C M G, R E

later (Note —If the moon had been near the western horizon, the effects of parallax in R A would have been much greater, and we would have selected the hour angles at T_1, T T_3 instead of the above Similarly if the moon had been slightly to the *east* of the meridian we would have substituted the hour angles at I 1 I and I_1 An indication of which hours to select is soon obtained with a little experience in predictions)

From these points on the line DD draw perpendiculars to CP, and call their lengths *a*, *l*, *c*

Thus *a l*, *c* are the values of the parallax in declination at I T_1 and T and will be transferred direct a little later, to Plate XXVIII

Parallax in R.A—For the values of parallax in R A set off the same hour angles along the latitude line, but this time from D towards D , then D*l* D and D*f* will be the values of parallax in R A to be transferred to Plate XXVIII

We have now done with Plate XXVII It might be noted here, that if the hour angle exceeds 6 hours, in plotting the parallax in R A on the latitude line we first move outwards to the circumference of the circle and then count inwards again from that point, for the excess on 6 hours Also the feet of the perpendiculars *a l c*, will be to the *right* of C if the moon is above the horizon and to the *left* if it is below

Hour Angles—To find the hour angles at T_0, T_1 and T we proceed as follows —

		h	m	s	
9h 26m 58s (T_0)		9	26	58	
accelerated for 9h	–		1	28 71	
accelerated for 26m	=			4 27	
accelerated for 58s				16	
		9	28	31 14	
Sidereal time at G M N (May 6th)	–	2	52	10 13	
Sidereal time at Greenwich		12	21	11 27	
East longitude in time	+		24	7 5	
Sidereal time at Bushire		12	45	18 77	
		12	45	0	(nearly)
as seconds cannot be plotted on Plate XXVII					
Moon's R A at T (p XI "Nautical Almanac")		11	14	0	(nearly)
☽'s hour angle at T		1	31	0 W	of the meridian

Now the variation in R A of the moon in 10 minutes at T 21 59s (about)

Variation in 1 hour = 129 54s 2m 9 5s

Whence ☽'s hour angle at I (1h 31m) + (1h)* – (2m 9 5s) 2h 29m (nearly)

Similarly ☽'s hour angle at I 3h 27m (nearly)

Next in Plate XXVIII —

Make OG_0 p_0† on the vertical line through O

Draw G_0M horizontally *to the left* of G and equal to *p*

Then plot MG_1 *p*‡

Join G G_1 and produce it both ways marking off equal intervals G G_1 G_1G and G G 1

Then G G_1 G &c are the geocentric places of the moon at the intervals T T_1 T &c respectively

* – because ☽ is W of meridian + if ☽ is E of meridian
† When *q* is + G_0 is above O when *q* is – G_0 is below O
‡ *q* + G_1 M, *q* – G M

Next, *to get the apparent position of the moon's centre at these times as seen from the station under consideration*, we plot at G_0, G_1, G_2 the parallaxes in declination and R.A. obtained in Plate XXVII.

When the points marked off on the latitude line in Plate XXVII are *above* the "moon line," the parallaxes in declination are plotted *downwards* from G_0, G_1, G_2; when *below*, the parallaxes are plotted upwards.

Similarly, if the hour angles at T_0, T_1, T_2 are *east*, the parallaxes in R.A. are plotted to the left of G_0, G_1, G_2; if *west*, to the right.

Then L_0, L_1, L_2 is the *apparent* path of the moon's centre, as seen at the station.

If this line cuts the circle in the centre of Plate XXVIII, the occultation will take place, and the points I, I_1 correspond to the immersion and emersion of the star respectively.

To find the times at these instants, select that part of the scale (fig. 2, Plate XXVIII) on which a distance equal to L_0, L_1 corresponds to 1 hour.

It will be seen that the distance L I corresponds to about 22m. 20s. Whence

	h	m	s
G, time of immersion =	9	26	58
+		22	20
	9	49	18
+		24	7.5
and L.M.T. of immersion =	10	13	25.5 (Long. E.)

(Note the actual L.M.T. of immersion was observed to be 10h. 14m. 21.8s.)

Treating L_1 L and L_1 I_1 similarly, we get the L.M.T. of emersion = 11h. 2m. 20s.

Angles from Moon's Vertex.—The angles on the graduated arc in Plate XXVIII, at I and I_1, are the angles from the moon's north point (always reckoned to the *left*) of immersion and emersion, (in the example 68° and 349°) respectively.

It will be easier in the observation to judge these angles by eye, however, if they are reckoned from the moon's vertex.

Now, at the time T_0, the line L_0G_0 passes through the zenith of the observer, and so by drawing a line through O, parallel to L_0G_0, we find that the vertex of the moon at T_0 is 60° to the *left* of the north point.

For exactness the line G_0 G_1 should be divided proportionally (at *g*) as L L_1 is by the point I, and a line should be then drawn through O parallel to I*g*. We thus get the moon's vertex *at the instant of immersion*, which in the example is 64° to the *left* of the moon's north point.

Whence the angle of immersion of the star is 68° − 64° = 4° to the left of the moon's vertex.

Similarly, the angle of emersion is 349° − 70° = 279° to the left of the moon's vertex.

Thus the occultation will occur as shown in fig. 65. It will of course appear, if seen through the diagonal eye-piece, as shown in fig. 66.

Moon's Limb—Illumination.—Further, we can foretell [illegible] immersion and emersion will take place in or from the bright or dark limb of the [illegible]

On p. XII, "Nautical Almanac" for May, 1903, under "[illegible]," we find that the moon is full on the 11th. Therefore on the 6th about [illegible] disc will remain unilluminated, the eastern limb bein[illegible] [illegible]

The immersion will therefore take p[illegible] dark limb, [illegible] from the bright [illegible] For all practical purposes this w[illegible] nt, but it sh[illegible] of the path of the star is alw[illegible] [illegible] to th[illegible] it is p[illegible]

[illegible] to take place in the same limb, [illegible]

In the f[illegible] such an occultation would [illegible] [illegible] the [illegible] will be [illegible] by errors in th[illegible]

method is here given by which the angle that the terminator (i.e. the line joining the ends of the bright limb) makes with the north point can be determined

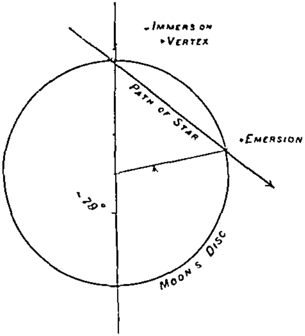


Fig 65

In the Ephemeris for Physical Observations of the Moon by Mr Crommelin published annually in the Monthly Notices of the Royal Astronomical Society the angle between the

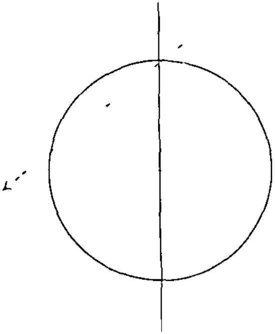


Fig 66

north point and the north pole of the moon is given If this ephemeris is available we could enter on Plate XXVIII the line joining the moon's poles which for all practical purposes could

be taken as the 'terminator', for as the inclination of the moon's axis to the plane of the ecliptic is small, the junctions of the bright limb and dark limb will in general be near the poles, and it may be taken that the angle between them will never exceed 5°, except when the moon is very young or old, or almost full

An exact solution can be arrived at by the following computation —

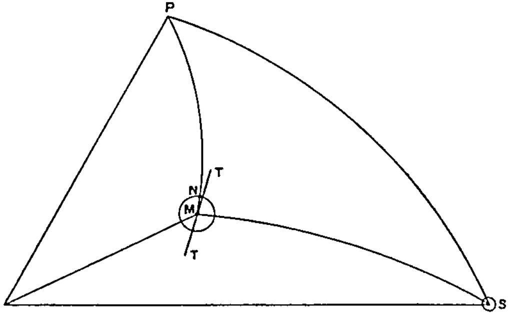


Fig 67

M is the moon, E the earth, S the sun, P the pole, and TT the terminator, at right angles to MS

Required to find the angle PMT —

Now, in the △ PMS, the two sides PM and PS (the north polar distances of the moon and sun) and the included angle MPS (difference of their R A) are known

Hence the triangle can be solved and PMS, and therefore PMT found (Note —N is the moon's north point)

To insert the amount of the illuminated disc on Plate XXVIII, use must be made of *Crommelin's ephemeris which gives the longitude of the terminator, or its distance from the centre of the disc*

Moon's Altitude—Plate XXVII can also be utilised to ascertain the *altitude* of the moon, at the instants of immersion and emersion as follows —

> Measure a distance equal to $L_0 G_0$ vertically upwards from C, in Plate XXVII, and note what parallel of latitude is reached (26° in the example)
>
> This gives us the zenith distance of the moon, and the altitude

$$= 90° - \text{zenith distance}$$
$$= 90° - 26° = 64° \text{ for immersion}$$
$$\text{and } = 90° - 38\tfrac{1}{2}° = 51\tfrac{1}{2}° \text{ for emersion}$$

The foregoing explanation is rather a long one, as the intention has been to provide for every example that may occur

Summary.—In brief, the procedure is as follows —

> On Plate XXVII draw the "moon line" and the latitude line and set off on the latter the various hour angles

Draw the perpendiculars for the parallaxes in declination

On Plate XXVIII plot q p, q, and the parallaxes in declination, and R A, and we thus get L_{0}, L_{1}, L.

The computation for longitude, from the *observed* immersion of the star dealt with in the prediction, is appended (See p 194)

LONGITUDE BY PHOTOGRAPHS OF THE MOON

(6) Longitudes by Photographs of the Moon —A photographic camera on a stable and rigid stand is set up so that the image of the moon is about in the centre of the plate, and a series of instantaneous exposures is made, allowing such an interval between the exposures that the moon images will not overlap, *i e*, from $1\frac{1}{2}$ minutes to $2\frac{1}{2}$ minutes according to the moon's age

After a set of, say, seven moon exposures the camera is left untouched until bright stars of approximately the same declination as the moon have arrived at the same point in the heavens The camera is then opened for periods of 15 to 30 seconds and the stars allowed to impress their trails on the plate The star exposures should be repeated four or five times

There should be at least the trails of two stars on the plate The local time of each moon and star exposure must be known

From such a negative it is clear that the right ascension of each moon image can be determined, and hence the G M T and longitude The positions of the centres of the moon images and of the star exposures are measured by a micrometer and are referred to two co-ordinate axes approximately parallel and perpendicular to the meridian through the centre

The method in its present form is due to Major E H Hills, C M G, R E Somewhat similar, though less practical, methods had been previously suggested by Dr Schlichter in England, and Dr Runge in Germany For a complete discussion see the Memoirs of the Royal Astronomical Society Vol LIII, 1897, or Wilson's Topographic Surveying' Wiley and Sons, New York

The accuracy of the method in the field has not been satisfactorily determined, it is perhaps fair to assume that under ordinary conditions the probable error of longitude by one plate would be less than a minute of arc If time permitted the building of an isolated pillar, this could be reduced The actual error of the mean of five plates was found at Chatham (in 1894, 1895) to be 0 71 second or about 10 seconds of arc The measurements and calculations are laborious, and are not readily made during an expedition

LONGITUDE BY MOON CULMINATING STARS

(7) Longitude by Moon Culminating Stars —The moon moves so rapidly in right ascension that, if at any instant we can determine its right ascension we can from this determine the Greenwich time at that instant and hence the longitude Its right ascension at meridian transit can be most accurately found by the interval in sidereal time between its transit and that of some neighbouring well known star

Such stars are called moon culminating stars and are those which have nearly the same right ascension and declination as the moon A list of these will be found for each day of the year in the Nautical Almanac (p 456 *et seq*)

The observation must be made with a transit instrument or transit theodolite, set in the plane of the meridian The diaphragm of the instrument should be fitted with at least three vertical wires

To prepare for the observation select a star from those given in the list of moon culminating stars and note its R A Next determine the moon's R A at the moment of transit Use an approximate value for the longitude and multiply the difference in longitude from Greenwich in hours by the variation in moon's R A, if longitude is L, subtract this

from R.A. at Greenwich, if W., add. The result will be the approximate R.A. of the bright limb at local transit. Having the right ascensions of star and moon at transit (i.e., the local sidereal times) the mean times can be computed if working with a M.T. chronometer.

Next compute the setting for zenith distance of the star in the usual way ($\zeta = \phi - \delta$).

Having prepared this information beforehand, note carefully the sidereal times to the nearest tenth of a second of the transits of star and bright limb of the moon over each wire. *From these deduce the sidereal interval between the star's transit and that of the moon's bright limb.*

Computation.—In the 'Nautical Almanac,' the R.A. of the bright limb only is given; the R.A. of the other limb may be obtained by adding or subtracting twice the "sidereal time of passage of semi-diameter." It may be necessary to do this, since near full moon the limb changes from I to II (these figures in the 'Nautical Almanac' indicate the limbs which come first or last to the meridian), and the R.A.'s taken from the 'Nautical Almanac' must be those of the limb observed.

Take the R.A.'s of the observed limb from the 'Nautical Almanac,' put them in order and take their differences as far as the fourth difference, thus:—

	R.A. ☽'s bright limb	Differences			
		1st	2nd	3rd	4th
Upper passage preceding day	A_{-2}				
		a_{-2}			
Lower " "	A_{-1}		b_{-1}		
		a_{-1}		c_{-1}	
Upper passage corresponding day	A_0		b_0		d_0
		a_1		c_1	
Lower following	A_1		b_1		
		a_2			
Upper "	A_2				

The first difference,

$$a_{-2} = A_{-1} - A_{-2}$$
$$a_{-1} = A_0 - A_{-1}, \text{ \&c.}$$

The second difference,

$$b_{-1} = a_{-1} - a_{-2}$$
$$b_0 = a_1 - a_{-1}, \text{ \&c.}$$

The R.A. of the moon's bright limb at transit in W. longitude L will be

$$A_0 + n a_1 + \frac{n(n-1)}{1 \cdot 2} b_0 + \frac{(n+1)(n)(n-1)}{1 \cdot 2 \cdot 3} c_1 + \frac{(n+1)(n)(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot 4} d_0,$$

where $n = \frac{L}{12}$, L being expressed in hours.

If the longitude is E., use the same formula but take the following list of R.A.'s of moon's bright limb:—

Lower passage two days before	A_{-2}
Upper passage preceding	A_{-1}
Lower " "	A_0
Upper corresponding day	A_1
Lower following	A_2

Then if the E. longitude is L, $n = \frac{12 - L}{12}$.

An approximate value for the longitude will be known, assume two values for the longitude one a few (1 to 5) minutes greater than the approximate value, and one a few minutes less. Work out the R A of moon's bright limb for these two values and find by proportional parts what alteration must be applied to the smaller longitude in order that the right ascension by formula may agree with the right ascension by observation.

The smaller longitude so corrected is the longitude required. *Great care must be taken to apply the signs strictly according to the rules of algebra.*

As regards the accuracy of a longitude by moon culminations an error of a second of time produces about 30 seconds error in the longitude. We may assume in the ordinary case an error of $\frac{1}{3}$ second and a resulting error of 10 seconds of longitude. As a rule, only about eight observations can be taken in a month so that after a month's work the mean of the results would be about 3 or 4 seconds in error or say nearly 1 minute of arc.

(8) **Longitude by Altitudes of the Moon.**—A fair method, but one not often used. For the details see Manual of Surveying for India. An error of a second of time in an altitude will produce at least 30 seconds of time in longitude sometimes considerably more. An error of 1 second in zenith distance produces at least 2 seconds of time in longitude. With a small theodolite we may expect an error in time of $\frac{1}{3}$ second and in arc (after four altitudes on two faces) of 5 seconds, so that the resulting error will be about $\sqrt{10 + 10^2}$, say 14 seconds of time.

(9) **Lunar Distances.**—A rough method, the observations cannot be taken with a theodolite.

(10) **Eclipses of Jupiter's Satellites.**—A very rough method.

These last two methods should never be used by the topographical surveyor or explorer.

NOTE.—It is of great importance that the explorer and surveyor should have clear ideas on the subject of the relative accuracy of the different methods available for determining longitude.

It should be understood that where longitude can be carried along by a triangulation this should always be done. Excellent results also can be got by telegraphic signals, the longitude of Mandalay was thus determined by Colonel Holdich, with an error of only 1·2 seconds of arc. (Such accuracy cannot, however, be always expected.)

Table showing the Terminal Error in Longitude which might be expected after a March of 300 miles in a Hilly Tropical Country

	Method	Probable error in longitude
Relative Methods	1 Triangulation	100 yards to $\frac{1}{4}$ mile
	2 Telegraph	$\frac{1}{8}$ to $\frac{1}{4}$ mile
	3 Latitudes and azimuths (N and S)	$\frac{1}{2}$ mile
	4 Chronometers (5)	1 mile
Absolute Methods	5 Occultation	$\frac{1}{4}$ mile
	6 Moon photographs (1 plate)	1 mile
	7 Moon culminations (8 nights)	1 mile
	8 Moon altitudes (1 night)	4 miles
	9 Lunar distance	10 miles
	10 Eclipses of Jupiter's Satellites	over 10 miles

CHAPTER XVII

THE THEORY OF ERRORS AS APPLIED TO TOPOGRAPHICAL WORK

An error of observation is the result of subtracting the true value from the measured value of a quantity. Errors are of three kinds:

(1) Mistakes, as for example that which results from reading a micrometer head the wrong way, and booking, say, 38 instead of 22 seconds.

(2) Systematic or constant errors, as for example that which results from using a steel tape without applying the correction for temperature, or the constant personal error of an observer in estimating the moment when a star crosses a wire in a theodolite.

(3) Errors which are neither mistakes nor systematic but which are the combined result of a number of small causes beyond the control of the observer, and which are subject to the laws of probability. Thus in intersecting a luminous trigonometrical signal the observer at each intersection tries to put the wire through the centre of the light, but the centre is an indefinite spot and no two intersections will be in precisely the same place.

Or, to take another example, suppose a base two miles long to be measured with a steel tape reading to hundredths of a foot; it is clear that it is exceedingly unlikely that a series of measurements will give exactly the same values, even when all the proper corrections have been applied.

It is to this third class that the name **error** is strictly to be applied, and the following remarks deal only with such errors.

Frequency of errors.—The following will be found on consideration to be the laws which govern the frequency of errors:—

(1) Small errors are more frequent than large ones.

(2) Positive and negative errors are equally likely.

(3) Very large errors do not occur.

Thus in measuring a topographical base with a steel tape it is to be expected that a measurement will be some hundredths of a foot in error, it is somewhat less likely that it will be tenths of a foot in error, and far less likely that it will be feet in error; also any measurement is just as likely to be in excess as in defect, and it is exceedingly unlikely that any measurement will be, say, ten yards in error.

From the second law it is clear that any expression giving the probability of an error of a given size will involve only even powers of the error.

It is also clear from the first and third laws that if a curve is drawn of which the abscissæ represent the sizes of the errors and the ordinates the probability of such errors occurring, the form of the curve would be that shown in fig. 68.

If we imagine every finite error to be composed of an infinitely great number of very small equal elementary errors which are equally likely to be positive as negative, it can be shown that y, the probability of the occurrence of an error x, can be represented by the expression:—

$$\frac{h}{\sqrt{\pi}} e^{-h^2x^2}\,dx \qquad (1)$$

where h is a constant.

This expression gives a small probability for any error however large, and to this extent differs from experience. This probability of a large error derived from this equation is so

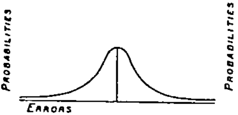


Fig 68

small as to be negligible and results from the hypothesis that finite errors are the result of the algebraic sum of an indefinitely large number of very small elementary errors.

The probability that an error lies between two limits x_1, x_2 is*

$$\frac{h}{\sqrt{\pi}}\int_{x_1}^{x_2} e^{-h^2x^2}\,dx \qquad \text{(ii)}$$

Residuals.—If a number of measurements are made of the same quantity, the sum of the measured values divided by the number of measurements is the **mean**, and a **residual** is the result of subtracting the mean from a measured value.

With a large number of observations the mean tends to become the true value, and the residuals to become errors.

Probable Error.—In an indefinitely large series of errors the **probable error** is that error than which there are as many errors greater as there are smaller; *i.e.*, the chance that any error taken at random is greater than the probable error is the same as that it is smaller.

The Probable Error of the Arithmetic Mean of a series of observations

$$= 0.674\sqrt{\frac{\Sigma r^2}{n(n-1)}},$$

when n is the number of observations and Σr^2 is the sum of the squares of the residuals.

The probable error is a valuable means of comparing the accuracy of sets of observations of the same class. Thus suppose two observers A and B have measured the same angle with the same instrument and have obtained the following results (seconds only are given):—

A	r	r^2	B	r	r^2
47	0	0	48	3	9
49	2	4	46	1	1
50	3	9	49	4	16
42	5	25	41	4	16
48	1	1	47	2	4
46	1	1	39	6	36
Mean 47	$\Sigma r^2 = 40$		45	$\Sigma r^2 = 82$	

The probable error of the mean of A's set is

$$.67\sqrt{\tfrac{40}{30}} = .78 \text{ and of B's set is } .67\sqrt{\tfrac{82}{30}} = 1.11$$

A's set is thus the most reliable.

* Note.—For the investigation of this expression and of the Theory of Errors generally, see 'Text Book on the Method of Least Squares.' Merriman.—J. Wiley & Sons, New York.

The means would be expressed as

$$47'' \pm 0{\cdot}78 \text{ and } 45 \pm 1{\cdot}11$$

Owing to the fact that it is very difficult to free observations of any kind from constant or systematic errors, the probable error obtained in this way is better considered as a test of the relative value of similar observations than as a statement of the amount by which the mean of a series is probably in error. Thus the probable error of angles in a geodetic triangulation derived from the inter-agreement of observed values of the angles is generally smaller than that derived from a consideration of the triangular errors.

Law of the Propagation of Error.—*The probable error of the sum or difference of two quantities is equal to the square root of the sum of the squares of their probable errors*, and so on for any number of quantities. This rule is of frequent use in all kinds of survey work.

As an example suppose that the probable error of a carefully measured base is ± 1·2 inches, and that subsequently an extension is measured with a probable error of ± 0·7 inch, then the probable error of the whole base will be $\sqrt{(1{\cdot}2^2 + {\cdot}7^2)} = 1{\cdot}4$.

The probable error of the arithmetic mean = probable error of a single observation divided by the square root of the number of observations.

The probable error of the mean therefore varies inversely as the square root of the number of observations.

Thus suppose the probable error of a determination of longitude by a single moon culmination is 3 miles, the probable error of the mean of the result of 9 culminations will be 1 mile.

If the probable error of longitude from 1 lunar distance is 10 miles, it will require 100 lunar distances to obtain a result with a probable error of 1 mile.

Examples.—

(1) *Latitudes and Azimuths.*—Suppose that it is found impossible to measure a base directly, and it is decided to use one determined by latitudes at two stations and the observed azimuth of the ray between them. It is required that this base shall have a probable error not greater than $\frac{1}{500}$. The azimuth of the ray is about 45°, how long must the ray be?

The probable error of each observed latitude is about 1 , say; the probable error due to local attraction is about 2 , the probable error of the difference of the two latitudes will therefore be

$$\sqrt{(1 + 1 + 4 + 4)} = 3{\cdot}16 = 316 \text{ feet (about)},$$

and as the azimuth of the ray is 45°, this will produce a probable error in the length of the ray of $316 \times \sqrt{2}$ feet = 442 feet, and if the probable error of the length of the ray is not to exceed $\frac{1}{500}$, the ray must not be less than 500 × 442 feet long or say 40 miles.

(2) *Long Traverses.*—American experience shows that the longitudinal error of a long traverse under favourable conditions is not greater than $\frac{1}{3000}$. Assuming that this error in the unfavourable conditions of the Gold Coast may amount to $\frac{1}{1000}$, and that a traverse is run approximately north and south, how long should the traverse be in order to obtain a check from latitude observations? The probable error of an observation for latitude with a zenith telescope may be put down as 0 1.

The probable error due to local attraction is about 2 0.

Total probable error in difference of latitude

$$= \sqrt{({\cdot}01 + {\cdot}01 + 4{\cdot}0 + 4{\cdot}0)} = 2{\cdot}83 \text{ or } 283 \text{ feet.}$$

If the latitudes are to be any check on the traverse, the probable error of their difference must not exceed that of the traverse, and the traverse must therefore not be shorter than 283,000 feet or 54 miles.

Probable Errors of a Triangulation.—If the probable error of each angle of a triangle is p then the probable error of the sum of the three angles is $p\sqrt{3}$.

If the triangular error is e the probable error of an angle is $\frac{e}{\sqrt{3}}$.

In N triangles the probable error of an angle is $\cdot 67\sqrt{\frac{\Sigma e^2}{3N}}$, since the probable value of a triangular error is $\cdot 67\sqrt{\frac{\Sigma e^2}{N}}$.

Example.—The following were the triangular errors of a chain of secondary triangles. What is the probable error of an angle?

Triangular errors (e).

e	e^2
2	4
5	25
3	9
4	16
6	36
1	1
4	16
3	9
2	4
6	36
4	16
0	0
5	25
7	49
	$\Sigma e^2 = 246$

The probable value of a triangular error $= \cdot 67\sqrt{\frac{246}{14}}$, and the probable error of an angle $= \cdot 67\sqrt{\frac{246}{42}} = 1{\cdot}62$.

Adjustment of the Errors in a Net-work of Long Traverses

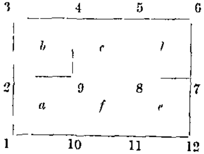


Fig. 69.

Let fig. 69 be a diagrammatic sketch of a net-work of traverses, each line being 50 to 70 miles long.

Each traverse circuit is given a letter, a, b, c, &c

Each latitude station has a number, 1, 2, 3, &c

Including the effect of local attraction each observed latitude has a probable error of about 1·8

First adjust the traverse errors, leaving out of consideration the observed latitudes

Thus —

For each traverse circuit such as a, we have, by computing round to the starting point, a closing error in the northings and southings, and one in the eastings and westings

Give each portion of the traverse from one junction point to the next an error e in the northings and southings, thus, e_{10-2}, e_{2-9}, &c

Then in each case such as a,

$$e_{10-2}+e_{2-9}+e_{9-10}=m, \text{ a known quantity,}$$

and so on for each traverse circuit, so that there will be f equations of this form, and by the ordinary method of least squares solve so that $e^2_{10-2}+e^2_{2-9}+ \ldots$ is a minimum

Do the same with the eastings and westings

The result will be the most probable consistent solution of the problem, so far only as concerns the traverse adjustment leaving out of consideration the observed latitudes

Next, with some initial latitude, say of 1, *deduce latitudes of all the junction points* Compare these with the astronomical (observed) latitudes Now, by inspection we can probably tell —

(1) Whether there are any large local attractions shown by abrupt discordance at a point surrounded by concordances

(2) Whether a northing in an E to W traverse is more or less accurate than a northing in a N to S traverse

It should now be possible to weight the traverse amplitudes as compared with the astronomical amplitudes

Then starting afresh we have for any line, such as 9–10

Error of observed latitude 9 − error of observed latitude 10
− Error in traverse northing 9–10 = a known quantity,

and so on for the other lines, added to the condition that the sum of the 'weighted squares' of the errors is to be a minimum

NOTE — *It is probable that this brief sketch of a particular case of an adjustment of errors by the method of least squares will not be very intelligible to those who have not studied the subject before* It is however impossible, even in a book devoted to topographical surveying, to omit all mention of the subject which enters largely into the highest field of surveying, viz, geodesy And it is very desirable that at least some of the main principles of the theory of errors should be appreciated by the topographer

The following books may be studied —

'Treatise on the Adjustment of Observations" Wright New York 1884
"Method of Least Squares" Merriman J Wiley & Sons New York 1884
'Geodesy" Col A R Clarke, C B, R.E, F R S Clarendon Press 1880

Examples of the application of the theory will be found in—

The various volumes of the ' G T Survey of India"
Report on the Geodetic Survey of S Africa" by Sir David Gill, K C B, F R S, 1896

TABLES.

Summary of Constants for Tables I, II, IX to XIII, XVII and XVIII

[illegible] Clarke [illegible] (1858)

a = Equatorial Radius = 20 926 348 feet

b = Polar Radius = 20 855 233 feet

= Compression or Ellipticity $= \frac{a-b}{a} = \frac{1}{294.26}$

e = Eccentricity $= \left\{\frac{a^2-b^2}{a^2}\right\}^{\frac{1}{2}}$

ν = Normal to meridian at latitude λ terminated by minor axis (or radius of curvature at right angles to meridian)

$$= \frac{a}{(1-e^2\sin^2\lambda)^{\frac{1}{2}}}$$

ρ = Radius of curvature to meridian at latitude λ

$$= \frac{a(1-e^2)}{(1-e^2\sin^2\lambda)^{\frac{3}{2}}}$$

Note.—In Clarke's later spheroid (1880) the values of a and b only differ from those given above by 140 feet and 138 feet respectively. These differences are so small that Tables based on Clarke's 1880 figure practically give the same results. The radii employed by the Survey of India differ from Clarke's by 3270 feet and 1520 feet respectively.

TABLE I.—Logarithms for facilitating the Computation of Terrestrial Latitudes, Longitudes, and Reverse Azimuths

Latitude		P		Q		R		S		T	
		$\frac{1}{\rho \sin 1''}$		$\frac{\rho}{\nu}$		$\frac{1}{2\rho\nu}$		$2 \sec\lambda \frac{\rho}{\nu}$		$\left(2\tan^2\lambda + \frac{\nu}{\rho}\right) \times \left(\frac{\cot\lambda \cos\lambda}{\ }\right)$	
°	′		Diff −		Diff +		Diff −		Diff +		Diff −
0	0	$\bar{3}$·9966885	1	$\bar{1}$·9970132	0	$\bar{8}$·38123	0	0·29807	0	Infinite	Infin
	10	6884	1	0132	1	23	0	807	1	2·23820	30101
	20	6883	2	0133	1	23	0	808	1	1·93719	17607
	30	6881	2	0134	2	23	0	809	1	76112	12491
	40	6879	4	0136	2	23	0	810	2	63621	9687
	50	6875	4	0138	3	23	0	812	2	53934	7914
1	0	$\bar{3}$·9966871	5	$\bar{1}$·9970141	3	$\bar{8}$·38123	0	0·29814	2	1·46020	6689
	10	6866	5	0144	4	23	0	16	3	39331	5793
	20	6861	6	0148	4	23	0	19	3	33538	5108
	30	6855	8	0152	5	23	0	22	4	28430	4628
	40	6847	7	0157	5	23	0	26	4	23802	4130
	50	6840	9	0162	6	23	0	30	4	10732	3769

TABLE I.—Logarithms for facilitating the Computation of

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
2	0	$\bar{3}$ 9966831		$\bar{1}$ 9970468		$\bar{8}$ 38123		0·29834		1·15963	
			9		6		0		5		3466
	10	6822		0474		23		39		12497	
			10		7		0		5		3207
	20	6812		0481		23		41		09290	
			11		7		0		5		2984
	30	6801		0488		23		49		06306	
			12		8		1		6		2790
	40	6789		0496		22		55		03516	
			12		8		0		6		2619
	50	6777		0504		22		61		00897	
			13		9		0		7		2468
3	0	$\bar{3}$ 9966764		$\bar{1}$ 9970513		$\bar{8}$ 38122		0 29868		0 98429	
			14		9		0		7		2333
	10	6750		0522		22		75		96096	
			15		10		0		7		2211
	20	6735		0532		22		82		93885	
			15		10		0		7		2102
	30	6720		0542		22		89		91783	
			16		10		0		9		2002
	40	6704		0552		22		98		89781	
			17		12		1		8		1912
	50	6687		0564		21		0 29906		87869	
			17		11		0		9		1828
4	0	$\bar{3}$ 9966670		$\bar{1}$ 9970575		$\bar{8}$ 38121		0 29915		0 86041	
			19		13		0		9		1752
	10	6651		0588		21		24		84289	
			19		12		0		9		1683
	20	6632		0600		21		33		82606	
			19		13		0		10		1617
	30	6613		0613		21		43		80989	
			21		14		1		10		1556
	40	6592		0627		20		53		79433	
			21		14		0		11		1500
	50	6571		0641		20		64		77933	
			22		15		0		11		1448
5	0	$\bar{3}$ 9966549		$\bar{1}$ 9970656		$\bar{8}$ 38120		0 29975		0 76485	
			23		15		0		12		1399
	10	6526		0671		20		87		75086	
			23		16		0		11		1352
	20	6503		0687		20		98		73734	
			24		16		1		12		1309
	30	6479		0703		19		0·30010		72425	
			25		16		0		13		1269
	40	6454		0719		19		23		71156	
			26		17		0		13		1230
	50	6428		0736		19		36		69926	
			26		18		0		13		1194
6	0	$\bar{3}$ 9966402		$\bar{1}$ 9970754		$\bar{8}$·38119		0 30049		0 68732	
			27		18		1		14		1160
	10	6375		0772		18		63		67572	
			28		19		0		14		1127
	20	6347		0791		18		77		66445	
			29		19		0		14		1096
	30	6318		0810		18		91		65349	
			29		19		1		15		1067
	40	6289		0829		17		0 30106		64282	
			30		20		0		15		1039
	50	6259		0849		17		21		63243	
			31		21		0		16		1013

Terrestrial Latitudes, Longitudes, and Reverse Azimuths

Latitude		P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
°	′										
7	0	$\bar{3}$.9966228	31	$\bar{1}$.9970870	21	$\bar{8}$.38117	1	0.30137	16	0.62230	986
	10	6197	32	0891	21	16	0	153	16	61244	963
	20	6165	33	0912	22	16	0	169	16	60281	939
	30	6132	34	0934	23	16	1	185	18	59342	917
	40	6098	34	0957	22	15	0	0.30203	17	58425	896
	50	6064	35	0979	24	15	0	220	18	57529	875
8	0	$\bar{3}$.9966029	36	$\bar{1}$.9971003	24	$\bar{8}$.38115	1	0.30238	18	0.56654	856
	10	5993	37	1027	24	14	0	256	18	55798	837
	20	5956	37	1051	25	14	0	274	19	54961	819
	30	5919	38	1076	25	14	1	293	20	54142	801
	40	5881	39	1101	26	13	0	0.30313	19	53341	784
	50	5842	39	1127	26	13	0	332	21	52557	769
9	0	$\bar{3}$.9965803	40	$\bar{1}$.9971153	27	$\bar{8}$.38113	1	0.30353	20	0.51788	752
	10	5763	41	1180	27	12	0	373	21	51036	737
	20	5722	41	1207	28	12	1	394	21	50299	723
	30	5681	43	1235	28	11	0	415	22	49576	709
	40	5638	43	1263	29	11	1	437	22	48867	695
	50	5595	43	1292	29	10	0	459	22	48172	682
10	0	$\bar{3}$.9965552	45	$\bar{1}$.9971321	29	$\bar{8}$.38110	0	0.30481	23	0.47490	669
	10	5507	45	1350	30	10	1	504	23	46821	656
	20	5462	45	1380	31	09	0	527	23	46165	645
	30	5417	47	1411	31	09	1	550	24	45520	633
	40	5370	47	1442	31	08	0	574	25	44887	621
	50	5323	48	1473	32	08	1	599	24	44266	611
11	0	$\bar{3}$.9965275	48	$\bar{1}$.9971505	32	$\bar{8}$.38107	0	0.30623	25	0.43655	600
	10	5227	49	1537	33	07	1	648	26	43055	589
	20	5178	50	1570	33	06	0	674	26	42466	579
	30	5128	51	1603	34	06	1	700	26	41887	570
	40	5077	51	1637	34	05	0	726	27	41317	560
	50	5026	52	1671	35	05	1	753	27	40757	550

TABLE I—Logarithms for facilitating the Computation of

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
12	0	$\bar{3}$.9964974	53	$\bar{1}$.9971706	35	$\bar{8}$.38104	0	0.30780	27	0.40207	542
	10	4921	53	1741	36	04	1	807	28	39665	532
	20	4868	54	1777	36	03	0	835	28	39133	524
	30	4814	55	1813	36	03	1	863	29	38609	516
	40	4759	55	1849	37	02	0	892	28	38093	507
	50	4704	56	1886	37	02	1	920	30	37586	500
13	0	$\bar{3}$.9964648	57	$\bar{1}$.9971923	38	$\bar{8}$.38101	1	0.30950	30	0.37086	492
	10	4591	57	1961	39	00	0	980	30	36594	484
	20	4534	58	2000	38	00	1	0.31010	30	36110	477
	30	4476	59	2038	39	$\bar{8}$.38099	0	040	31	35633	469
	40	4417	60	2077	40	99	1	071	31	35164	462
	50	4357	60	2117	40	98	1	102	32	34702	455
14	0	$\bar{3}$.9964297	60	$\bar{1}$.9972157	40	$\bar{8}$.38097	0	0.31134	32	0.34247	448
	10	4237	62	2197	41	97	1	166	33	33799	442
	20	4175	62	2238	42	96	0	199	33	33357	435
	30	4113	62	2280	41	96	1	232	33	32922	429
	40	4051	64	2321	43	95	1	265	34	32493	422
	50	3987	64	2364	42	94	0	299	34	32071	416
15	0	$\bar{3}$.9963923	64	$\bar{1}$.9972406	44	$\bar{8}$.38094	1	0.31333	34	0.31655	410
	10	3859	66	2450	43	93	1	367	35	31245	404
	20	3793	66	2493	44	92	0	402	35	30841	399
	30	3727	66	2537	44	92	1	437	36	30442	393
	40	3661	67	2581	45	91	1	473	36	30049	388
	50	3594	68	2626	45	90	0	509	37	29661	383
16	0	$\bar{3}$.9963526	69	$\bar{1}$.9972671	46	$\bar{8}$.38090	1	0.31546	36	0.29278	377
	10	3457	69	2717	46	89	1	582	38	28901	371
	20	3388	70	2763	47	88	0	620	37	28530	367
	30	3318	70	2810	46	88	1	657	38	28163	361
	40	3248	71	2856	45	87	1	695	39	27802	357
	50	3177	72	2901	48	86	0	734	39	2744[illegible]	[illegible]

Terrestrial Latitudes, Longitudes, and Reverse Azimuths

Latitude		P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
17	0	$\bar{3}$·9963105	72	$\bar{1}$·9972952	48	$\bar{8}$·38086	1	0·31773	39	0·27094	347
	10	3033	73	3000	48	85	1	812	40	26747	342
	20	2960	73	3048	49	84	1	852	40	26405	338
	30	2887	74	3097	50	83	0	892	41	26067	333
	40	2813	75	3147	50	83	1	933	40	25734	329
	50	2738	75	3197	50	82	1	973	42	25405	324
18	0	$\bar{3}$·9962663	76	$\bar{1}$·9973247	50	$\bar{8}$·38081	1	0·32015	42	0·25081	320
	10	2587	77	3297	51	80	0	057	42	24761	316
	20	2510	77	3348	52	80	1	099	42	24445	312
	30	2433	78	3400	52	79	1	141	43	24133	307
	40	2355	78	3452	52	78	1	184	44	23826	304
	50	2277	79	3504	52	77	1	228	44	23522	300
19	0	$\bar{3}$·9962198	80	$\bar{1}$·9973556	54	$\bar{8}$·38076	0	0·32272	44	0·23222	295
	10	2118	80	3610	53	76	1	316	44	22927	292
	20	2038	80	3663	54	75	1	360	45	22635	289
	30	1958	82	3717	54	74	1	405	46	22346	284
	40	1876	81	3771	55	73	1	451	46	22062	281
	50	1795	83	3826	54	72	0	497	46	21781	278
20	0	$\bar{3}$·9961712	83	$\bar{1}$·9973880	56	$\bar{8}$·38072	1	0·32543	47	0·21503	273
	10	1629	83	3936	56	71	1	590	47	21230	271
	20	1546	85	3992	56	70	1	637	48	20959	267
	30	1461	84	4048	56	69	1	685	48	20692	263
	40	1377	86	4104	57	68	1	733	48	20429	261
	50	1291	85	4161	57	67	0	781	49	20168	257
21	0	$\bar{3}$·9961206	87	$\bar{1}$·9974218	58	$\bar{8}$·38067	1	0·32830	49	0·19911	253
	10	1119	87	4276	58	66	1	879	50	19658	251
	20	1032	87	4334	58	65	1	929	50	19407	248
	30	0945	88	4392	59	64	1	979	51	19159	244
	40	0857	89	4451	59	63	1	0·33030	51	18915	242
	50	0768	89	4510	59	62	1	081	51	18673	239

TABLE I.—Logarithms for facilitating the Computation of

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
22	0	$\bar{3}$ 9960679	90	$\bar{1}$ 9974569	60	$\bar{8}$ 38061	1	0 33132	52	0 18434	235
	10	0589	90	4629	60	60	0	184	52	18199	233
	20	0499	91	4689	61	60	1	236	53	17966	230
	30	0408	91	4750	60	59	1	289	53	17736	226
	40	0317	92	4810	62	58	1	342	54	17510	225
	50	0225	92	4872	61	57	1	396	54	17285	221
23	0	$\bar{3}$ 9960133	93	$\bar{1}$·9974933	62	$\bar{8}$ 38056	1	0 33450	54	0 17064	219
	10	0040	93	4995	62	55	1	504	55	16845	216
	20	$\bar{3}$ 9959947	94	5057	63	54	1	559	55	16629	213
	30	9853	94	5120	63	53	1	614	56	16416	211
	40	9759	95	5183	63	52	1	670	56	16205	208
	50	9664	96	5246	64	51	1	726	57	15997	205
24	0	$\bar{3}$ 9959568	96	$\bar{1}$ 9975310	64	$\bar{8}$ 38050	1	0 33783	57	0 15792	204
	10	9472	96	5374	64	49	1	840	58	15588	200
	20	9376	97	5438	64	48	1	898	58	15388	199
	30	9279	97	5502	65	47	1	956	58	15189	195
	40	9182	98	5567	66	46	1	0 34014	59	14994	194
	50	9084	99	5633	65	45	1	073	59	14800	191
25	0	$\bar{3}$·9958985	98	$\bar{1}$ 9975698	66	$\bar{8}$ 38044	1	0 34132	60	0 14609	189
	10	8887	100	5764	66	43	1	192	60	14420	187
	20	8787	100	5830	67	42	1	252	61	14233	184
	30	8687	100	5897	67	41	1	313	61	14049	182
	40	8587	101	5964	67	40	1	374	62	13867	180
	50	8486	101	6031	67	39	1	436	62	13687	178
26	0	$\bar{3}$·9958385	102	$\bar{1}$ 9976098	68	$\bar{8}$ 38038	1	0 34498	62	0 13509	175
	10	8283	102	6166	68	37	1	560	63	13334	174
	20	8181	102	6234	69	36	1	623	64	13160	172
	30	8079	103	6303	68	35	1	687	64	12988	169
	40	7976	104	6371	69	34	1	751	64	12819	167
	50	7872	104	6440	70	33	1	815	65	12652	166

Terrestrial Latitudes, Longitudes, and Reverse Azimuths.

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
27	0	$\bar{3}$ 9957768		$\bar{1}$ 9976510		$\bar{8}$ 38032		0 34880		0 12486	
			104		69		1		65		163
	10	7664		6579		31		945		12323	
			105		70		1		66		162
	20	7559		6648		30		0 35011		12161	
			105		70		1		66		160
	30	7454		6719		29		077		12001	
			106		71		1		67		158
	40	7348		6790		28		144		11843	
			106		71		1		67		155
	50	7242		6861		27		211		11688	
			107		71		1		68		154
28	0	$\bar{3}$ 9957135		$\bar{1}$ 9976932		$\bar{8}$ 38026		0 35279		0 11534	
			107		71		1		68		152
	10	7028		7003		25		347		11382	
			107		71		1		69		151
	20	6921		7074		24		416		11231	
			108		72		1		69		148
	30	6813		7146		23		485		11083	
			108		72		1		69		147
	40	6705		7218		22		554		10936	
			109		73		2		70		145
	50	6596		7291		20		624		10791	
			109		73		1		71		143
29	0	$\bar{3}$ 9956487		$\bar{1}$ 9977364		$\bar{8}$ 38019		0 35695		0 10648	
			109		72		1		71		141
	10	6378		7436		18		766		10507	
			110		74		1		71		140
	20	6268		7510		17		837		10367	
			110		73		1		72		138
	30	6158		7583		16		909		10229	
			111		74		1		73		137
	40	6047		7657		15		982		10092	
			111		74		1		73		134
	50	5936		7731		14		0 36055		09958	
			111		74		1		73		134
30	0	$\bar{3}$ 9955825		$\bar{1}$ 9977805		$\bar{8}$ 38013		0 36128		0 09824	
			112		75		1		74		131
	10	5713		7880		12		202		09693	
			112		74		1		74		130
	20	5601		7954		11		276		09563	
			112		75		2		75		129
	30	5489		8029		09		351		09434	
			113		76		1		76		127
	40	5376		8105		08		427		09307	
			113		75		1		76		125
	50	5263		8180		07		503		09182	
			114		76		1		76		124
31	0	$\bar{3}$ 9955149		$\bar{1}$ 9978256		$\bar{8}$ 38006		0 36579		0 09058	
			114		76		1		77		123
	10	5035		8332		05		656		08935	
			114		76		1		77		121
	20	4921		8408		04		733		08814	
			115		76		1		78		119
	30	4806		8484		03		811		08695	
			115		77		2		79		118
	40	4691		8561		01		890		08577	
			115		77		1		79		117
	50	4576		8638		00		969		08460	
			115		77		1		79		116

TABLE I.—Logarithms for facilitating the Computation of

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
32	0	$\bar{3}$ 9954461		$\bar{1}$ 9978715		$\bar{8}$ 37999		0·37048		0 08344	
			116		77		1		80		114
	10	4345		8792		98		128		08230	
			117		78		1		81		113
	20	4228		8870		97		209		08117	
			116		77		1		81		111
	30	4112		8947		96		290		08006	
			117		78		2		81		110
	40	3995		9025		94		371		07896	
			117		78		1		82		108
	50	3878		9103		93		453		07788	
			118		79		1		83		107
33	0	$\bar{3}$ 9953760		$\bar{1}$ 9979182		$\bar{8}$ 37992		0·37536		0·07681	
			118		78		1		83		106
	10	3642		9260		91		619		07575	
			118		79		1		83		105
	20	3524		9339		90		702		07470	
			118		79		1		85		104
	30	3406		9418		89		787		07366	
			119		79		2		84		102
	40	3287		9497		87		871		07264	
			119		80		1		85		101
	50	3168		9577		86		956		07163	
			119		79		1		86		99
34	0	$\bar{3}$·9953049		$\bar{1}$ 9979656		$\bar{8}$ 37985		0 38012		0 07064	
			120		80		1		86		99
	10	2929		9736		84		128		06965	
			120		80		1		87		97
	20	2809		9816		83		215		06868	
			120		80		2		88		96
	30	2689		9896		81		303		06772	
			121		80		1		87		95
	40	2568		9976		80		390		06677	
			120		81		1		89		94
	50	2448		$\bar{1}$·9980057		79		470		06583	
			121		80		1		89		93
35	0	$\bar{3}$·9952327		$\bar{1}$·9980137		$\bar{8}$ 37978		0 38568		0 06490	
			121		81		1		89		91
	10	2206		0218		77		657		06399	
			122		81		2		91		91
	20	2084		0299		75		748		06308	
			122		81		1		90		89
	30	1962		0380		74		838		06210	
			122		82		1		91		88
	40	1840		0462		73		929		06131	
			122		81		1		92		87
	50	1718		0543		72		0 39021		06044	
			122		82		2		92		86
36	0	$\bar{3}$·9951596		$\bar{1}$·9980625		$\bar{8}$ 37970		0 39113		0·05958	
			123		82		1		93		85
	10	1473		0707		69		206		05874	
			123		82		1		94		84
	20	1350		0789		68		300		05789	
			123		82		1		94		83
	30	1227		0871		67		394		05706	
			123		82		1		94		82
	40	1104		0953		66		493		05624	
			124		82		2		99		81
	50	0980		1035		64		594		05543	
			124		83		1		95		80

Terrestrial Latitudes, Longitudes, and Reverse Azimuths.

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
37	0	$\bar{3}$·9950856	124	$\bar{1}$·9981118	82	$\bar{8}$·37963	1	0·39679	97	0·05463	79
	10	0732	124	1200	83	62	1	776	96	05384	78
	20	0608	124	1283	83	61	2	872	98	05306	77
	30	0484	125	1366	83	59	1	970	98	05229	76
	40	0359	125	1449	83	58	1	0·40068	99	05153	75
	50	0234	125	1532	84	57	1	167	99	05078	75
38	0	$\bar{3}$·9950109	125	$\bar{1}$·9981616	83	$\bar{8}$·37956	2	0·40266	100	0·05003	74
	10	$\bar{3}$·9949984	125	1699	84	54	1	366	100	04929	72
	20	9859	126	1783	83	53	1	466	101	04857	71
	30	9733	126	1866	84	52	1	567	102	04786	71
	40	9607	125	1950	84	51	2	669	102	04715	70
	50	9482	126	2034	84	49	1	771	103	04645	68
39	0	$\bar{3}$·9949356	127	$\bar{1}$·9982118	84	8·37948	1	0·40874	103	0·04577	68
	10	9229	126	2202	85	47	1	977	104	04509	67
	20	9103	126	2287	84	46	2	0·41081	105	04442	67
	30	8977	127	2371	84	44	1	186	105	04375	65
	40	8850	127	2455	85	43	1	291	106	04310	65
	50	8723	127	2540	84	42	2	397	107	04245	63
40	0	$\bar{3}$·9948596	127	$\bar{1}$·9982624	85	$\bar{8}$·37940	1	0·41504	107	0·04182	63
	10	8469	127	2709	85	39	1	611	108	04119	62
	20	8342	127	2794	85	38	1	719	108	04057	61
	30	8215	128	2879	85	37	2	827	109	03996	61
	40	8087	127	2964	85	35	1	936	110	03935	60
	50	7960	128	3049	85	34	1	0·42046	110	03875	59
41	0	$\bar{3}$·9947832	128	$\bar{1}$·9983134	85	$\bar{8}$·37933	1	0·42156	111	0·03816	59
	10	7704	127	3219	85	32	2	267	112	03757	58
	20	7577	128	3304	86	30	1	379	112	03699	57
	30	7449	128	3390	85	29	1	491	113	03642	56
	40	7321	129	3475	85	28	2	604	114	03586	55
	50	7192	128	3560	86	26	1	718	114	03531	54

TABLE I.—Logarithms for facilitating the Computation of

LATITUDE		P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
°	′										
42	0	$\bar{3}$ 9947064		$\bar{1}$ 9983646		$\bar{8}$ 37925		0 42832		0 03477	
			128		85		1		115		54
	10	6936		3731		24		917		03423	
			129		86		2		116		53
	20	6807		3817		22		0 43063		03370	
			128		85		1		116		53
	30	6679		3902		21		179		03317	
			128		86		1		117		52
	40	6551		3988		20		296		03265	
			129		86		1		118		51
	50	6422		4074		19		414		03214	
			129		86		2		118		50
43	0	$\bar{3}$ 9946293		$\bar{1}$ 9984160		$\bar{8}$ 37917		0 43532		0·03164	
			128		85		1		119		50
	10	6165		4245		16		651		03114	
			129		86		1		120		49
	20	6036		4331		15		771		03065	
			129		86		1		120		49
	30	5907		4417		14		891		03016	
			129		86		2		121		48
	40	5778		4503		12		0 44012		02968	
			129		86		1		122		47
	50	5649		4589		11		134		02921	
			129		86		1		122		47
44	0	$\bar{3}$ 9945520		$\bar{1}$ 9984675		$\bar{8}$ 37910		0 44256		0 02874	
			128		86		2		124		46
	10	5392		4761		08		380		02828	
			129		86		1		123		45
	20	5263		4847		07		503		02783	
			129		86		1		125		45
	30	5134		4933		06		628		02738	
			129		86		1		125		44
	40	5005		5019		05		753		02694	
			129		86		2		126		44
	50	4876		5105		03		879		02650	
			129		86		1		127		44
45	0	$\bar{3}$ 9944747		$\bar{1}$ 9985191		$\bar{8}$ 37902		0 45006		0 02606	
			129		86		1		128		43
	10	4618		5277		01		134		02563	
			129		86		2		128		42
	20	4489		5363		$\bar{8}$ 37899		262		02521	
			129		86		1		129		41
	30	4360		5449		98		391		02480	
			129		86		1		130		40
	40	4231		5535		97		521		02440	
			129		86		2		131		40
	50	4102		5621		95		652		02400	
			129		86		1		131		39
46	0	$\bar{3}$ 9943973		$\bar{1}$ 9985707		$\bar{8}$ 37894		0 45783		0 02361	
			129		86		1		132		39
	10	3844		5793		93		915		02322	
			129		86		1		133		39
	20	3715		5879		92		0 46018		02283	
			129		86		2		133		38
	30	3586		5965		90		181		02245	
			120		85		1		135		38
	40	3457		6050		89		316		02207	
			129		86		1		135		37
	50	3328		6136		88		451		02170	
			129		86		2		136		37

Terrestrial Latitudes, Longitudes, and Reverse Azimuths.

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
47	0	$\bar{3}$.9913100	128	$\bar{1}$.9986222	86	$\bar{8}$.37886	1	0.46987	137	0.02133	36
	10	3071	129	6308	86	85	1	724	137	02097	36
	20	2842	128	6394	85	84	1	861	138	02061	35
	30	2814	129	6479	86	83	2	999	140	02026	34
	40	2585	128	6565	86	81	1	0.47139	140	01992	34
	50	2557	129	6651	85	80	1	279	140	01958	33
48	0	$\bar{3}$.9912428	128	$\bar{1}$.9986736	86	$\bar{8}$.37879	2	0.47419	142	0.01925	33
	10	2300	128	6822	85	77	1	561	142	01892	33
	20	2172	129	6907	86	76	1	703	143	01859	32
	30	2043	128	6993	85	75	1	846	145	01827	31
	40	1915	128	7078	86	74	2	991	144	01796	31
	50	1787	127	7164	85	72	1	0.48135	146	01765	30
49	0	$\bar{3}$.9911660	128	$\bar{1}$.9987249	85	$\bar{8}$.37871	1	0.48281	147	0.01735	31
	10	1532	128	7334	85	70	1	428	147	704	31
	20	1404	127	7419	85	69	2	575	149	673	30
	30	1277	128	7504	85	67	1	724	149	643	29
	40	1149	127	7589	85	66	1	873	150	614	29
	50	1022	127	7674	85	65	2	0.49023	151	585	29
50	0	$\bar{3}$.9910895	127	$\bar{1}$.9987759	85	$\bar{8}$.37863	1	0.49174	152	0.01556	28
	10	0768	127	7844	84	62	1	326	152	528	27
	20	0641	127	7928	85	61	1	478	154	501	27
	30	0514	127	8013	84	60	2	632	155	474	27
	40	0387	126	8097	84	58	1	787	155	447	26
	50	0261	127	8181	85	57	1	942	156	421	26
51	0	$\bar{3}$.9910134	126	$\bar{1}$.9988266	84	$\bar{8}$.37856	1	0.50098	158	0.01395	26
	10	0008	126	8350	84	55	2	256	158	369	25
	20	$\bar{3}$.9909882	126	8434	84	53	1	414	159	344	25
	30	9756	126	8518	84	52	1	573	160	319	24
	40	9630	125	8602	83	51	2	733	161	295	24
	50	9505	125	8685	84	49	1	894	162	271	24

TABLE I — Logarithms for facilitating the Computation of

Latitude °	′	P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
52	0	$\bar{3}$ 9939380	126	$\bar{1}$ 9988769	83	$\bar{8}$ 37848	1	0 51056	163	0 01247	24
	10	9254	125	8852	84	47	1	219	164	223	23
	20	9129	124	8936	83	46	1	383	165	200	23
	30	9005	125	9019	83	45	2	548	166	177	22
	40	8880	125	9102	83	43	1	714	167	155	22
	50	8755	124	9185	83	42	1	881	168	133	21
53	0	$\bar{3}$ 9938631	124	$\bar{1}$ 9989268	82	$\bar{8}$ 37841	1	0 52049	169	0 01112	22
	10	8507	124	9350	83	40	2	218	170	090	21
	20	8383	123	9433	82	38	1	388	171	069	21
	30	8260	124	9515	83	37	1	559	172	048	20
	40	8136	123	9598	82	36	1	731	174	028	20
	50	8013	123	9680	82	35	2	905	174	008	19
54	0	$\bar{3}$ 9937890	122	$\bar{1}$ 9989762	82	$\bar{8}$ 37833	1	0 53079	175	0 00989	20
	10	7768	123	9844	81	32	1	254	176	969	19
	20	7645	122	9925	82	31	1	430	178	950	19
	30	7523	122	$\bar{1}$ 9990007	81	30	1	608	178	931	18
	40	7401	122	0088	81	29	2	786	180	913	18
	50	7279	121	0169	81	27	1	966	180	895	17
55	0	$\bar{3}$ 9937158	122	$\bar{1}$ 9990250	81	$\bar{8}$ 37826	1	0 54146	182	0 00878	18
	10	7036	121	0331	81	25	1	328	183	860	18
	20	6915	120	0412	80	24	2	511	184	842	17
	30	6795	121	0492	80	22	1	695	185	825	17
	40	6674	120	0572	81	21	1	880	187	808	16
	50	6554	120	0653	80	20	1	0 55067	187	792	16
56	0	$\bar{3}$ 9936434	120	$\bar{1}$ 9990733	79	$\bar{8}$ 37819	1	0 55254	189	0 00776	16
	10	6314	119	0812	80	18	2	443	190	760	16
	20	6195	119	0892	79	16	1	633	191	744	16
	30	6076	119	0971	79	15	1	824	192	728	15
	40	5957	118	1050	79	14	1	0 56016	193	713	15
	50	5839	118	1129	79	13	1	209	195	698	14

Terrestrial Latitudes, Longitudes and Reverse Azimuths

Latitude		P	Diff −	Q	Diff +	R	Diff −	S	Diff +	T	Diff −
57	0	$\bar{3}$ 9935721		$\bar{1}$ 0091208		$\bar{8}$ 37812		0 56404		0 00684	
			118		79		1		196		15
	10	5603		1287		11		600		669	
			118		78		2		197		14
	20	5485		1365		09		797		655	
			117		78		1		199		14
	30	5368		1443		08		996		641	
			117		78		1		199		14
	40	5251		1521		07		0 57195		627	
			117		78		1		201		13
	50	5134		1599		06		396		614	
			116		78		1		203		13
58	0	$\bar{3}$ 9935018		$\bar{1}$ 0091677		$\bar{8}$ 37805		0 57599		0 00601	
			116		77		1		203		13
	10	4902		1754		04		802		588	
			116		77		2		205		13
	20	4786		1831		02		0 58007		575	
			115		77		1		207		13
	30	4671		1908		01		214		562	
			115		76		1		207		12
	40	4556		1984		00		421		550	
			114		77		1		209		12
	50	4442		2061		$\bar{8}$ 37799		630		538	
			115		76		1		210		12
59	0	$\bar{3}$ 9934327		$\bar{1}$ 0092137		$\bar{8}$ 37798		0 58840		0 00526	
			114		76		1		212		12
	10	4213		2213		97		0 59052		514	
			113		76		1		213		12
	20	4100		2289		96		265		502	
			113		75		2		215		12
	30	3987		2364		94		480		490	
			113		75		1		216		11
	40	3874		2439		93		696		479	
			113		75		1		217		11
	50	3761		2514		92		913		468	
			112		75		1		219		10
60	0	$\bar{3}$ 9933649		$\bar{1}$ 0092589		$\bar{8}$ 37791		0 60132		0 00458	

TABLE II (A)

Directions for applying the signs to the terms of the Latitude, Longitude, and Azimuth formulæ, when the fixed station A lies North of the Equator

The Azimuth is reckoned from South by West. The Latitude North, and Longitude East, should always be considered positive.

Terms of the Formulæ	Magnitude of the given Azimuth A			
	0 to 90°	90 to 180°	180 to 270	270° to 360°
$\delta_1\lambda$	−	+	+	−
$\delta_1 L$	−	−	+	+
$\delta_1 A$	−	−	+	+
$\delta_2\lambda$	−	−	−	−
$\delta_2 L$	+	−	+	−
$\delta_2 A$	+	−	+	−

For use in Distance and Azimuth Computation —

$A + \frac{\Delta A}{2}$ is in	1st	2nd	3rd	4th Quadrant
when $\frac{\Delta\lambda}{\Delta L}$ are	$\frac{-}{-}$	$\frac{+}{-}$	$\frac{+}{+}$	$\frac{-}{+}$

TABLE II (B)

Directions for applying the signs to the terms of the Latitude, Longitude, and Azimuth formulæ, when the fixed station A lies to the South of the Equator.

The Latitude South, and the Longitude East, should be considered positive.

Terms of the Formulæ	Magnitude of the given Azimuth A			
	0 to 90	90 to 180	180° to 270	270 to 360
$\delta_1\lambda$	+	−	−	+
$\delta_1 L$	−	−	+	+
$\delta_1 A$	+	+	−	−
$\delta_2\lambda$	−	−	−	−
$\delta_2 L$	−	+	−	+
$\delta_2 A$	+	−	+	−

For use in Distance and Azimuth Computation —

$A + \frac{\Delta A}{2}$ is in	1st	2nd	3rd	4th Quadrant
when $\frac{\Delta\lambda}{\Delta L}$ are	$\frac{+}{-}$	$\frac{-}{-}$	$\frac{-}{+}$	$\frac{+}{+}$

TABLE III

Computation of Heights from Vertical Angles

Table K, for Use in Computing Heights when only One Angle has been Observed

Distance between Stations in feet	K
1000	0 4
2000	0 9
3000	0 13
4000	0 17
5000	0 21
6000	0 26
7000	0 30
8000	0 34
9000	0 38
10000	0 43
20000	1 25
30000	2 8
40000	2 50
50000	3 33
60000	4 15
70000	4 58
80000	5 40
90000	6 23
100000	7 5

The value of the coefficient of refraction has been found on the average to be about 07 It is with this value that the above table has been computed

TABLE IV

Computation of Heights

To facilitate finding the Subtended Angle when only one Angle has been observed

Latitude	Adopted Refraction Expressed in Decimals of Contained Arc								
	0 20	*0 10*	*0 09*	*0 08*	*0 07*	*0 06*	*0 05*	*0 04*	*0 03*
°									
0	2 5276	2 4027	2 3920	2 3815	2 3713	2 3613	2 3515	2 3420	2 3327
5	5277	4027	3920	3815	3713	3613	3516	3420	3327
10	5277	4028	3921	3816	3714	3614	3516	3421	3327
15	5278	4029	3922	3817	3715	3615	3517	3422	3328
20	5280	4030	3924	3818	3716	3616	3519	3423	3330
25	5281	4032	3925	3820	3718	3618	3521	3425	3332
30	5284	4034	3927	3822	3720	3620	3523	3427	3334
35	5286	4036	3929	3825	3722	3623	3525	3429	3336
40	5288	4039	3932	3827	3725	3625	3527	3432	3338
45	5291	4042	3935	3830	3728	3628	3530	3435	3342
50	5294	4045	3937	3833	3731	3631	3533	3438	3344
55	5297	4047	3940	3835	3733	3633	3536	3440	3347
60	5299	4049	3942	3838	3736	3636	3538	3442	3349

Note —If nothing is known of the Refraction, adopt 0 07 as the value of n

TABLES V. and VI.

For Determining Differences of Height with the Barometer.

(Baily.)

TABLE V.—Part I. Thermometers in the Open Air in Degrees Fahrenheit.

$t + t'$	A	$t + t'$	A	$t + t'$	A
0	4·74913	65	4·78113	130	4·81138
5	4·75190	70	353	135	362
10	377	75	592	140	585
15	623	80	830	145	807
20	888	85	4·79066	150	4·82028
25	4·76141	90	301	155	248
30	382	95	535	160	466
35	642	100	768	165	684
40	891	105	4·80000	170	900
45	4·77138	110	4·80229	175	4·83115
50	384	115	458	180	330
55	628	120	686		
60	871	125	913		

Part II. Attached Thermometers (not required with aneroid).

$\tau - \tau'$	B	$\tau - \tau'$	B	$\tau - \tau'$ °	B
0	0·00000	25	0·00109	50	0·00217
5	22	30	130	55	238
10	43	35	152	60	259
15	65	40	174		
20	87	45	195		

TABLE VI.—Latitude of the place.

Lat. °	C	Lat.	C	Lat.	C
0	0·00117	35	0·00040	70	$\bar{1}$·99910
5	115	40	20	75	00
10	110	45	0·00000	80	$\bar{1}$·99890
15	101	50	$\bar{1}$·99980	85	85
20	90	55	60	90	83
25	75	60	42		
30	58	65	25		

TABLES V and VI (*continued*)

If β be the reading of barometer at lower station
β' ,, ,, ,, upper ,,
t ,, detached thermometer in air at lower station
t' ,, ,, ,, ,, upper ,,
τ ,, attached ,, ,, lower ,,
τ' ,, ,, ,, ,, upper ,,
A, the number in the table corresponding to $t + t'$
B, ,, ,, ,, $\tau - \tau'$
Make $D = \log \beta - (\log \beta' + B)$

then

$\log D + A + C = \log h$ of the difference in altitude

Example.—In lat 51° 30 the barometer at upper station read 25 55 in and 29 81 at lower station The readings of the detached thermometers were 50° 5 and 63, and those of the attached thermometers 52 and 62° at the upper and lower stations respectively What was the difference of altitude?

Log 29 81 =	1 47436
− Log 25 55 =	− 1 40739
B =	0 00043
D	0 06654
Log D	2 82308
A	4 80389
C	1 99973
Log h =	3 62670

h = 4234 feet the required difference

NOTE.—In the case of aneroids the term B is 0 as they are compensated for change of temperature of the *instrument* The term A, however, which depends on the *air* temperatures, must always be included in the computation

TABLES VII and VIII

For Determining Heights with the Boiling Point Thermometer

Table VII

Approximate Height* in feet Corresponding to the Observed Boiling Point in Degrees Fahrenheit and Corresponding Barometer Readings

Boiling Point	Barometer Reading	Approximate Height	Boiling Point	Barometer Reading	Approximate Height
°			°		
212	29 921	—	195	21 133	9 084
211	29 329	522	194	20 690	9 638
210	28 746	1 046	193	20 255	10 193
209	28 174	1 571	192	19 828	10 750
208	27 613	2 097	191	19 409	11 307
207	27 062	2 624	190	18 998	11 867
206	26 521	3 151	189	18 595	12 426
205	25 989	3 680	188	18 199	12 988
204	25 466	4 210	187	17 809	13 556
203	24 952	4 741	186	17 426	14 124
202	24 447	5 278	185	17 050	14 694
201	23 950	5 815	184	16 681	15 266
200	23 461	6 354	183	16 319	15 839
199	22 980	6 895	182	15 964	16 412
198	22 507	7 439	181	15 616	16 987
197	22 042	7 984	180	15 275	17 564
196	21 584	8 533			

* Accurately—this table gives the altitude above level at which water boils at 212° temperature of intermediate air being 32° F

Table VIII

Correction for Temperature of Intermediate Air			
Mean Temperature	Multiplier	Mean Temperature	Multiplier
°		°	
20	0 975	55	1 048
25	0 985	60	1 058
30	0 996	65	1 069
35	1 006	70	1 079
40	1 017	75	1 090
45	1 027	80	1 100
50	1 038	85	1 110

Example —The boiling point at upper station was 204° 2 and at lower station 208° 7, and the respective air temperatures were 75 and 83°

Tabular number for 204 2 = 4107
208 7 = 1726

2381

Temperature at upper station, 75
lower 83

Mean temperature 79, multiplier, 1 1044

And difference of height = 2381 × 1 044 = 2628 feet

BLE IX — Graticules of Maps Sides and Diagonals of Squares of a quarter of a degree of Lat. and Long on the scale of 1 inch to 2 miles

Latitude		Length in Inches			
		m	n	p	q
		on meridian	on lower parallel	on upper parallel	on diagonal
		$\frac{\pi}{180} \times \frac{\rho}{[illegible]}$	$\frac{1}{4} \times \frac{\pi}{180 \times 4}$ [illegible]	$\times \frac{\rho \cos \lambda}{[illegible]}$	$\sqrt{m + [illegible] p}$
° ′					
From 0 0 to	0 1	8 88	8 617	8 617	12 187
0 1	0 30	588	617	616	187
0 30	0 4	88	616	616	18
0 4	1 0	88	61	61	18
1 0	1 1	88	61	615	186
1 1	1 30	588	61	614	18
1 30	1 4	88	614	613	184
1 4	0	588	613	611	183
2 0	1	8 88	8 611	8 610	12 18
1	30	88	610	638	181
30	4	88	638	637	180
4	3	88	63	63	1 9
3	3 1	88	63	633	1 8
3 1	3 30	88	633	631	176
3 30	3 4	88	631	6 8	17
3 4	4 0	88	6 8	626	173
4 0	4 1	8 88	8 6	8 6 3	12 171
4 1	4 30	88	6 3	6	169
4 30	4 4	81	6	617	167
4 4		81	617	614	16
0	1	81	614	611	163
1	30	81	611	607	161
30	4	81	607	603	1 8
4	6 0	580	603	600	15
0	1	8 589	8 600	8 9 3	12 1 3
6 1	6 30	81	6 3	591	150
30	4	87	6 1	587	147
6 4	0	81	8	583	144
7 0	1	581	83	578	141
7 1	30	589	8	573	137
30	4	81	5 3	568	134
4	8 0	591	568	5 3	131
8 0	8 1	8 5 1	8 63	8 5 8	12 127
8 1	8 30	91	5 8		123
8 30	8 4	591	5 2	547	119
8 4	9 0	591	547	541	116
9 0	1	90	41	53	111
1	30	591	3		1 7
30	9 4	91		5 3	1 3
4	10 0	591	5 3	516	099
10	10 1	8 91	8 16	8 1	12 094
10 1	10 30	591	510	513	089
10 30	10 4	591	503	4 6	085
10 4	11 0	91	4	480	081
11 0	11 1	91	489	482	0 7
11 1	11 30	591	48	4 4	070
11 30	11 4		474	467	06
11 4	12 0		46	4 9	061

TABLE IX.—Graticules of Maps. Sides and Diagonals of Squares of

Latitude.	Length in Inches.			
	m on meridian	n on lower parallel.	p on upper parallel	q on diagonal
° ′ ° ′				
From 12 0 to 12 15	8·592	8·459	8·451	12·054
12 15 „ 12 30	592	451	443	049
12 30 „ 12 45	592	443	435	043
12 45 „ 13 0	592	435	426	037
13 0 „ 13 15	592	426	418	031
13 15 „ 13 30	593	418	409	026
13 30 „ 13 45	593	409	400	020
13 45 „ 14 0	593	400	391	014
14 0 „ 14 15	8 593	8 391	8 382	12·007
14 15 „ 14 30	593	382	373	001
14 30 „ 14 45	594	373	364	11·995
14 45 „ 15 0	594	364	354	989
15 0 „ 15 15	594	354	344	982
15 15 „ 15 30	594	344	334	974
15 30 „ 15 45	594	334	324	968
15 45 „ 16 0	595	324	314	962
16 0 „ 16 15	8·595	8·314	8 303	11·954
15 15 „ 16 30	595	303	293	947
16 30 „ 16 45	595	293	282	940
16 45 „ 17 0	595	282	271	932
17 0 „ 17 15	596	271	260	925
17 15 „ 17 30	596	260	249	918
17 30 „ 17 45	596	249	238	910
17 45 „ 18 0	596	238	226	902
18 0 „ 18 15	8 596	8·226	8·214	11·894
18 15 „ 18 30	597	214	203	886
18 30 „ 18 45	597	203	191	878
18 45 „ 19 0	597	191	179	870
19 0 „ 19 15	597	178	166	861
19 15 „ 19 30	598	166	154	853
19 30 „ 19 45	598	154	141	845
19 45 „ 20 0	598	141	128	836
20 0 „ 20 15	8 598	8·128	8 115	11 827
20 15 „ 20 30	599	115	102	819
20 30 „ 20 45	599	102	089	810
20 45 „ 21 0	599	089	076	801
21 0 „ 21 15	599	076	062	792
21 15 „ 21 30	600	062	049	783
21 30 „ 21 45	600	049	035	774
21 45 „ 22 0	600	035	021	765
22 0 „ 22 15	8·600	8·021	8 007	11·755
22 15 „ 22 30	601	007	7 992	746
22 30 „ 22 45	601	7 992	978	736
22 45 „ 23 0	601	978	963	726
23 0 „ 23 15	601	963	949	716
23 15 „ 23 30	602	949	934	707
23 30 „ 23 45	602	934	919	697
23 45 „ 24 0	602	919	904	687

a quarter of a degree of Lat and Long on the scale of 1 inch to 2 miles

Latitude	Length in Inches			
	m	n	p	q
	on meridian	on lower parallel	on upper parallel	on diagonal
° °				
From 24 0 to 24 15	8·603	7·904	7·888	11·677
24 15 „ 24 30	603	888	873	667
24 30 „ 24 45	603	873	857	656
24 45 „ 25 0	604	857	841	646
25 0 „ 25 15	604	841	825	636
25 15 „ 25 30	604	825	809	626
25 30 „ 25 45	604	809	793	615
25 45 „ 26 0	605	793	777	604
26 0 „ 26 15	8·605	7·777	7·760	11·593
26 15 „ 26 30	605	760	743	582
26 30 „ 26 45	606	743	727	571
26 45 „ 27 0	606	727	709	559
27 0 „ 27 15	606	709	692	548
27 15 „ 27 30	606	692	675	537
27 30 „ 27 45	607	675	658	526
27 45 „ 28 0	607	658	640	515
28 0 „ 28 15	8·607	7·640	7·623	11·503
28 15 „ 28 30	608	623	605	492
28 30 „ 28 45	608	605	587	480
28 45 „ 29 0	608	587	569	468
29 0 „ 29 15	609	569	550	457
29 15 „ 29 30	609	551	532	445
29 30 „ 29 45	609	532	513	433
29 45 „ 30 0	610	513	495	421
30 0 „ 30 15	8·610	7·495	7·476	11·409
30 15 „ 30 30	610	476	457	397
30 30 „ 30 45	611	457	438	385
30 45 „ 31 0	611	438	419	372
31 0 „ 31 15	611	419	399	360
31 15 „ 31 30	612	399	379	347
31 30 „ 31 45	612	379	360	335
31 45 „ 32 0	612	360	340	323
32 0 „ 32 15	8·613	7·340	7·320	11·310
32 15 „ 32 30	613	320	300	297
32 30 „ 32 45	613	300	279	284
32 45 „ 33 0	614	279	259	271
33 0 „ 33 15	614	259	238	258
33 15 „ 33 30	614	238	218	244
33 30 „ 33 45	615	218	197	232
33 45 „ 34 0	615	197	175	219
34 0 „ 34 15	8·616	7·175	7·155	11·206
34 15 „ 34 30	616	155	134	193
34 30 „ 34 45	616	134	112	179
34 45 „ 35 0	617	112	091	166
35 0 „ 35 15	617	091	069	153
35 15 „ 35 30	617	069	047	139
35 30 „ 35 45	618	047	025	125
35 45 „ 36 0	618	025	003	112

TABLE IX.—Graticules of Maps. Sides and Diagonals of Squares of

Latitude	Length in Inches			
	m on meridian	n on lower parallel	p on upper parallel	q on diagonal
° ′ ° ′				
From 36 0 to 36 15	8·618	7·003	6·981	11·098
36 15 „ 36 30	619	6·981	959	085
36 30 „ 36 45	619	959	937	071
36 45 „ 37 0	620	937	914	057
37 0 „ 37 15	620	914	891	043
37 15 „ 37 30	620	891	868	029
37 30 „ 37 45	621	868	846	015
37 45 „ 38 0	621	846	822	001
38 0 „ 38 15	8·621	6·822	6·799	10·987
38 15 „ 38 30	622	799	776	973
38 30 „ 38 45	622	776	752	959
38 45 „ 39 0	623	752	729	945
39 0 „ 39 15	623	729	705	930
39 15 „ 39 30	623	705	681	916
39 30 „ 39 45	624	681	657	902
39 45 „ 40 0	624	657	633	887
40 0 „ 40 15	8·625	6·633	6·609	10·873
40 15 „ 40 30	625	609	584	858
40 30 „ 40 45	625	584	560	843
40 45 „ 41 0	626	560	535	829
41 0 „ 41 15	626	535	510	814
41 15 „ 41 30	626	510	486	800
41 30 „ 41 45	627	486	461	785
41 45 „ 42 0	627	461	435	770
42 0 „ 42 15	8·627	6·435	6·410	10·755
42 15 „ 42 30	628	410	385	741
42 30 „ 42 45	628	385	359	726
42 45 „ 43 0	629	359	334	712
43 0 „ 43 15	629	334	308	696
43 15 „ 43 30	629	308	282	681
43 30 „ 43 45	630	282	256	667
43 45 „ 44 0	630	256	230	651
44 0 „ 44 15	8·631	6·230	6·204	10·637
44 15 „ 44 30	631	204	177	621
44 30 „ 44 45	631	177	151	606
44 45 „ 45 0	632	151	124	591
45 0 „ 45 15	632	124	098	576
45 15 „ 45 30	632	098	071	561
45 30 „ 45 45	633	071	044	546
45 45 „ 46 0	633	044	017	531
46 0 „ 46 15	8·634	6·017	5·990	10·516
46 15 „ 46 30	634	5·990	963	501
46 30 „ 46 45	634	963	935	485
46 45 „ 47 0	635	935	908	470
47 0 „ 47 15	635	908	880	455
47 15 „ 47 30	635	880	852	439
47 30 „ 47 45	636	852	824	424
47 45 „ 48 0	636	824	797	409

a quarter of a degree of Lat. and Long on the scale of 1 inch to 2 miles

Latitude	Length in Inches			
	m on meridian	n on lower parallel	p on upper parallel	q on diagonal
From 48° 0′ to 48° 15′	8·637	5 797	5 769	10 394
48 15 ,, 48 30	637	769	741	378
48 30 48 45	637	741	712	363
48 45 49 0	638	712	684	348
49 0 ,, 49 15	638	684	655	333
49 15 49 30	639	655	627	318
49 30 49 45	639	627	598	302
49 45 50 0	640	598	569	[illegible]
50 0 50 15	8 640	5 569	5 540	10 271
50 15 50 30	640	540	511	256
50 30 50 45	640	511	482	240
50 45 51 0	641	482	453	225
51 0 51 15	641	453	423	210
51 15 51 30	642	423	394	195
51 30 51 45	642	394	364	179
51 45 ,, 52 0	642	364	335	164
52 0 52 15	8 643	5 335	5·305	10 149
52 15 ,, 52 30	643	305	275	133
52 30 ,, 52 45	643	275	245	118
52 45 53 0	644	245	215	103
53 0 53 15	644	215	185	088
53 15 53 30	645	185	155	073
53 30 53 45	645	155	124	057
53 45 54 0	645	124	094	042
54 0 54 15	8 646	5 094	5 063	10 027
54 15 54 30	646	063	032	012
54 30 54 45	646	032	002	9 996
54 45 55 0	647	002	4 971	981
55 0 55 15	647	4 971	940	966
55 15 55 30	647	940	909	951
55 30 55 45	648	909	878	936
55 45 56 0	648	878	846	921
56 0 56 15	8 649	4 846	4 815	9 906
56 15 56 30	649	815	784	891
56 30 ,, 56 45	649	784	752	876
56 45 57 0	650	752	721	861
57 0 57 15	650	721	689	847
57 15 57 30	[illegible]	689	657	832
57 30 57 45	651	657	[illegible]	817
57 45 58 0	651	[illegible]	593	802
58 0 58 15	8 651	4 593	4 561	9 787
58 15 58 30	652	561	529	773
58 30 58 45	652	529	497	758
58 45 59 0	652	497	464	743
59 0 59 15	653	464	432	720
59 15 ,, 59 30	653	432	400	715
59 30 59 45	653	400	367	700
59 45 60 0	654	367	334	686

TABLE X —Length in Miles of

Latitude	Length of 15 of Latitude	Length of 15 of Longitude	Latitude	Length of 15 of Latitude	Length of 15′ of Longitude
° ′			° ′		
0 0 to 0 15	17 176	17 294	16 0 to 16 15	17 190	16 628
1 0 to 1 15	176	·290	17 0 to 17 15	·191	542
2 0 to 2 15	176	·282	18 0 to 18 15	192	452
3 0 to 3 15	176	270	19 0 to 19 15	194	356
4 0 to 4 15	176	252	20 0 to 20 15	196	256
5 0 to 5 15	178	228	21 0 to 21 15	198	·152
6 0 to 6 15	178	200	22 0 to 22 15	200	·042
7 0 to 7 15	178	·166	23 0 to 23 15	·202	15 926
8 0 to 8 15	180	126	24 0 to 24 15	206	808
9 0 to 9 15	180	082	25 0 to 25 15	208	·682
10 0 to 10 15	182	032	26 0 to 26 15	210	·554
11 0 to 11 15	182	16·978	27 0 to 27 15	212	·418
12 0 to 12 15	183	918	28 0 to 28 15	214	·280
13 0 to 13 15	184	852	29 0 to 29 15	218	·138
14 0 to 14 15	186	782	30 0 to 30 15	220	14 900
15 0 to 15 15	188	708	31 0 to 31 15	222	838

a quarter of a degree (15′) of Latitude and Longitude.

Latitude	Length of 15′ of Latitude	Length of 15′ of Longitude	Latitude	Length of 15′ of Latitude	Length of 15′ of Longitude.
° ′			° ′		
32 0 to 32 15	17 226	14 680	47 0 to 47 15	17 270	11 816
33 0 to 33 15	228	518	48 0 to 48 15	274	·594
34 0 to 34 15	232	·350	49 0 to 49 15	276	368
35 0 to 35 15	234	182	50 0 to 50 15	280	138
36 0 to 36 15	·236	006	51 0 to 51 15	282	10 906
37 0 to 37 15	240	13 828	52 0 to 52 15	285	·670
38 0 to 38 15	242	·644	53 0 to 53 15	288	·430
39 0 to 39 15	246	458	54 0 to 54 15	292	·188
40 0 to 40 15	250	266	55 0 to 55 15	294	9 942
41 0 to 41 15	252	·070	56 0 to 56 15	298	692
42 0 to 42 15	254	12 870	57 0 to 57 15	300	442
43 0 to 43 15	258	668	58 0 to 58 15	303	·186
44 0 to 44 15	262	·460	59 0 to 59 15	·306	8 928
45 0 to 45 15	264	248	60 0 to 60 15	308	669
46 0 to 46 15	268	034			

The length of the diagonal, q, of a space bounded by meridians and parallels m, n, p,

$$= \sqrt{m^2 + np}$$

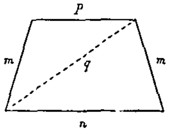


TABLE XI—Linear value in feet of one second of Arc

Formula: Length in feet $= \rho \sin 1''$

Latitude	Length in feet	Logarithm	Diff.	Latitude	Length in feet	Logarithm	Diff.
° ′			+	° ′			+
0 0	100·7654	2·0033115	0	2 30	100·7674	2·0033199	6
5	7654	115	1	35	7675	205	6
10	7654	116	0	40	7676	211	6
15	7654	116	1	45	7678	217	6
20	7654	117	1	50	7679	223	6
25	7654	118	1	55	7681	229	7
30	7655	119	1	3 0	100·7682	2·0033236	7
35	7655	120	1	5	7684	243	7
40	7656	121	1	10	7685	250	7
45	7656	122	2	15	7687	257	8
50	7656	124	2	20	7689	265	7
55	7657	126	3	25	7690	272	8
1 0	100·7657	2·0033129	2	30	7692	280	8
5	7658	131	3	35	7694	288	8
10	7658	134	2	40	7696	296	8
15	7659	136	3	45	7698	304	9
20	7660	139	3	50	7700	313	8
25	7660	142	3	55	7702	321	9
30	7661	145	4	4 0	100·7704	2·0033330	9
35	7662	149	4	5	7706	339	9
40	7663	153	3	10	7708	348	9
45	7664	156	4	15	7710	357	10
50	7665	160	4	20	7713	367	10
55	7666	164	5	25	7715	377	10
2 0	100·7667	2·0033169	4	30	7717	387	10
5	7668	173	5	35	7719	397	11
10	7669	178	5	40	7722	408	10
15	7670	183	5	45	7724	418	11
20	7671	188	5	50	7727	429	11
25	7672	193	6	55	7729	440	11

and its Logarithm, measured along the Meridian.

Latitude	Length in feet.	Logarithm	Diff.	Latitude	Length in feet.	Logarithm	Diff.
° '			+	° '			+
5 0	100·7732	2·0033451	11	7 30	100·7829	2·0033868	17
5	7735	462	12	35	7833	885	17
10	7737	474	11	40	7837	902	17
15	7740	485	12	45	7841	919	17
20	7743	497	12	50	7845	936	17
25	7745	509	12	55	7849	953	18
30	7748	521	12	8 0	100·7853	2·0033971	18
35	7751	533	13	5	7857	989	18
40	7754	546	13	10	7861	2·0034007	18
45	7757	559	13	15	7865	025	19
50	7760	572	13	20	7870	044	18
55	7763	585	13	25	7874	062	19
6 0	100·7766	2·0033598	13	30	7878	081	19
5	7769	611	14	35	7883	100	19
10	7772	625	14	40	7887	119	19
15	7776	639	14	45	7892	138	19
20	7779	653	14	50	7896	157	20
25	7782	667	15	55	7901	177	20
30	7786	682	14	9 0	100·7905	2·0034197	20
35	7789	696	15	5	7910	217	20
40	7792	711	15	10	7914	237	20
45	7796	726	15	15	7919	257	21
50	7799	741	15	20	7924	278	20
55	7803	756	16	25	7929	298	21
7 0	100·7806	2·0033772	15	30	7934	319	21
5	7810	787	16	35	7939	340	22
10	7814	803	16	40	7944	362	21
15	7817	819	16	45	7949	383	22
20	7821	835	16	50	7953	405	21
25	7825	851	17	55	7959	426	22

TABLE XI—Linear value in feet of one second of Arc

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff.
° ′			+	° ′			+
10 0	100 7964	2 0034448	22	12 30	100 8135	2 0035186	27
5	7969	470	22	35	8141	213	28
10	7974	492	22	40	8148	241	27
15	7979	514	33	45	8154	268	28
20	7984	537	23	50	8160	296	28
25	7990	560	23	55	8167	324	28
30	7995	583	23	13 0	100 8173	2 0035352	28
35	8000	606	24	5	8180	380	29
40	8006	630	23	10	8187	409	28
45	8011	653	24	15	8193	437	29
50	8017	677	24	20	8200	466	29
55	8022	701	24	25	8206	495	29
11 0	100 8028	2 0034725	24	30	8213	524	29
5	8034	749	24	35	8220	553	30
10	8039	773	24	40	8227	583	30
15	8045	797	25	45	8234	613	30
20	8050	822	25	50	8241	643	30
25	8056	847	25	55	8248	673	30
30	8062	872	25	14 0	100 8255	2·0035703	30
35	8068	897	26	5	8262	733	30
40	8074	923	25	10	8269	763	31
45	8080	948	26	15	8276	794	31
50	8086	974	26	20	8283	825	31
55	8092	2 0035000	26	25	8290	856	31
12 0	100 8098	2 0035026	26	30	8297	887	31
5	8104	052	27	35	8305	918	31
10	8110	079	26	40	8312	949	32
15	8116	105	27	45	8319	981	32
20	8122	132	27	50	8327	2 0036013	32
25	8129	159	27	55	8334	045	32

and its Logarithm, measured along the Meridian.

Latitude °	′	Length in feet.	Logarithm	Diff.
15	0	100·8342	2·0036077	+ 32
	5	8349	109	32
	10	8357	141	33
	15	8364	174	33
	20	8372	207	33
	25	8379	240	33
	30	8387	273	33
	35	8395	306	33
	40	8403	339	33
	45	8410	372	34
	50	8418	406	34
	55	8426	440	34
16	0	100·8434	2·0036474	34
	5	8442	508	35
	10	8450	543	34
	15	8458	577	35
	20	8466	612	35
	25	8474	647	35
	30	8482	682	35
	35	8490	717	35
	40	8498	752	35
	45	8506	787	36
	50	8515	823	36
	55	8523	859	36
17	0	100·8531	2·0036895	36
	5	8540	931	36
	10	8548	967	36
	15	8557	2·0037003	37
	20	8565	040	36
	25	8574	076	37
17	30	100·8582	2·0037113	37
	35	8591	150	37
	40	8599	187	37
	45	8608	224	38
	50	8617	262	37
	55	8626	299	38
18	0	100·8634	2·0037337	38
	5	8643	375	38
	10	8652	413	38
	15	8661	451	39
	20	8670	490	38
	25	8679	528	39
	30	8688	567	39
	35	8697	606	39
	40	8706	645	39
	45	8715	684	39
	50	8724	723	39
	55	8733	763	40
19	0	100·8742	2·0037802	40
	5	8752	842	40
	10	8761	882	40
	15	8770	922	40
	20	8779	962	40
	25	8789	2·0038002	40
	30	8798	042	40
	35	8807	082	41
	40	8817	123	41
	45	8826	164	41
	50	8836	205	41
	55	8845	246	42

TABLE XI —Linear value in feet of one second of Arc

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff
° ′			+	° ′			+
20 0	100 8855	2 0038288	41	22 30	100 9158	2 0039592	45
5	8865	329	42	35	9169	637	46
10	8874	371	41	40	9179	683	46
15	8884	412	42	45	9190	729	46
20	8894	454	42	50	9200	775	46
25	8903	496	42	55	9211	821	46
30	8913	538	42	23 0	100 9222	2 0039867	46
35	8923	580	43	5	9233	913	47
40	8933	623	42	10	9244	960	46
45	8943	665	43	15	9254	2 0040006	47
50	8953	708	43	20	9265	053	47
55	8963	751	43	25	9276	100	47
21 0	100 8973	2 0038794	43	30	9287	147	47
5	8983	837	44	35	9298	194	47
10	8993	881	43	40	9309	241	47
15	9003	924	44	45	9320	258	48
20	9013	968	43	50	9331	336	48
25	9023	2 0039011	44	55	9342	384	48
30	9033	055	44	24 0	100 9353	2 0040432	48
35	9043	099	44	5	9364	480	48
40	9054	143	44	10	9375	528	48
45	9064	187	45	15	9387	576	48
50	9074	232	44	20	9398	624	48
55	9085	276	45	25	9409	672	49
22 0	100 9095	2 0039321	45	30	9420	721	48
5	9106	366	45	35	9432	769	49
10	9116	411	45	40	9443	818	49
15	9127	456	45	45	9454	867	49
20	9137	501	45	50	9466	916	49
25	9147	546	46	55	9477	965	50

and its Logarithm, measured along the Meridian.

Latitude		Length in feet.	Logarithm	Diff
°	′			+
25	0	100·9489	2·0041015	49
	5	9500	064	49
	10	9512	113	50
	15	9523	163	50
	20	9535	213	50
	25	9547	263	50
	30	9558	313	50
	35	9570	363	50
	40	9581	413	50
	45	9593	463	51
	50	9605	514	50
	55	9616	564	51
26	0	100·9628	2·0041615	51
	5	9640	666	51
	10	9652	717	51
	15	9664	768	51
	20	9676	819	51
	25	9687	870	51
	30	9699	921	51
	35	9711	972	52
	40	9723	2·0042024	52
	45	9735	076	52
	50	9747	128	52
	55	9760	180	52
27	0	100·9772	2·0042232	52
	5	9784	284	52
	10	9796	336	52
	15	9808	388	53
	20	9820	441	52
	25	9832	493	53

Latitude		Length in feet	Logarithm	Diff
°	′			+
27	30	100·9845	2·0042546	53
	35	9857	599	53
	40	9869	652	53
	45	9882	705	53
	50	9894	758	53
	55	9906	811	54
28	0	100·9919	2·0042865	53
	5	9931	918	54
	10	9944	972	53
	15	9956	2·0043025	54
	20	9969	079	54
	25	9981	133	54
	30	9994	187	54
	35	101·0006	241	54
	40	0019	295	54
	45	0031	349	54
	50	0044	403	54
	55	0056	457	55
29	0	101·0069	2·0043512	55
	5	0082	567	55
	10	0095	622	55
	15	0107	677	55
	20	0120	732	55
	25	0133	787	55
	30	0146	842	55
	35	0159	897	55
	40	0172	952	55
	45	0185	2·0044007	56
	50	0198	063	56
	55	0210	119	56

TABLE XI—Linear value in feet of one second of Arc

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff
° ′			+	° ′			+
30 0	101 0223	2 0044175	56	32 30	101 0622	2 0045888	58
5	0236	231	56	35	0635	946	59
10	0249	287	56	40	0649	2 0046005	58
15	0262	343	56	45	0663	063	59
20	0275	399	56	50	0677	122	59
25	0289	455	56	55	0690	181	59
30	0302	511	56	33 0	101 0704	2 0046240	59
35	0315	567	57	5	0718	299	59
40	0328	624	56	10	0731	358	59
45	0341	680	57	15	0745	417	59
50	0354	737	57	20	0759	476	59
55	0367	794	57	25	0773	535	59
31 0	101 0381	2 0044851	57	30	0787	594	59
5	0394	908	57	35	0800	653	60
10	0407	965	57	40	0814	713	59
15	0421	2 0045022	57	45	0828	772	60
20	0434	079	57	50	0842	832	59
25	0447	136	57	55	0856	891	60
30	0461	193	57	34 0	101 0870	2 0046951	60
35	0474	250	58	5	0883	2·0047011	60
40	0487	308	58	10	0897	071	60
45	0501	366	58	15	0911	131	60
50	0514	424	57	20	0925	191	60
55	0527	481	58	25	0939	251	60
32 0	101 0541	2 0045539	58	30	0953	311	60
5	0554	597	58	35	0967	371	60
10	0568	655	58	40	0981	431	60
15	0581	713	59	45	0995	491	61
20	0595	772	58	50	1009	552	60
25	0608	830	58	55	1023	612	61

and its Logarithm, measured along the Meridian.

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff
° ′			+	° ′			+
35 0	101 1038	2·0047673	60	37 30	101 1467	2·0049516	62
5	1052	733	61	35	1481	578	63
10	1066	794	61	40	1496	641	62
15	1080	855	61	45	1510	703	63
20	1094	916	61	50	1525	766	62
25	1108	977	61	55	1539	828	63
30	1122	2·0048038	61	38 0	101 1554	2·0049891	62
35	1137	099	61	5	1569	953	63
40	1151	160	61	10	1583	2·0050016	62
45	1165	221	61	15	1598	078	63
50	1179	282	61	20	1612	141	63
55	1193	343	61	25	1627	204	63
36 0	101 1208	2·0048404	61	30	1642	267	62
5	1222	465	62	35	1656	329	63
10	1236	527	61	40	1671	392	63
15	1250	588	62	45	1686	455	63
20	1265	650	61	50	1700	518	63
25	1279	711	62	55	1715	581	63
30	1294	773	61	39 0	101 1730	2·0050644	63
35	1308	834	62	5	1744	707	64
40	1322	896	62	10	1759	771	63
45	1337	958	62	15	1774	834	63
50	1351	2·0049020	62	20	1788	897	63
55	1366	082	62	25	1803	960	63
37 0	101 1380	2·0049144	62	30	1818	2·0051023	63
5	1395	206	62	35	1832	086	64
10	1409	268	62	40	1847	150	63
15	1423	330	62	45	1862	213	64
20	1438	392	62	50	1877	277	63
25	1452	454	62	55	1891	340	64

TABLE XI.—Linear value in feet of one second of Arc

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff
° ′			+	° ′			+
40 0	101·1906	2·0051404	63	42 30	101·2353	2·0053321	64
5	1921	467	64	35	2368	385	64
10	1936	531	63	40	2383	449	64
15	1951	594	64	45	2398	513	65
20	1966	658	63	50	2413	578	64
25	1980	721	64	55	2428	642	65
30	1995	785	64	43 0	101·2443	2·0053707	64
35	2010	849	64	5	2458	771	64
40	2025	913	63	10	2473	835	64
45	2040	976	64	15	2488	899	65
50	2055	2·0052040	64	20	2503	964	64
55	2070	104	64	25	2518	2·0054028	65
41 0	101·2085	2·0052168	64	30	2533	093	64
5	2100	232	64	35	2548	157	65
10	2114	296	63	40	2563	222	64
15	2129	359	64	45	2578	286	65
20	2144	423	64	50	2593	351	64
25	2159	487	64	55	2608	415	64
30	2174	551	64	44 0	101·2623	2·0054479	64
35	2189	615	64	5	2638	543	65
40	2204	679	64	10	2653	608	64
45	2219	743	65	15	2668	672	65
50	2234	808	64	20	2684	737	64
55	2249	872	64	25	2699	801	65
42 0	101·2264	2·0052936	64	30	2714	866	64
5	2279	2·0053000	64	35	2729	930	65
10	2294	064	64	40	2744	995	64
15	2308	128	64	45	2759	2·0055059	65
20	2323	192	64	50	2774	124	64
25	2338	256	65	55	2789	188	65

and its Logarithm, measured along the Meridian.

Latitude		Length in feet.	Logarithm	Diff.	Latitude		Length in feet.	Logarithm	Diff.
°	′			+	°	′			+
45	0	101·2804	2·0055253	64	47	30	101·3255	2·0057186	64
	5	2819	317	65		35	3270	250	65
	10	2834	382	64		40	3285	315	64
	15	2849	446	65		45	3300	379	64
	20	2864	511	64		50	3315	443	64
	25	2879	575	65		55	3330	507	65
	30	2894	640	64	48	0	101·3345	2·0057572	64
	35	2909	704	65		5	3360	636	64
	40	2924	769	64		10	3375	700	64
	45	2939	833	65		15	3390	764	64
	50	2954	898	64		20	3405	828	64
	55	2969	962	65		25	3419	892	64
46	0	101·2984	2·0056027	64		30	3434	956	64
	5	2999	091	65		35	3449	2·0058020	65
	10	3015	156	64		40	3464	085	64
	15	3030	220	65		45	3479	149	64
	20	3045	285	64		50	3494	213	63
	25	3060	349	65		55	3509	276	64
	30	3075	414	64	49	0	101·3524	2·0058340	64
	35	3090	478	65		5	3539	404	64
	40	3105	543	64		10	3554	468	64
	45	3120	607	65		15	3569	532	64
	50	3135	672	64		20	3584	596	63
	55	3150	736	64		25	3598	659	64
47	0	101·3165	2·0056800	64		30	3613	723	64
	5	3180	864	65		35	3628	787	64
	10	3195	929	64		40	3643	851	63
	15	3210	993	65		45	3658	914	64
	20	3225	2·0057058	64		50	3673	978	63
	25	3240	122	64		55	3688	2·0059041	64

TABLE XI.—Linear value in feet of one second of Arc

Latitude	Length in feet	Logarithm	Diff	Latitude	Length in feet	Logarithm	Diff
° ′			+	° ′			+
50 0	101·3703	2·0059105	63	52 30	101·4144	2·0060995	62
5	3717	168	64	35	4158	2·0061057	63
10	3732	232	63	40	4173	120	62
15	3747	295	64	45	4187	182	63
20	3762	359	63	50	4202	245	62
25	3776	422	64	55	4217	307	62
30	3791	486	63	53 0	101·4231	2·0061369	62
35	3806	549	64	5	4246	431	62
40	3821	613	63	10	4260	493	62
45	3836	676	63	15	4275	555	62
50	3851	739	63	20	4289	617	61
55	3865	802	64	25	4303	678	62
51 0	101·3880	2·0059866	63	30	4318	740	62
5	3895	929	63	35	4332	802	62
10	3909	992	63	40	4347	864	61
15	3924	2·0060055	63	45	4361	925	62
20	3939	118	63	50	4375	987	61
25	3954	181	63	55	4390	2·0062048	62
30	3968	244	62	54 0	101·4404	2·0062110	61
35	3983	306	63	5	4418	171	61
40	3998	369	63	10	4433	232	61
45	4012	432	63	15	4447	293	62
50	4027	495	62	20	4461	355	61
55	4041	557	63	25	4476	416	61
52 0	101·4056	2·0060620	63	30	4490	477	61
5	4071	683	63	35	4504	538	61
10	4085	746	62	40	4518	599	61
15	4100	808	63	45	4532	660	61
20	4115	871	62	50	4547	721	60
25	4129	933	62	55	4561	781	61

and its Logarithm, measured along the Meridian.

Latitude °	′	Length in feet	Logarithm	Diff +
55	0	101 4575	2·0062842	61
	5	4589	903	61
	10	4603	964	60
	15	4618	2·0063024	61
	20	4632	085	60
	25	4646	145	60
	30	4660	205	60
	35	4674	265	61
	40	4688	326	60
	45	4702	386	60
	50	4716	446	60
	55	4730	506	60
56	0	101 4744	2·0063566	60
	5	4758	626	60
	10	4772	686	59
	15	4786	745	60
	20	4800	805	59
	25	4814	864	60
	30	4828	924	59
	35	4842	983	60
	40	4856	2·0064043	59
	45	4869	102	59
	50	4883	161	59
	55	4897	220	59
57	0	101 4911	2·0064279	59
	5	4925	338	59
	10	4938	397	59
	15	4952	456	59
	20	4966	515	58
	25	4980	573	59
57	30	101 4993	2·0064632	58
	35	5007	690	59
	40	5021	749	58
	45	5034	807	59
	50	5048	866	58
	55	5062	924	58
58	0	101 5075	2·0064982	58
	5	5089	2·0065040	58
	10	5102	098	58
	15	5116	156	58
	20	5129	214	57
	25	5143	271	58
	30	5156	329	57
	35	5170	386	58
	40	5183	444	57
	45	5197	501	57
	50	5201	558	57
	55	5223	615	58
59	0	101 5237	2·0065673	57
	5	5250	730	57
	10	5263	787	56
	15	5276	843	57
	20	5289	900	56
	25	5303	956	57
	30	5316	2·0066013	56
	35	5329	069	57
	40	5343	126	56
	45	5356	182	57
	50	5369	239	56
	55	5382	295	56
60	0	101 5395	2·0066351	

TABLE XII.—Linear value in feet of one second of Arc

Formula: Length in feet $= \frac{\pi}{180 \times 3600} \nu \cos \lambda$.

Latitude.	Length in feet	Diff.	Logarithm.	Diff.	Latitude.	Length in feet.	Diff.	Logarithm.	Diff.
° ′		—		—	° ′		—		—
0 0	101·4538	1	2·0062683	5	2 30	101·3579	65	2·0058576	279
5	4537	4	2678	13	35	3514	67	8297	287
10	4533	5	2665	23	40	3417	69	8010	297
15	4528	7	2642	31	45	3378	72	7713	306
20	4521	10	2611	42	50	3306	74	7407	316
25	4511	11	2569	50	55	3232	75	7091	324
30	4500	14	2519	59	3 0	191·3157	78	2·0056767	333
35	4486	16	2460	69	5	3079	79	6434	342
40	4470	18	2391	77	10	3000	83	6092	353
45	4452	21	2314	87	15	2917	84	5739	360
50	4431	22	2227	96	20	2833	86	5379	370
55	4409	24	2131	105	25	2747	88	5009	379
1 0	101·4385	27	2·0062026	114	30	2659	91	4630	388
5	4358	29	1912	123	35	2568	93	4242	398
10	4329	31	1789	133	40	2475	95	3844	407
15	4298	33	1656	141	45	2380	96	3437	415
20	4265	35	1515	151	50	2284	99	3022	425
25	4230	37	1364	159	55	2185	102	2597	434
30	4193	40	1205	169	4 0	101·2083	103	2·0052163	443
35	4153	41	1036	178	5	1980	105	1720	453
40	4112	44	0858	187	10	1875	108	1267	462
45	4068	46	0671	197	15	1767	110	0805	471
50	4022	48	0474	204	20	1657	112	0334	480
55	3974	50	0270	215	25	1545	113	2·0049854	489
2 0	101·3924	52	2·0060055	224	30	1432	116	9365	498
5	3872	54	2·0059831	233	35	1316	119	8867	507
10	3818	57	9598	242	40	1197	120	8360	518
15	3761	58	9356	251	45	1077	122	7842	525
20	3703	61	9105	260	50	0955	125	7317	536
25	3642	63	8845	269	55	0830	127	6781	544

and its Logarithm, measured along Parallels of Latitude.

Latitude ° ′	Length in feet	Diff	Logarithm	Diff
5 0	101·0703	—	2·0046237	—
5	0575	128	5684	553
10	0444	131	5121	563
15	0310	134	4549	572
20	0175	135	3968	581
25	0038	137	3377	591
30	100·9899	139	2778	599
35	9757	142	2170	608
40	9614	143	1552	618
45	9468	146	0924	628
50	9320	148	0288	636
55	9170	150	2·0039643	645
6 0	100·9017	153	2·0038987	656
5	8863	154	8324	663
10	8707	156	7650	674
15	8548	159	6968	682
20	8388	160	6276	692
25	8225	163	5575	701
30	8060	165	4865	710
35	7893	167	4145	720
40	7724	169	3417	728
45	7553	171	2679	738
50	7380	173	1932	747
55	7204	176	1175	757
7 0	100·7026	178	2·0030409	766
5	6847	179	2·0029634	775
10	6665	182	8849	785
15	6481	184	8056	793
20	6295	186	7253	803
25	6107	188	6442	811
		190		822

Latitude ° ′	Length in feet	Diff	Logarithm	Diff
7 30	100·5917	—	2·0025620	—
35	5724	193	4789	[illegible]
40	5530	194	3949	[illegible]
45	5333	197	3100	[illegible]
50	5134	199	2241	[illegible]
55	4934	200	1374	867
8 0	100·4731	203	2·0020498	[illegible]
5	4525	206	2·0019611	887
10	4318	207	8714	[illegible]
15	4109	209	7808	[illegible]
20	3898	211	6895	913
25	3684	214	5971	924
30	3469	215	5038	933
35	3251	218	4095	943
40	3031	220	3144	951
45	2809	222	2182	962
50	2585	224	1212	970
55	2359	226	0232	980
9 0	100·2130	229	2·0009243	989
5	1900	230	8244	999
10	1668	232	7237	1007
15	1433	235	6220	1017
20	1196	237	5193	1027
25	0958	238	4157	1036
30	0717	241	3111	1046
35	0474	243	2056	1055
40	0229	245	0993	1063
45	99·9981	248	1·9999919	1074
50	9732	249	8836	1083
55	9481	251	7744	1092
		254		1102

TABLE XII.—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff.	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
10 0	99 9227	255	1 9990642	1111	12 30	99 0647	319	1 9959118	1396
5	8972	258	5531	1121	35	0328	320	57792	1405
10	8714	260	4410	1129	40	0008	323	56387	1415
15	8454	262	3281	1140	45	98 9685	324	54972	1425
20	8192	264	2141	1149	50	9361	327	53547	1435
25	7928	266	0992	1159	55	9034	329	52123	1444
30	7662	268	1 9989833	1167	13 0	98 8705	331	1 9950668	1454
35	7394	271	8666	1177	5	8374	333	49214	1463
40	7123	272	7489	1187	10	8041	335	47751	1473
45	6851	275	6302	1196	15	7706	337	46278	1482
50	6576	276	5106	2205	20	7369	339	44796	1493
55	6300	279	3901	1215	25	7030	341	43303	1502
11 0	99 6021	281	1 9982686	1225	30	6689	344	41801	1511
5	5740	283	81461	1234	35	6345	345	40290	1521
10	5457	285	80227	1244	40	6000	347	38769	1531
15	5172	287	78983	1253	45	5653	350	37238	1541
20	4885	289	77730	1263	50	5303	352	35697	1550
25	4596	291	76467	1271	55	4951	354	34147	1560
30	4305	294	75196	1282	14 0	98·4597	355	1 9932587	1570
35	4011	295	73914	1290	5	4212	358	31017	1579
40	3716	297	72624	1301	10	3884	360	29438	1589
45	3419	300	71323	1310	15	3524	362	27849	1600
50	3119	302	70013	1320	20	3162	364	26249	1607
55	2817	304	68693	1329	25	2798	366	24642	1619
12 0	99 2513	305	1·9967364	1338	30	2432	369	23023	1629
5	2208	308	66026	1349	35	2063	370	21394	1637
10	1900	311	64677	1358	40	1693	373	19757	1648
15	1589	312	63319	1367	45	1320	374	18109	1657
20	1277	314	61952	1377	50	0946	376	16452	1667
25	0963	316	60575	1387	55	0570	379	14785	1677

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		–		–	° ′		–		–
15 0	98·0191	381	1·9913108	1687	17 30	96·7879	442	1·9858211	1985
5	97·9810	382	11421	1696	35	7437	444	56226	1994
10	9428	385	09725	1706	40	6993	446	54232	2004
15	9043	387	08019	1716	45	6547	449	52228	2015
20	8656	388	06303	1726	50	6098	450	50213	2024
25	8268	391	04577	1736	55	5648	452	48189	2036
30	7877	393	02841	1746	18 0	96·5196	455	1·9846153	2044
35	7484	395	01095	1755	5	4741	456	44109	2055
40	7089	397	1·9899340	1766	10	4285	459	42054	2066
45	6692	400	97574	1775	15	3826	460	39988	2075
50	6292	401	95799	1785	20	3366	462	37913	2085
55	5891	403	94014	1795	25	2904	465	35828	2095
16 0	97·5488	405	1·9892219	1805	30	2439	466	33733	2106
5	5083	408	90414	1814	35	1973	469	31627	2116
10	4675	409	88600	1825	40	1504	471	29511	2126
15	4266	412	86775	1834	45	1033	472	27385	2136
20	3854	413	84941	1845	50	0561	475	25249	2147
25	3441	416	83096	1854	55	0086	476	23102	2156
30	3025	417	81242	1864	19 0	95·9610	479	1·9820946	2166
35	2608	420	79378	1875	5	9131	481	18780	2178
40	2188	422	77503	1885	10	8650	483	16602	2188
45	1766	423	75618	1894	15	8167	484	14414	2197
50	1343	426	73724	1904	20	7683	487	12217	2208
55	0917	428	71820	1914	25	7196	489	10009	2218
17 0	97·0489	430	1·9869906	1924	30	6707	490	07791	2228
5	0059	432	67982	1934	35	6217	493	05563	2239
10	96·9627	434	66048	1945	40	5724	495	03324	2249
15	9193	436	64103	1954	45	5229	497	01075	2260
20	8757	438	62149	1964	50	4732	499	1·9798815	2270
25	8319	440	60185	1974	55	4233	501	96545	2280

TABLE XII—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
20 0	95 3732	502	1 9794265	2290	22 30	93 7777	563	1 9720995	2606
5	3230	505	91975	2301	35	7214	564	18389	2618
10	2725	507	89674	2311	40	6650	567	15771	2627
15	2218	509	87363	2321	45	6083	569	13144	2639
20	1709	511	85042	2332	50	5514	570	10505	2650
25	1198	513	82710	2343	55	4944	573	07855	2660
30	0685	515	80367	2353	23 0	93 4371	574	1 9705195	2672
35	0170	517	78014	2363	5	3797	577	02523	2681
40	94 9653	519	75651	2374	10	3220	578	1 9699842	2694
45	9134	520	73277	2384	15	2642	581	97148	2703
50	8614	523	70893	2394	20	2061	582	94445	2715
55	8091	525	68499	2406	25	1479	584	91730	2725
21 0	94 7566	527	1 9766093	2416	30	0895	587	89005	2736
5	7039	529	63677	2425	35	0308	588	86269	2748
10	6510	531	61252	2437	40	92 9720	590	83521	2758
15	5979	533	58815	2447	45	9130	593	80763	2769
20	5446	535	56368	2458	50	8537	594	77994	2780
25	4911	537	53910	2468	55	7943	596	75214	2790
30	4374	538	51442	2478	24 0	92 7347	598	1 9672424	2802
35	3836	541	48964	2491	5	6749	600	69622	2813
40	3295	543	46473	2499	10	6149	602	66809	2824
45	2752	545	43974	2511	15	5547	604	63985	2835
50	2207	547	41463	2521	20	4943	606	61150	2846
55	1660	548	38942	2531	25	4337	608	58304	2857
22 0	94 1112	551	1 9736411	2543	30	3729	609	55447	2867
5	0561	553	33868	2553	35	3120	612	52580	2879
10	0008	555	31315	2564	40	2508	614	49701	2891
15	93 9453	556	28751	2574	45	1894	615	46810	2901
20	8897	559	26177	2586	50	1279	618	43909	2912
25	8338	561	23591	2596	55	0661	620	40997	2924

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet.	Diff	Logarithm.	Diff.	Latitude.	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
25 0	92·0041	621	1·9638073	2934	27 30	90·0558	679	1·9545116	3277
5	91·9420	624	35139	2946	35	89·9879	681	41839	3288
10	8796	625	32193	2957	40	9198	683	38551	3299
15	8171	627	29236	2968	45	8515	685	35252	3312
20	7544	629	26268	2979	50	7830	687	31940	3323
25	6915	632	23289	2991	55	7143	689	28617	3335
30	6283	633	20298	3002	28 0	89·6454	690	1·9525282	3346
35	5650	635	17296	3013	5	5764	692	21936	3358
40	5015	637	14283	3024	10	5072	695	18578	3372
45	4378	638	11259	3035	15	4377	696	15206	3381
50	3740	641	08224	3048	20	3681	698	11825	3394
55	3099	643	05176	3058	25	2983	700	08431	3406
26 0	91·2456	645	1·9602118	3070	30	2283	702	05025	3418
5	1811	646	1·9599048	3080	35	1581	704	01607	3429
10	1165	649	95968	3093	40	0877	706	1·9498178	3441
15	0516	650	92875	3103	45	0171	707	94737	3453
20	90·9866	653	89772	3116	50	88·9464	709	91284	3465
25	9213	654	86656	3126	55	8755	712	87819	3477
30	8559	656	83530	3137	29 0	88·8043	713	1·9484342	3490
35	7903	659	80393	3150	5	7330	715	80852	3501
40	7244	660	77243	3161	10	6615	717	77351	3513
45	6584	662	74082	3172	15	5898	719	73838	3525
50	5922	663	70910	3183	20	5179	720	70313	3538
55	5259	666	67727	3196	25	4459	723	66775	3548
27 0	90·4593	668	1·9564531	3207	30	3736	724	63227	3562
5	3925	670	61324	3218	35	3012	726	59665	3573
10	3255	671	58106	3231	40	2286	728	56092	3585
15	2584	674	54875	3241	45	1558	730	52507	3598
20	1910	675	51634	3253	50	0828	732	48909	3611
25	1235	677	48381	3265	55	0096	734	45298	3622

TABLE XII —Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
30 0	87 9362	736	1 9441676	3635	32 30	85 6192	791	1 9327233	4012
5	8626	737	38041	3646	35	5701	792	23221	4023
10	7889	739	34395	3658	40	4909	795	19198	4038
15	7150	742	30737	3672	45	4114	796	15160	4049
20	6408	743	27065	3684	50	3318	798	11111	4063
25	5665	744	23381	3695	55	2520	799	07048	4076
30	4921	747	19686	3708	33 0	85 1721	802	1 9302972	4089
35	4174	749	15978	3721	5	0919	803	1 9298883	4102
40	3425	750	12257	3733	10	0116	805	94781	4115
45	2675	752	08524	3745	15	84 9311	807	90666	4128
50	1923	755	04779	3758	20	8504	809	86538	4142
55	1168	756	01021	3770	25	7695	811	82396	4154
31 0	87 0412	757	1 9397251	3783	30	6884	812	78242	4168
5	86 9655	760	93468	3795	35	6072	814	74074	4180
10	8895	762	89673	3808	40	5258	816	69894	4195
15	8133	763	85865	3820	45	4442	817	65699	4207
20	7370	765	82045	3833	50	3625	820	61492	4221
25	6605	767	78212	3845	55	2805	821	57271	4234
30	5838	769	74367	3858	34 0	84 1984	823	1 9253037	4247
35	5069	770	70509	3871	5	1161	825	48790	4261
40	4299	773	66638	3882	10	0336	827	44529	4274
45	3526	774	62756	3897	15	83 9509	828	40255	4287
50	2752	776	58859	3909	20	8681	830	35968	4301
55	1976	778	54950	3921	25	7851	832	31667	4315
32 0	86 1198	780	1 9351029	3934	30	7019	834	27352	4329
5	0418	782	47095	3947	35	6185	835	23024	4341
10	85 9636	783	43148	3960	40	5350	838	18683	4356
15	8853	785	39188	3972	45	4512	839	14327	4368
20	8068	787	35216	3985	50	3673	841	09959	4382
25	7281	789	31231	3998	55	2832	842	05577	4396

and its Logarithm, measured along Parallels of Latitude

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′					° ′				
35 0	83·1990	–	1·9201181	–	37 30	80·3901	–	1·9062817	–
5	1146	844	1·9196771	4410	35	5004	897	57983	4834
10	0300	846	92348	4423	40	4106	898	53136	4847
15	82·9452	848	87911	4437	45	3206	900	48272	4864
20	8602	850	83461	4450	50	2305	901	43395	4877
25	7751	851	78996	4465	55	1402	903	38503	4892
30	6898	853	74517	4479	38 0	80·0497	905	1·9033596	4907
35	6043	855	70026	4491	5	79·9590	907	28675	4921
40	5186	857	65519	4507	10	8682	908	23739	4936
45	4328	858	60999	4520	15	7772	910	18788	4951
50	3468	860	56466	4533	20	6860	912	13822	4966
55	2606	862	51918	4548	25	5947	913	08840	4982
36 0	82·1742	864	1·9147356	4562	30	5032	915	03844	4996
5	0877	865	42780	4576	35	4115	917	1·8998833	5011
10	0010	867	38190	4590	40	3196	919	93807	5026
15	81·9141	869	33586	4604	45	2276	920	88766	5041
20	8270	871	28968	4618	50	1354	922	83710	5056
25	7398	872	24336	4632	55	0431	923	78638	5072
30	6524	874	19689	4647	39 0	78·9506	925	1·8973552	5086
35	5648	876	15029	4660	5	8579	927	68450	5102
40	4770	878	10354	4675	10	7650	929	63333	5117
45	3891	879	05665	4689	15	6720	930	58201	5132
50	3011	880	00962	4703	20	5788	932	53054	5147
55	2128	883	1·9096244	4718	25	4854	934	47891	5163
37 0	81·1243	885	1·9091512	4732	30	3919	935	42713	5178
5	0357	886	86766	4746	35	2982	937	37520	5193
10	80·9469	888	82005	4761	40	2044	938	32311	5209
15	8580	889	77230	4775	45	1104	940	27086	5225
20	7689	891	72440	4790	50	0162	942	21846	5240
25	6796	893	67635	4805	55	77·9218	944	16590	5256
		895		4818			945		5271

TABLE XII—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
°		—		—			—		—
40 0	7 8 73	917	1 8911319	5287	42 30	74 9157	996	1 874 727	5 5
5	326	918	06032	530	35	8161	997	399	5792
10	63 8	9 1	00730	5319	40	7164	999	34169	5809
15	54 7	9 2	1 889 411	5334	4	6165	1000	28351	5826
20	44 5	9 3	90077	53 0	50	51?5	1002	22525	5844
25	352	9	847 7	5366	5	4163	1003	1?681	5859
30	2567	957	79361	5381	43 0	74 3160	1005	1 8 10822	58 8
35	1610	9 8	73980	5307	5	2155	1007	04944	?894
40	06 2	960	68583	5414	10	1148	1008	1 8699350	5913
45	76 5692	962	63169	5430	15	0140	1010	59?8	
50	8730	963	5 39	5415	20	5 9130	1011	8 200	5947
55	7767	965	5 294	5461	25	8119	1013	81262	5964
41 0	76 6802	966	1 8846833	178	30	7106	1015	75298	598_
5	5836	968	41355	5493	35	6091	1016	69316	5898
10	4868	9 0	3 862	5511	40	5075	1018	63318	6017
15	3898	9 2	30351	5526	4.	4057	1019	5 301	6034
20	5 6	9 3	24825	5543	50	3038	1020	51267	605
25	1953	975	19 82	5559	55	2018	1023	45215	60 0
30	09 8	9 6	13723	55 5	44 0	73 5095	1024	1 8639145	6087
35	0002	9 8	08148	5592	5	7 59 1	10 5	33058	0105
40	90 4	9 9	0 5 6	5608	10	8946	10 7	26953	6123
4	8015	981	1 8 96048	56 5	15	919	10 9	20830	6141
50	70?4	983	913 3	5640	20	6890	1030	14689	61 8
55	6081	984	8 683	5?58	2	5860	1031	08531	61 7
42 (	5 5097	986	1 8 800 5	5? 5	30	4829	1033	0 354	6195
	4111	988	13 0	5?31	35	3 96	103	1 8596159	6 13
1	3123	989	?8 9	5 08	40	2761	1036	89946	6 31
15	2134	991	6 551	5 5	45	1 5	1038	83715	6 50
2)	1143	99	5 26	5 11	50	0687	1039	77465	6_67
25	(151	994	51185	5 58	55	71 3649	1041	71198	6_86

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff.
° ′		—		—	° ′		—		—
45 0	71·8007	1043	1·8561912	6304	47 30	68·6679	1087	1·8367540	6882
5	7564	1044	55608	6323	35	5592	1089	60658	6902
10	6520	1045	52285	6341	40	4503	1090	53756	6922
15	5475	1047	45944	6360	45	3413	1091	46834	6943
20	4428	1048	39584	6378	50	2322	1093	39891	6962
25	3380	1050	33206	6397	55	1229	1095	32929	6985
30	2330	1052	26809	6415	48 0	68·0134	1096	1·8325944	7004
35	1278	1053	20394	6435	5	67·9038	1098	18940	7025
40	0225	1055	13959	6453	10	7940	1099	11915	7046
45	70·9170	1056	07506	6472	15	6841	1100	04869	7065
50	8114	1057	01034	6491	20	5741	1102	1·8297804	7087
55	7057	1059	1·8494543	6510	25	4639	1103	90717	7108
46 0	70·5998	1061	1·8488033	6529	30	3536	1105	83609	7129
5	4937	1062	81504	6548	35	2431	1106	76480	7149
10	3875	1064	74956	6568	40	1325	1107	69331	7171
15	2811	1065	68388	6586	45	0218	1109	62160	7192
20	1746	1066	61802	6605	50	66·9109	1110	54968	7212
25	0680	1068	55197	6626	55	7999	1112	47756	7236
30	69·9612	1070	48571	6644	49 0	66·6887	1113	1·8240520	7255
35	8542	1071	41927	6664	5	5774	1115	33265	7277
40	7471	1072	35263	6683	10	4659	1116	25988	7299
45	6399	1074	28580	6704	15	3543	1118	18689	7320
50	5325	1076	21876	6722	20	2425	1119	11369	7342
55	4249	1077	15154	6743	25	1306	1120	04027	7364
47 0	69·3172	1078	1·8408411	6761	30	0186	1122	1·8196663	7385
5	2094	1080	01650	6783	35	65·9064	1123	89278	7407
10	1014	1081	1·8394867	6801	40	7941	1124	81871	7429
15	68·9933	1083	88066	6822	45	6817	1126	74442	7452
20	8850	1085	81214	6842	50	5691	1127	66990	7472
25	7765	1086	74402	6862	55	4564	1129	59518	7496

TABLE XII.—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm.	Diff.	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
40 0	77 8273	947	1 8911319	5287	42 30	74 9157	996	1 8745727	5775
5	7326	948	06032	5302	35	8161	997	39952	5792
10	6378	951	00730	5319	40	7164	999	34160	5809
15	5427	952	1 8895411	5334	45	6165	1000	28351	5826
20	4475	953	90077	5350	50	5165	1002	22525	5844
25	3522	955	84727	5366	55	4163	1003	16681	5859
30	2567	957	79361	5381	43 0	74 3160	1005	1 8710822	5878
35	1610	958	73980	5397	5	2155	1007	04944	5894
40	0652	960	68583	5414	10	1148	1008	1 8699050	5913
45	76 9692	962	63169	5430	15	0140	1010	93137	5928
50	8730	963	57739	5445	20	73 9130	1011	87209	5947
55	7767	965	52294	5461	25	8119	1013	81262	5964
41 0	76 6802	966	1 8846833	5478	30	7106	1015	75298	5982
5	5836	968	41355	5493	35	6091	1016	69316	5998
10	4868	970	35862	5511	40	5075	1018	63318	6017
15	3898	972	30351	5526	45	4057	1019	57301	6034
20	2926	973	24825	5543	50	3038	1020	51267	6052
25	1953	975	19282	5559	55	2018	1023	45215	6070
30	0978	976	13723	5575	44 0	73 0995	1024	1 8639145	6087
35	0002	978	08148	5592	5	72 9971	1025	33058	6105
40	75·9024	979	02556	5608	10	8946	1027	26953	6123
45	8045	981	1 8796948	5625	15	7919	1029	20830	6141
50	7064	983	91323	5640	20	6890	1030	14689	6158
55	6081	984	85683	5658	25	5860	1031	08531	6177
42 0	75 5097	986	1 8780025	5675	30	4829	1033	02354	6195
5	4111	988	74350	5691	35	3796	1035	1 8596159	6213
10	3123	989	68659	5708	40	2761	1036	89946	6231
15	2134	991	62951	5725	45	1725	1038	83715	6250
20	1143	992	57226	5741	50	0687	1039	77465	6267
25	0151	994	51485	5758	55	71·9648	1041	71198	6280

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		–		–	° ′		–		–
45 0	71 8607		1 8564912		47 30	68 6679		1 8367540	
		1043		6304			1087		6882
5	7564		58608		35	5592		60658	
		1044		6323			1089		6902
10	6520		52285		40	4503		53756	
		1045		6341			1090		6922
15	5475		45944		45	3413		46834	
		1047		6360			1091		6943
20	4428		39584		50	2322		39891	
		1048		6378			1093		6962
25	3380		33206		55	1229		32929	
		1050		6397			1095		6985
30	2330		26809		48 0	68 0134		1 8325944	
		1052		6415			1096		7004
35	1278		20394		5	67 9038		18940	
		1053		6435			1098		7025
40	0225		13959		10	7940		11915	
		1055		6453			1099		7046
45	70 9170		07506		15	6841		04869	
		1056		6472			1100		7065
50	8114		01034		20	5741		1 8297804	
		1057		6491			1102		7087
55	7057		1 8494543		25	4639		90717	
		1059		6510			1103		7108
46 0	70 5998		1 8488033		30	3536		83609	
		1061		6529			1105		7129
5	4937		81504		35	2431		76480	
		1062		6548			1106		7149
10	3875		74956		40	1325		69331	
		1064		6568			1107		7171
15	2811		68388		45	0218		62160	
		1065		6586			1109		7192
20	1746		61802		50	66 9109		54968	
		1066		6605			1110		7212
25	0680		55197		55	7999		47756	
		1068		6626			1112		7236
30	69 9612		48571		49 0	66 6887		1 8240520	
		1070		6644			1113		7255
35	8542		41927		5	5774		33265	
		1071		6664			1115		7277
40	7471		35263		10	4659		25988	
		1072		6683			1116		7299
45	6399		28580		15	3543		18689	
		1074		6704			1118		7320
50	5325		21876		20	2425		11369	
		1076		6722			1119		7342
55	4249		15154		25	1306		04027	
		1077		6743			1120		7364
47 0	69 3172		1 8408411		30	0186		1 8196663	
		1078		6761			1122		7385
5	2094		01650		35	65 9064		89278	
		1080		6783			1123		7407
10	1014		1 8394867		40	7941		81871	
		1081		6801			1124		7429
15	68 9933		88066		45	6817		74442	
		1083		6822			1126		7452
20	8850		81244		50	5691		66990	
		1085		6842			1127		7472
25	7765		74402		55	4564		59518	
		1086		6862			1129		7496

TABLE XII.—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	° ′		—		—
50 0	65 3435	1130	1 8152022	7518	52 30	61 8935	1171	1·7916447	8223
5	2305	1132	14504	7540	35	7764	1172	08224	8248
10	1173	1133	36964	7563	40	6592	1174	1 7899976	8273
15	0040	1134	29401	7585	45	5418	1175	91703	8299
20	64 8906	1136	21816	7607	50	4243	1176	83404	8324
25	7770	1137	14209	7631	55	3067	1177	75080	8349
30	6633	1138	06578	7652	53 0	61·1890	1179	1 7866731	8374
35	5495	1140	1 8098926	7675	5	0711	1180	58357	8400
40	4355	1141	91251	7699	10	60 9531	1181	49957	8425
45	3214	1143	83552	7722	15	8350	1183	41532	8451
50	2071	1144	75830	7744	20	7167	1184	33081	8478
55	0927	1145	68086	7768	25	5983	1185	24603	8502
51 0	63·9782	1147	1 8060318	7791	30	4798	1187	16101	8529
5	8635	1148	52527	7814	35	3611	1188	07572	8555
10	7487	1149	44713	7838	40	2423	1189	1·7799017	8581
15	6338	1151	36875	7861	45	1234	1190	90436	8607
20	5187	1152	29014	7884	50	0044	1192	81829	8634
25	4035	1154	21130	7908	55	59 8852	1193	73195	8660
30	2881	1155	13222	7932	54 0	59 7659	1194	1 7764535	8687
35	1726	1156	05290	7956	5	6465	1196	55848	8713
40	0570	1157	1 7997334	7979	10	5269	1197	47135	8741
45	62 9413	1159	89355	8004	15	4072	1198	38394	8767
50	8254	1160	81351	8027	20	2874	1199	29627	8795
55	7094	1162	73324	8053	25	1675	1201	20832	8822
52 0	62 5932	1163	1·7965271	8076	30	0474	1202	12010	8849
5	4769	1164	57195	8100	35	58 9272	1203	03161	8875
10	3605	1166	49095	8125	40	8069	1204	1·7694286	8904
15	2439	1167	40970	8149	45	6865	1206	85382	8931
20	1272	1168	32821	8174	50	5659	1207	76451	8959
25	0104	1169	24647	8200	55	4452	1208	67492	8987

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet	Diff	Logarithm	Diff.	Latitude	Length in feet	Diff	Logarithm	Diff.
° ′		—		—	° ′		—		—
55 0	58 3244	1209	1 7658505	9015	57 30	54 6431	1246	1 7375354	9912
5	2035	1211	49490	9042	35	5185	1246	65442	9943
10	0824	1212	40448	9071	40	3939	1248	55499	9976
15	57 9612	1213	31377	9100	45	2691	1249	45523	10007
20	8399	1214	22277	9127	50	1442	1251	35516	10041
25	7185	1216	13150	9157	55	0191	1251	25475	10073
30	5969	1217	03993	9185	58 0	53 8940	1253	1 7315402	10105
35	4752	1218	1 7594808	9213	5	7687	1254	05297	10139
40	3534	1219	85595	9243	10	6433	1255	1 7295158	10172
45	2315	1220	76352	9272	15	5178	1256	84986	10205
50	1095	1222	67080	9300	20	3922	1257	74781	10237
55	56 9873	1223	57780	9330	25	2665	1258	64544	10272
56 0	56 8650	1224	1 7548450	9360	30	1407	1259	54272	10305
5	7426	1226	39090	9389	35	0148	1261	43967	10340
10	6200	1226	29701	9418	40	52 8887	1262	33627	10373
15	4974	1228	20283	9449	45	7625	1263	23254	10408
20	3746	1229	10834	9478	50	6362	1264	12846	10440
25	2517	1230	01356	9508	55	5098	1265	02406	10477
30	1287	1231	1 7491848	9538	59 0	52 3833	1266	1 7191929	10511
35	0056	1233	82310	9570	5	2567	1267	81418	10545
40	55 8823	1234	72740	9599	10	1300	1269	70873	10581
45	7589	1235	63141	9630	15	0031	1270	60292	10617
50	6354	1236	53511	9661	20	51 8761	1271	49675	10651
55	5118	1238	43850	9691	25	7490	1271	39024	10686
57 0	55 3880	1238	1 7434159	9723	30	6219	1273	1 7128338	10723
5	2642	1240	24436	9754	35	4946	1274	17615	10758
10	1402	1241	14682	9784	40	3672	1275	06857	10794
15	0161	1242	04898	9816	45	2397	1277	1 7096063	10831
20	54 8919	1243	1 7395082	9848	50	1120	1277	85232	10867
25	7676	1245	85234	9880	55	50 9843	1278	74365	10903
					60 0	50 8565		1 7063462	

TABLE XII—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—	°		—		—
50 0	65 3135	1130	1 8152022	7518	52 30	61 8935	1171	1 7916447	8223
5	2305	1132	44504	7540	35	7764	1172	08224	8248
10	1173	1133	36964	7563	40	6592	1174	1 7899976	8273
15	0040	1134	29401	7585	45	5418	1175	91703	8299
20	64 8906	1136	21816	7607	50	4243	1176	83404	8324
25	7770	1137	14209	7631	55	3067	1177	75080	8349
30	6633	1138	06578	7652	53 0	61 1890	1179	1 7866731	8374
35	5495	1140	1 8098926	7675	5	0711	1180	58357	8400
40	4355	1141	91251	7699	10	60 9531	1181	49957	8425
45	3214	1143	83552	7722	15	8350	1183	41532	8451
50	2071	1144	75830	7744	20	7167	1184	33081	8478
55	0927	1145	68086	7768	25	5983	1185	24603	8502
51 0	63·9782	1147	1 8060318	7791	30	4798	1187	16101	8529
5	8635	1148	52527	7814	35	3611	1188	07572	8555
10	7487	1149	44713	7838	40	2423	1189	1·7799017	8581
15	6338	1151	36875	7861	45	1234	1190	90436	8607
20	5187	1152	29014	7884	50	0044	1192	81829	8634
25	4035	1154	21130	7908	55	59 8852	1193	73195	8660
30	2881	1155	13222	7932	54 0	59 7659	1194	1 7764535	8687
35	1726	1156	05290	7956	5	6465	1196	55848	8713
40	0570	1157	1·7997334	7979	10	5269	1197	47135	8741
45	62 9413	1159	89355	8004	15	4072	1198	38394	8767
50	8254	1160	81351	8027	20	2874	1199	29627	8795
55	7094	1162	73324	8053	25	1675	1201	20832	8822
52 0	62 5932	1163	1·7965271	8076	30	0474	1202	12010	8849
5	4769	1164	57195	8100	35	58 9272	1203	03161	8875
10	3605	1166	49095	8125	40	8069	1204	1·7694286	8904
15	2439	1167	40970	8149	45	6865	1206	85382	8931
20	1272	1168	32821	8174	50	5659	1207	76451	8959
25	0104	1169	24647	8200	55	4452	1208	67492	8987

and its Logarithm, measured along Parallels of Latitude.

Latitude	Length in feet	Diff	Logarithm	Diff
° ′		—		—
55 0	58 3244	1210	1 7658505	9015
5	2035	1211	49490	9042
10	0824	1212	40448	9071
15	57 9612	1213	31377	9100
20	8399	1214	22277	9127
25	7185	1216	13150	9157
30	5969	1217	03993	9185
35	4752	1218	1 7494808	9214
40	3534	1219	85595	9243
45	2315	1220	76352	9272
50	1095	1222	67080	9300
55	56 9873	1223	57780	9330
56 0	56 8650	1224	1 7548450	9360
5	7426	1225	39090	9389
10	6201	1226	29701	9418
15	4974	1228	20283	9449
20	3746	1229	10834	9478
25	2517	1230	01356	9508
30	1287	1231	1 7491848	9538
35	0056	1233	82310	9570
40	55 8823	1234	72740	9599
45	7589	1235	63141	9630
50	6354	1236	53511	9661
55	5118	1238	43850	9691
57 0	55 3880	1238	1 7434159	9723
5	2642	1240	24436	9754
10	1402	1241	14682	9784
15	0161	1242	04898	9816
20	54 8919	1243	1 7395082	9848
25	7676	1245	85234	9880
57 30	54 6431	1246	1 7375354	9912
35	5185	1246	65442	9943
40	3939	1248	55499	9976
45	2691	1249	45523	10007
50	1442	1251	35516	10041
55	0191	1251	25475	10073
58 0	53 8940	1253	1 7315402	10105
5	7687	1254	05297	10139
10	6433	1255	1 7295158	10172
15	5178	1256	84986	10205
20	3922	1257	74781	10237
25	2665	1258	64544	10272
30	1407	1259	54272	10305
35	0148	1261	43967	10340
40	52 8887	1262	33627	10373
45	7625	1263	23254	10408
50	6362	1264	12846	10440
55	5098	1265	02406	10477
59 0	52 3833	1266	1 7191929	10511
5	2567	1267	81418	10545
10	1300	1269	70873	10581
15	0031	1270	60292	10617
20	51 8761	1271	49675	10651
25	7490	1271	39024	10686
30	6219	1273	1 7128338	10723
35	4946	1274	17615	10758
40	3672	1275	06857	10794
45	2397	1277	1 7096063	10831
50	1120	1277	85232	10867
55	50 9843	1278	74365	10903
60 0	50 8565		1 7063462	

TABLE XII.—Linear value in feet of one second of Arc

Latitude	Length in feet	Diff	Logarithm	Diff	Latitude	Length in feet	Diff	Logarithm	Diff
°		—		—	°		—		—
50 0	65 313	1130	1 81 2022	7518	52 30	61 8935	1171	1·7916447	8?3
5	2305	1132	14504	7540	35	7761	1172	08?24	8248
10	1173	1133	36964	7563	40	659_	1174	1 78999?6	82?3
15	0040	1134	29401	7585	45	5418	1175	91705	8?99
20	64 8906	1136	21816	7607	50	4243	1176	83404	8324
25	77 0	1137	14?09	7631	55	3067	1177	75080	834?
30	6633	1138	06578	7652	53 0	61 1890	1179	1 7866731	8374
35	5495	1140	1 80989 6	7675	5	0711	1180	58357	8400
40	4355	1141	91251	7699	10	60 9531	1181	49957	8425
45	3214	1143	83552	7722	15	83 0	1183	41532	8451
50	?071	1144	75830	7744	20	7167	1184	33081	8478
55	0927	1145	68086	7768	25	5983	1185	24603	850_
51 0	63 ·9 8?	1147	1 8060318	7791	30	4?98	1187	16101	85?9
5	8635	1148	52527	7814	35	3611	1188	07572	8555
10	7487	1149	44713	7838	40	2423	1189	1 7799017	8581
15	6338	1151	36875	7861	45	1?34	1190	90436	8607
20	5187	1152	29014	7884	50	0044	1192	81829	8634
25	4035	1154	21130	7908	55	59 8852	1193	73195	8660
30	2881	1155	13??2	7932	54 0	59 7659	1194	1·7764535	8687
35	17?6	1156	05?90	7956	5	6465	1196	55848	8713
40	05?0	1157	1·799 334	7979	10	5?69	1197	4 135	8741
45	62·9413	1159	89355	8004	15	4072	1198	38394	8767
50	8254	1160	81351	8027	20	_874	1199	29627	8 95
55	?094	1162	733 4	8053	25	1675	1?01	20832	8822
52 0	62 593	1163	1 ?96 271	8076	30	0474	120_	1?010	8849
5	4 ?0	1164	5?195	8100	35	58·9272	1?03	03101	88 5
10	3005	1166	49095	8125	40	8069	1204	1·?694?86	8904
15	2139	1167	409 0	8149	45	?8?5	1?06	85382	8931
20	1272	1168	3??_1	8174	50	5659	1?07	76451	8959
25	0104	1169	24647	8200	55	4452	1208	67492	8987

TABLE XIV.—Astronomical Refraction

The Mean Refraction is for Barometer 30 inches and Temperature 50° Fahr

Apparent Altitude		Mean Refraction	Diff for 1′ Alt	Diff for +1 in Baro	Diff for 1 Temp
°	′	′ ″	″	″	″
0	0	33 51	11 7	74	8 1
	5	32 53	11·3	71	7 6
	10	31 58	10·9	69	7 3
	15	31 5	10 5	67	7·0
	20	30 13	10 1	65	6 7
	25	29 24	9 7	63	6 4
	30	28 37	9 4	61	6 1
	35	27 51	9·0	59	5·9
	40	27 6	8 7	58	5 6
	45	26 24	8 4	56	5 4
	50	25 43	8 0	55	5 1
	55	25 3	7 7	53	4·9
1	0	24 25	7 4	52	4 7
	5	23 48	7 1	50	4 6
	10	23 13	6·9	49	4 5
	15	22 40	6 6	48	4 4
	20	22 8	6 3	46	4 2
	25	21 37	6 1	45	4 0
	30	21 7	5·9	44	3 9
	35	20 38	5 7	43	3 8
	40	20 10	5 5	42	3 6
	45	19 43	5 3	40	3 5
	50	19 17	5 1	39	3 4
	55	18 52	4 9	39	3 3
2	0	18 29	4 8	38	3 2
	5	18 5	4 6	37	3 1
	10	17 43	4 4	36	3 0
	15	17 21	4 3	36	2 9
	20	17 0	4 1	35	2 8
	25	16 40	4 0	34	2 8
	30	16 21	3 9	33	2 7
	35	16 2	3 7	33	2 7
	40	15 43	3 6	32	2 6
	45	15 25	3 5	32	2 5
	50	15 8	3 4	31	2 4
	55	14 51	3 3	30	2 3
3	0	14 35	3 2	30	2 3
	5	14 19	3 1	29	2 2
	10	14 4	3 0	29	2 2
	15	13 50	2 9	28	2 1
	20	13 35	2 8	28	2 1
	25	13 21	2 7	27	2 0
	30	13 7	2 7	27	2 0
	35	12 53	2 6	26	2 0
	40	12 41	2 5	26	1 9
	45	12 28	2 4	25	1 9
	50	12 16	2 4	25	1 9
	55	12 3	2 3	25	1 8

Apparent Altitude		Mean Refraction	Diff for 1′ Alt	Diff for +1 in Baro	Diff for − 1° Temp
°	′	′ ″	″	″	″
4	0	11 52	2 2	24 1	1·70
	10	11 30	2 1	23 4	1 64
	20	11 10	2 0	22 7	1 58
	30	10 50	1 9	22 0	1 53
	40	10 32	1 8	21 3	1 48
	50	10 15	1 7	20 7	1 43
5	0	9 58	1 6	20 1	1·38
	10	9 42	1 5	19 6	1 34
	20	9 27	1 5	19 1	1 30
	30	9 11	1 4	18 6	1 26
	40	8 58	1 3	18 1	1 22
	50	8 45	1 3	17 6	1 19
6	0	8 32	1 2	17 2	1 15
	10	8 20	1 2	16 8	1 11
	20	8 9	1 1	16 4	1 09
	30	7 58	1 1	16 0	1 06
	40	7 47	1 0	15 7	1 03
	50	7 37	1 0	15 3	1 00
7	0	7 27	1 0	15 0	0 98
	10	7 17	0 9	14 6	95
	20	7 8	9	14 3	93
	30	6 59	8	14 1	91
	40	6 51	8	13 8	89
	50	6 43	8	13 5	87
8	0	6 35	0 7	13 3	0 85
	10	6 28	7	13 1	83
	20	6 21	7	12 8	82
	30	6 14	7	12 6	80
	40	6 7	7	12 3	79
	50	6 0	6	12 1	77
9	0	5 54	0 6	11 9	0 76
	10	5 47	6	11 7	74
	20	5 41	6	11 5	73
	30	5 36	6	11 3	71
	40	5 30	5	11 1	71
	50	5 25	5	11 0	70
10	0	5 20	0 5	10 8	0 69
	10	5 15	5	10 6	67
	20	5 10	5	10 4	65
	30	5 5	5	10 2	64
	40	5 0	5	10 1	63
	50	4 56	4	9 9	62
11	0	4 51	0 4	9 8	0 60
	10	4 47	4	9 6	59
	20	4 43	4	9 5	58
	30	4 39	4	9 4	57
	40	4 35	4	9 2	56
	50	4 31	4	9 1	55

NOTE —If the Barometer is over 30 inches the Barometer correction is *additive* to the Mean Refraction
If the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction
And *vice versa*

TABLE XIII

Spherical Excess

Area of Triangle in Square Miles	Spherical Excess	Area of Triangle in Square Miles	Spherical Excess	Area of Triangle in Square Miles	Spherical Excess
					″
5	0.07	60	0.70	400	5.27
10	.13	70	.92	500	6.59
15	.20	80	1.05	600	7.91
20	.26	90	1.19	700	9.23
30	.40	100	1.32	800	10.54
40	.53	200	2.64	900	11.86
50	.66	300	3.95	1000	13.18

Computed from formula, spherical excess $= \frac{A}{r^2 \sin 1''}$ where A = area of triangle, r = mean radius of curvature

The table has been made out for latitude 30°, but may be used for all latitudes for topographical, tertiary, and secondary work

TABLE XIV.—Astronomical Refraction

The Mean Refraction is for Barometer 30 inches and Temperature 50° Fahr

Apparent Altitude		Mean Refraction		Diff for 1′ Alt	Diff for +1 in Baro	Diff for −1° Temp
°	′	′	″	″	″	″
0	0	33	51	11 7	74	8 1
	5	32	53	11·3	71	7·6
	10	31	58	10·9	69	7 3
	15	31	5	10 5	67	7·0
	20	30	13	10 1	65	6 7
	25	29	24	9 7	63	6 4
	30	28	37	9 4	61	6 1
	35	27	51	9·0	59	5·9
	40	27	6	8 7	58	5 6
	45	26	24	8 4	56	5 4
	50	25	43	8 0	55	5 1
	55	25	3	7 7	53	4·9
1	0	24	25	7 4	52	4 7
	5	23	48	7 1	50	4 6
	10	23	13	6·9	49	4 5
	15	22	40	6 6	48	4 4
	20	22	8	6 3	46	4 2
	25	21	37	6 1	45	4 0
	30	21	7	5·9	44	3 9
	35	20	38	5 7	43	3 8
	40	20	10	5 5	42	3 6
	45	19	43	5 3	40	3 5
	50	19	17	5 1	39	3 4
	55	18	52	4 9	39	3 3
2	0	18	29	4 8	38	3 2
	5	18	5	4 6	37	3 1
	10	17	43	4 4	36	3 0
	15	17	21	4 3	36	2 9
	20	17	0	4 1	35	2 8
	25	16	40	4 0	34	2 8
	30	16	21	3 9	33	2 7
	35	16	2	3 7	33	2 7
	40	15	43	3 6	32	2 6
	45	15	25	3 5	32	2 5
	50	15	8	3 4	31	2 4
	55	14	51	3 3	30	2 3
3	0	14	35	3 2	30	2 3
	5	14	19	3 1	29	2 2
	10	14	4	3 0	29	2 2
	15	13	50	2 9	28	2 1
	20	13	35	2 8	28	2 1
	25	13	21	2 7	27	2 0
	30	13	7	2 7	27	2 0
	35	12	53	2 6	26	2 0
	40	12	41	2 5	26	1 9
	45	12	28	2 4	25	1 9
	50	12	16	2 4	25	1 9
	55	12	3	2 3	25	1 8

Apparent Altitude		Mean Refraction		Diff for 1′ Alt	Diff for +1 in Baro	Diff for −1° Temp
°	′	′	″		″	″
4	0	11	52	2 2	24 1	1·70
	10	11	30	2 1	23 4	1 64
	20	11	10	2 0	22 7	1 58
	30	10	50	1 9	22 0	1 53
	40	10	32	1 8	21 3	1 48
	50	10	15	1 7	20 7	1 43
5	0	9	58	1 6	20 1	1 38
	10	9	42	1 5	19 6	1 34
	20	9	27	1 5	19 1	1 30
	30	9	11	1 4	18 6	1 26
	40	8	58	1 3	18 1	1 22
	50	8	45	1 3	17 6	1 19
6	0	8	32	1 2	17 2	1 15
	10	8	20	1 2	16 8	1 11
	20	8	9	1 1	16 4	1 09
	30	7	58	1 1	16 0	1 06
	40	7	47	1·0	15 7	1 03
	50	7	37	1 0	15 3	1 00
7	0	7	27	1 0	15 0	0 98
	10	7	17	0 9	14 6	95
	20	7	8	9	14 3	93
	30	6	59	8	14 1	91
	40	6	51	8	13 8	89
	50	6	43	8	13 5	87
8	0	6	35	0 7	13 3	0 85
	10	6	28	7	13 1	83
	20	6	21	7	12 8	82
	30	6	14	7	12 6	80
	40	6	7	7	12 3	79
	50	6	0	6	12 1	77
9	0	5	54	0 6	11 9	0 76
	10	5	47	6	11 7	74
	20	5	41	6	11 5	73
	30	5	36	6	11 3	71
	40	5	30	5	11 1	71
	50	5	25	5	11 0	70
10	0	5	20	0 5	10 8	0 69
	10	5	15	5	10 6	67
	20	5	10	5	10 4	65
	30	5	5	5	10 2	64
	40	5	0	5	10 1	63
	50	4	56	4	9 9	62
11	0	4	51	0 4	9 8	0 60
	10	4	47	4	9 6	59
	20	4	43	4	9 5	58
	30	4	39	4	9 4	57
	40	4	35	4	9 2	56
	50	4	31	4	9 1	55

NOTE —If the Barometer is over 30 inches the Barometer correction is *additive* to the Mean Refraction
If the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction
And *vice versa*

TABLE XIII

Spherical Excess

Area of Triangle in Square Miles	Spherical Excess	Area of Triangle in Square Miles	Spherical Excess	Area of Triangle in Square Miles	Spherical Excess
	′		′		′
5	0 07	60	0 79	400	5 27
10	13	70	92	500	6 59
15	20	80	1 05	6 0	7 01
20	26	90	1 19	700	9 23
30	40	100	1 32	800	10 54
40	53	200	2 64	900	11 86
50	66	300	3 95	1000	13 18

Computed from formula spherical excess $= \frac{A}{r \sin 1}$ where A = area of triangle, r = mean radius of curvature

The table has been made out for latitude 30°, but may be used for all latitudes for topographical, tertiary, and secondary work

TABLE XV

Parallax of the Sun

Altitude	Parallax	Altitude	Parallax
°		°	
0	8·8	50	5·7
5	8·8	55	5·1
10	8·7	60	4·4
15	8·5	65	3·7
20	8·3	70	3·0
25	8·0	75	2·3
30	7·7	80	1·5
35	7·2	85	0·8
40	6·8	90	0·0
45	6·2		

Computed from the formula

Parallax in altitude = horizontal parallax × cos altitude

The sun's horizontal parallax varies during the year from 8·95 to 8·65

The above is a mean value, sufficiently exact for all topographical purposes

TABLE XIV.—Astronomical Refraction.

The Mean Refraction is for Barometer 30 inches and Temperature 50° Fahr

Apparent Altitude	Mean Refraction	Diff for 1 Alt	Diff for + 1 in Baro	Diff for − 1° Temp	Apparent Altitude	Mean Refraction	Diff for 1′ Alt	Diff for + 1 in Baro	Diff for − 1° Temp
°				′	° ′	′ ″			′
12 0	4 28 1	0 38	9 00	0 556	42 0	1 4 6	0 038	2 16	0 130
10	4 24 4	37	8 86	548	43 0	1 2 4	036	2 09	125
20	4 20 8	36	8 74	541	44 0	1 0 3	034	2 02	120
30	4 17 3	35	8 63	533	45 0	0 58 1	034	1 94	117
40	4 13 9	33	8 51	524	46 0	56 1	033	1 88	112
50	4 10 7	32	8 41	517	47 0	54 2	032	1 81	108
13 0	4 7 5	0 31	8 30	0 509	48 0	0 52 3	0·031	1·75	0 104
10	4 4 4	31	8 20	503	49 0	50 5	030	1 69	101
20	4 1 4	30	8 10	496	50 0	48 8	029	1 63	097
30	3 58 4	30	8 00	490	51 0	47 1	028	1 58	094
40	3 55 5	29	7 89	482	52 0	45 4	027	1 52	000
50	3 52 6	29	7 79	476	53 0	43 8	026	1 47	088
14 0	3 49·0	0 28	7 70	0 469	54 0	0 42 2	0·026	1 41	0 085
10	3 47 1	28	7 61	464	55 0	40 8	025	1 36	082
20	3 44 4	27	7 52	458	56 0	39 3	025	1 31	079
30	3 41 8	26	7 43	453	57 0	37 8	025	1 26	076
40	3 39 2	26	7 34	448	58 0	36 4	024	1 22	073
50	3 36 7	25	7 26	444	59 0	35 0	024	1 17	070
15 0	3 34 3	0 24	7 18	0 439	60 0	33 6	0 023	1 12	0 067
30	3 27 3	22	6 95	424	61 0	32 3	022	1·08	065
16 0	3 20 6	21	6 73	411	62 0	31 0	022	1·04	062
30	3 14 4	20	6 51	399	63 0	29·7	021	0 99	060
17 0	3 8 5	19	6 31	386	64 0	28 4	021	95	057
30	3 2 9	18	6 12	374	65 0	27 2	020	91	055
18 0	2 57 6	0 17	5 98	0 362	66 0	25·9	0 020	0 87	0 052
19 0	2 47·7	16	5 61	340	67 0	24 7	020	83	050
20 0	2 38 7	15	5 31	322	68 0	23 5	020	79	047
21 0	2 30 5	13	5·04	305	69 0	22 4	020	75	045
22 0	2 23 2	12	4 79	290	70 0	21 2	020	71	043
23 0	2 16 5	11	4 57	276	71 0	19 9	020	67	040
24 0	2 10 1	0 10	4 35	0 264	72 0	18 8	0·019	0 63	0·038
25 0	2 4 2	09	4 16	252	73 0	17 7	018	59	036
26 0	1 58 8	09	3·97	241	74 0	16 6	018	56	033
27 0	1 53 8	08	3 81	230	75 0	15 5	018	52	031
28 0	1 49 1	08	3 65	219	76 0	14 4	018	48	029
29 0	1 44·7	07	3 50	209	77 0	13 4	017	45	027
30 0	1 40 5	0·07	3 36	0 201	78 0	12 3	0·017	0 41	0 025
31 0	1 36 6	06	3 23	193	79 0	11 2	017	38	023
32 0	1 33·0	06	3 11	186	80 0	10 2	017	34	021
33 0	1 29 5	06	2·99	179	81 0	9 2	017	31	018
34 0	1 26 1	05	2 88	173	82 0	8 2	017	27	016
35 0	1 23·0	05	2·78	167	83 0	7 1	017	24	014
36 0	1 20·0	0·05	2 68	0 161	84 0	6 1	0 017	0 20	0·012
37 0	1 17 1	05	2 58	155	85 0	5 1	017	17	010
38 0	1 14 4	05	2 49	149	86 0	4 1	017	14	008
39 0	1 11 8	04	2 40	144	87 0	3 1	017	10	006
40 0	1 9 3	04	2 32	139	88 0	2·0	017	07	004
41 0	1 6·0	04	2 24	134	89 0	1·0	017	03	002

Note —If the Barometer is over 30 inches the Barometer correction is *additive* to the Mean Refraction
If the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction
And *vice versa*

25	0 3	3 9	11 5	22 9	38 3	57 6	80 8	108 0	139 1	174 1	213 0	255 8	302 6	353 3	408 0	466 5	529 0	595 3	665 6	739 8
26	0 4	4 0	11 6	23 1	38 6	58 0	81 3	108 5	139 6	174 7	213 7	256 6	303 4	354 2	408 9	467 5	530 1	596 5	666 8	741 1
27	0 4	4 1	11 8	23 4	38 9	58 3	81 7	109 0	140 2	175 3	214 4	257 4	304 2	355 1	409 8	468 5	531 1	597 6	668 0	742 4
28	0 4	4 2	11 9	23 6	39 2	58 7	82 1	109 5	140 7	175 9	215 1	258 1	305 1	356 0	410 8	469 5	532 2	598 7	669 2	743 6
29	0 5	4 3	12 1	23 8	39 5	59 0	82 5	110 0	141 3	176 6	215 8	258 9	305 9	356 9	411 7	470 5	533 3	599 9	670 4	744 9
30	0 5	4 4	12 3	24 0	39 8	59 4	83 0	110 4	141 8	177 2	216 4	259 6	306 7	357 7	412 7	471 5	534 3	601 0	671 6	746 2
31	0 5	4 5	12 4	24 3	40 1	59 8	83 4	110 9	142 4	177 8	217 1	260 4	307 5	358 6	413 6	472 6	535 4	602 2	672 9	747 4
32	0 6	4 6	12 6	24 5	40 3	60 1	83 8	111 4	143 0	178 4	217 8	261 1	308 4	359 5	414 6	473 6	536 5	603 3	674 1	748 7
33	0 6	4 7	12 8	24 7	40 6	60 5	84 2	111 9	143 5	179 0	218 5	261 9	309 2	360 4	415 5	474 6	537 6	604 5	675 3	750 0
34	0 6	4 8	12 9	25 0	40 9	60 8	84 7	112 4	144 1	179 7	219 2	262 6	310 0	361 3	416 5	475 6	538 7	605 6	676 5	751 3
35	0 7	4 9	13 1	25 2	41 2	61 2	85 1	112 9	144 6	180 3	219 9	263 4	310 8	362 2	417 4	476 6	539 7	606 8	677 7	752 6
36	0 7	5 0	13 3	25 4	41 5	61 6	85 5	113 4	145 2	180 9	220 6	264 1	311 6	363 1	418 4	477 6	540 8	607 9	678 9	753 9
37	0 7	5 1	13 4	25 7	41 8	61 9	86 0	113 9	145 8	181 6	221 3	264 9	312 5	364 0	419 4	478 7	541 9	609 1	680 1	755 1
38	0 8	5 2	13 6	25 9	42 1	62 3	86 4	114 4	146 3	182 2	222 0	265 7	313 3	364 8	420 3	479 7	543 0	610 2	681 3	756 4
39	0 8	5 3	13 8	26 2	42 5	62 7	86 8	114 9	146 9	182 8	222 7	266 4	314 1	365 7	421 3	480 7	544 1	611 4	682 6	757 7
40	0 9	5 4	14 0	26 4	42 8	63 0	87 3	115 4	147 5	183 5	223 4	267 2	315 0	366 6	422 2	481 7	545 2	612 5	683 8	759 0
41	0 9	5 6	14 1	26 6	43 1	63 4	87 7	115 9	148 0	184 1	224 1	268 0	315 8	367 5	423 2	482 8	546 3	613 7	685 0	760 2
42	1 0	5 7	14 3	26 9	43 4	63 8	88 1	116 4	148 6	184 7	224 8	268 7	316 6	368 4	424 2	483 8	547 4	614 8	686 2	761 5
43	1 0	5 8	14 5	27 1	43 7	64 2	88 6	116 9	149 2	185 4	225 5	269 5	317 4	369 3	425 1	484 8	548 4	616 0	687 4	762 8
44	1 1	5 9	14 7	27 4	44 0	64 5	89 0	117 4	149 7	186 0	226 2	270 3	318 3	370 2	426 1	485 8	549 5	617 2	688 7	764 1
45	1 1	6 0	14 9	27 6	44 3	64 9	89 5	117 9	150 3	186 6	226 9	271 0	319 1	371 1	427 0	486 9	550 6	618 3	689 9	765 4
46	1 2	6 1	15 0	27 9	44 6	65 3	89 9	118 4	150 9	187 3	227 6	271 8	319 9	372 0	428 0	487 9	551 7	619 5	691 1	766 7
47	1 2	6 2	15 2	28 1	44 9	65 7	90 3	118 9	151 5	187 9	228 3	272 6	320 8	372 9	429 0	488 9	552 8	620 6	692 3	768 0
48	1 3	6 4	15 4	28 3	45 2	66 0	90 8	119 5	152 0	188 5	229 0	273 3	321 6	373 8	429 9	490 0	553 9	621 8	693 6	769 3
49	1 3	6 5	15 6	28 6	45 5	66 4	91 2	120 0	152 6	189 2	229 7	274 1	322 4	374 7	430 9	491 0	555 0	623 0	694 8	770 6
50	1 4	6 6	15 8	28 8	45 9	66 8	91 7	120 5	153 2	189 8	230 4	274 9	323 3	375 6	431 9	492 0	556 1	624 1	696 0	771 9
51	1 4	6 7	15 9	29 1	46 2	67 2	92 1	121 0	153 8	190 5	231 1	275 6	324 1	376 5	432 8	493 1	557 2	625 3	697 3	773 1
52	1 5	6 8	16 1	29 4	46 5	67 6	92 6	121 5	154 3	191 1	231 8	276 4	325 0	377 4	433 8	494 1	558 3	626 5	698 5	774 5
53	1 5	7 0	16 3	29 6	46 8	68 0	93 0	122 0	154 9	191 8	232 5	277 2	325 8	378 3	434 8	495 2	559 4	627 6	699 7	775 7
54	1 6	7 1	16 5	29 9	47 1	68 3	93 5	122 5	155 5	192 4	233 2	278 0	326 7	379 3	435 8	496 2	560 5	628 8	701 0	777 1
55	1 6	7 2	16 7	30 1	47 5	68 7	93 9	123 1	156 1	193 1	234 0	278 8	327 5	380 2	436 7	497 2	561 6	630 0	702 2	778 4
56	1 7	7 3	16 9	30 4	47 8	69 1	94 4	123 6	156 7	193 7	234 7	279 5	328 4	381 1	437 7	498 3	562 8	631 2	703 5	779 7
57	1 8	7 5	17 1	30 6	48 1	69 5	94 8	124 1	157 3	194 4	235 4	280 3	329 2	382 0	438 7	499 3	563 9	632 3	704 7	781 0
58	1 8	7 6	17 3	30 9	48 4	69 9	95 3	124 6	157 8	195 0	236 1	281 1	330 0	382 9	439 7	500 4	565 0	633 5	705 9	782 3
59	1 9	7 7	17 5	31 2	48 8	70 3	95 7	125 1	158 4	195 7	236 8	281 9	330 9	383 8	440 6	501 4	566 1	634 7	707 1	783 6

TABLE XVI

Latitude by Circum Meridian Observations

Reduction to Meridian

$$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

t	m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m	13m	14m	15m	16m	17m	18m	19m
s	′	″																	′	″
0	0.0	2.0	7.8	17.7	31.4	49.1	70.7	96.2	125.7	159.0	196.3	237.5	282.7	331.7	384.7	441.6	502.5	567.2	635.9	708.4
1	0.0	2.0	8.0	17.9	31.7	49.4	71.1	96.7	126.2	159.6	197.0	238.3	283.5	332.6	385.6	442.6	503.5	568.3	637.0	709.7
2	0.0	2.1	8.1	18.1	31.9	49.7	71.5	97.1	126.7	160.2	197.6	239.0	284.3	333.4	386.6	443.6	504.6	569.4	638.2	710.9
3	0.0	2.2	8.2	18.3	32.2	50.1	71.9	97.6	127.2	160.8	198.3	239.7	285.0	334.3	387.5	444.6	505.6	570.5	639.4	712.1
4	0.0	2.2	8.4	18.5	32.5	50.4	72.3	98.0	127.8	161.4	198.9	240.4	285.8	335.2	388.4	445.6	506.7	571.6	640.6	713.4
5	0.0	2.3	8.5	18.7	32.7	50.7	72.7	98.5	128.3	162.0	199.6	241.1	286.6	336.0	389.3	446.6	507.7	572.8	641.7	714.6
6	0.0	2.4	8.7	18.9	33.0	51.1	73.1	99.0	128.8	162.6	200.3	241.9	287.4	336.8	390.2	447.5	508.8	573.9	642.9	715.9
7	0.0	2.4	8.8	19.1	33.3	51.4	73.5	99.4	129.3	163.2	200.9	242.6	288.2	337.7	391.2	448.5	509.8	575.0	644.1	717.1
8	0.0	2.5	8.9	19.3	33.5	51.7	73.9	99.9	129.9	163.8	201.6	243.3	289.0	338.6	392.1	449.5	510.9	576.1	645.3	718.4
9	0.0	2.6	9.1	19.5	33.8	52.1	74.3	100.4	130.4	164.4	202.2	244.1	289.8	339.4	393.0	450.5	511.9	577.2	646.5	719.6
10	0.1	2.7	9.2	19.7	34.1	52.4	74.7	100.8	130.9	165.0	202.9	244.8	290.6	340.3	393.9	451.5	513.0	578.4	647.7	720.9
11	0.1	2.7	9.4	19.9	34.4	52.7	75.1	101.3	131.5	165.6	203.6	245.5	291.4	341.2	394.9	452.5	514.0	579.5	648.9	722.1
12	0.1	2.8	9.5	20.1	34.6	53.1	75.5	101.8	132.0	166.2	204.2	246.3	292.2	342.0	395.8	453.5	515.1	580.6	650.0	723.4
13	0.1	2.9	9.6	20.3	34.9	53.4	75.9	102.3	132.6	166.8	204.9	247.0	293.0	342.9	396.7	454.5	516.1	581.7	651.2	724.6
14	0.1	3.0	9.8	20.5	35.2	53.8	76.3	102.7	133.1	167.4	205.6	247.7	293.8	343.7	397.6	455.5	517.2	582.9	652.4	725.9
15	0.1	3.1	9.9	20.7	35.5	54.1	76.7	103.2	133.6	168.0	206.3	248.5	294.6	344.6	398.6	456.5	518.3	584.0	653.3	727.2
16	0.1	3.1	10.1	20.9	35.7	54.5	77.1	103.7	134.2	168.6	206.9	249.2	295.4	345.5	399.5	457.5	519.3	585.1	654.8	728.4
17	0.2	3.2	10.2	21.2	36.0	54.8	77.5	104.2	134.7	169.2	207.6	249.9	296.2	346.4	400.5	458.5	520.4	586.2	656.0	729.7
18	0.2	3.3	10.4	21.4	36.3	55.1	77.9	104.6	135.3	169.8	208.3	250.7	297.0	347.2	401.4	459.5	521.5	587.4	657.2	730.9
19	0.2	3.4	10.5	21.6	36.6	55.5	78.3	105.1	135.8	170.4	208.9	251.4	297.8	348.1	402.3	460.5	522.5	588.5	658.4	732.2
20	0.2	3.5	10.7	21.8	36.9	55.8	78.8	105.6	136.3	171.0	209.6	252.2	298.6	349.0	403.3	461.5	523.6	589.6	659.6	733.5
21	0.2	3.6	10.8	22.0	37.2	56.2	79.2	106.1	136.9	171.6	210.3	252.9	299.4	349.8	404.2	462.5	524.7	590.8	660.8	734.7
22	0.3	3.7	11.0	22.3	37.4	56.5	79.6	106.6	137.4	172.2	211.0	253.6	300.2	350.7	405.1	463.5	525.7	591.9	662.0	736.0
23	0.3	3.8	11.2	22.5	37.7	56.9	80.0	107.0	138.0	172.9	211.7	254.4	301.0	351.6	406.1	464.5	526.8	593.0	663.2	737.3
24	0.3	3.8	11.3	22.7	38.0	57.3	80.4	107.5	138.5	173.5	212.3	255.1	301.8	352.5	407.0	465.5	527.9	594.2	664.4	738.5

TABLE XVIII

Longitude by Occultations

Reduction of Latitude Compression $\frac{1}{294}$					
Lat	Red	Lat	Red	Lat	Red
°	′ ″	°	′ ″	°	′ ″
0	0 0	30	10 7	60	10 9
1	0 24	31	10 19	61	9 56
2	0 4[illegible]	32	10 30	62	9 43
3	1 13	33	10 40	63	9 29
4	1 37	34	10 50	64	9 14
5	2 1	35	10 59	65	8 59
6	2 25	36	11 7	66	8 43
7	2 49	37	11 14	67	8 26
8	3 13	38	11 21	68	8 9
9	3 36	39	11 2[illegible]	69	7 51
10	3 59	40	11 31	70	7 32
11	4 22	41	11 35	71	7 13
12	4 45	42	11 38	72	6 54
13	5 7	43	11 40	73	6 34
14	5 28	44	11 41	74	6 13
15	5 50	45	11 42	75	5 52
16	6 11	46	11 42	76	5 31
17	6 31	47	11 41	77	5 9
18	6 51	48	11 38	78	4 46
19	7 11	49	11 36	79	4 24
20	7 30	50	11 32	80	4 1
21	7 49	51	11 27	81	3 38
22	8 8	52	11 22	82	3 14
23	8 24	53	11 15	83	2 50
24	8 41	54	11 8	84	2 26
25	8 57	55	11 0	85	2 2
26	9 12	56	10 52	86	1 38
27	9 27	57	10 42	87	1 14
28	9 41	58	10 32	88	0 49
29	9 54	59	10 21	89	0 24

Formula

$$\phi - \phi' = \frac{e^2}{(2 - e^2)\sin 1''}\sin 2\phi - \left\{ \ldots \right\} \frac{\sin 4\phi}{2\sin 1''},$$

$$e = \sqrt{2c - c^2}$$

$$c = \frac{1}{294}$$

TABLE XVII

Longitude by Occultations

Correction of the Moon's Equatorial Parallax for the figure of the Earth

(Compression $\frac{1}{294 \cdot 6}$)

Lat	Horizontal Parallax 54	56	58	60	62′
°					″
0	0	0	0	0	0
8	2	2	2	2	2
16	8	9	9	9	9
20	1 3	1 3	1 4	1 4	1 5
24	1 8	1 9	2·0	2 0	2·0
28	2 5	2 5	2 6	2 7	2 8
32	3 1	3 2	3 3	3 4	3 5
36	3 8	4 0	4 1	4 3	4 4
40	4 6	4 7	4 9	5 1	5 2
44	5 3	5 5	5 7	5 9	6 1
48	6 4	6 7	7 0	7 2	7 4
52	6 9	7 1	7 4	7 6	7 9
56	7 6	7 9	8 2	8 5	8·7
60	8 3	8 6	8 9	9 2	9 6
64	9 0	9 3	9 6	10 0	10 3
68	9 5	9 9	10 3	10 6	11·0
72	10 0	10 4	10 8	11 2	11 6
76	10 5	10 9	11 2	11 6	12·0
80	10 8	11 2	11 6	12 0	12 4

APPENDIX I

TABLE I.—LATITUDES, LONGITUDES AND REVERSE AZIMUTHS

This Table gives the logarithms of the quantities P, Q, R, S, T used in the forms employed for the computation of Terrestrial Latitudes, Longitudes, and Reverse Azimuths (*vide* Form 2).

Suppose that **A** and **B** are two stations and that we have given—

λ = Latitude of **A**

L = Longitude of **A**,

A = Azimuth of **B** from **A** (measured from S. to W.)

c = Distance in feet at mean sea level between **A** and **B**,

and we require

λ_b = Latitude of **B**

L_b = Longitude of **B**

B = Azimuth of **A** from **B**

If we put

$$\lambda_b = \lambda + \Delta\lambda, \quad L_b = L + \Delta L, \quad \text{and} \quad B = \pi + A + \Delta A$$

it will be seen that we require the differences

$$\Delta\lambda, \quad \Delta L \quad \text{and} \quad \Delta A$$

The values of these quantities are found as follows:—

Putting

$$\Delta\lambda = \delta_1\lambda + \delta\lambda$$
$$\Delta L = \delta_1 L + \delta L$$
$$\Delta A = \delta_1 A + \delta A$$

then

$\delta_1\lambda = P \cos A \cdot c$	$\delta\lambda = \delta_1 A R \sin A \cdot c$
$\delta_1 L = \delta_1\lambda Q \sec\lambda \tan A$	$\delta L = \delta\lambda S \cot A$
$\delta_1 A = \delta_1 L \sin\lambda$	$\delta A = \delta L T$

which are the expressions used in the form for computation.

It will be noticed that each term is found consecutively from the preceding one.

The proper signs to be given to the quantities will be found in Tables II (A) and II (B).

For latitudes other than those given in the Table the quantities must be found by interpolation.

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$$\Delta A = \delta_1 A + \delta_2 A$$

then

$\delta_1\lambda = P c \cos A \cdot c$		$\delta_2\lambda = \delta_1 A R \sin A \cdot c$	
$\delta_1 L = \delta_1\lambda Q \sec\lambda \tan A$		$\delta_2 L = \delta_2\lambda S \cot A$	
$\delta_1 A = \delta_1 L \sin\lambda$		$\delta_2 A = \delta_2 L T$	

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then

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$\delta_1 L = \delta_1\lambda Q \sec\lambda \tan A$	$\delta_2 L = \delta_2\lambda S \cot A$
$\delta_1 A = \delta_1 L \sin\lambda$	$\delta_2 A = \delta_2 L T,$

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APPENDIX II

COMPUTATION OF DISTANCES AND AZIMUTHS

Given the latitudes and longitudes of two points A and B

REQUIRED	Azimuths **B** from **A** (A) and **A** from **B** (π + A + ΔA) and length in feet between Station **A** and Station **B** (c)	
Data	**Latitude**	**Longitude**
Station **A**	λ_a –	L_a –
Station **B**	λ_b	L_b –
$\Delta\lambda$	±	ΔL – ±
$\frac{\Delta\lambda}{2}$	±	$\frac{\Delta L}{2}$ – ±
λ_m – Middle Latitude $\left(\lambda_a + \frac{\Delta\lambda}{2}\right)$		

$\Delta\lambda$ When **A** and **B** are both N or S of the Equator

$\Delta\lambda$ is $\frac{+}{-}$ when λ_b is $\frac{\text{greater}}{\text{less}}$ than λ_a and $\lambda_m = \frac{1}{2}(\lambda_a + \lambda_b)$

If the ray crosses the Equator

$\Delta\lambda$ is always negative and $= \lambda_a + \lambda_b$, and $\lambda_m = \frac{1}{2}$ diff between λ_a and λ_b

ΔL When **A** and **B** are both W of Greenwich

ΔL is $\frac{+}{-}$ when L_b is $\frac{\text{less}}{\text{greater}}$ than L_a

When **A** and **B** are both E of Greenwich

ΔL is $\frac{+}{-}$ when L_b is $\frac{\text{greater}}{\text{less}}$ than L_a

If the ray crosses the meridian of Greenwich —

$\Delta L = L_a + L_b$ and is $\frac{+}{-}$ if L_b is $\frac{E}{W}$ of Greenwich

Formulæ	Logarithms	Results
	To 5 places of Decimals	
$\frac{\Delta A}{2} = \frac{\Delta L}{2} \sin \lambda_m$	$\log \frac{\Delta L}{2}$ (in secs) —	$\frac{\Delta A}{2}$ —
	$\log \sin \lambda_m$ —	[When λ_m is $\frac{N}{S}$ of equator ΔA has the $\frac{\text{same}}{\text{opposite}}$ sign as ΔL.]
	$\log \frac{\Delta A}{2}$ —	
$\log \tan\left(A + \frac{\Delta A}{2}\right) = \log$ Table XII† − log Table XI† + log ΔL − $\log \Delta\lambda$	log Table XII	$A + \frac{\Delta A}{2}$
	Colog Table XI	[Quadrant obtained from Table II (A) or II (B) with given signs of $\Delta\lambda$ and ΔL as above.]
	$\log \Delta L$ (in secs)	
	Colog $\Delta\lambda$	
	$\log \tan\left(A + \frac{\Delta A}{2}\right)$	
$\log c = \log \sec\left(A + \frac{\Delta A}{2}\right) + \log$ Table XI† + $\log \Delta\lambda$	$\log \sec\left(A + \frac{\Delta A}{2}\right)$	
	log Table XI =	
	$\log \Delta\lambda$ (in secs)	
	$\log c$	c
Required Azimuths	$A + \frac{\Delta A}{2}$	
A	$\frac{\Delta A}{2}$ + or −	
	Diff	Azimuth of **B** from **A**
$\pi + A + \Delta A$	π + Sum	Azimuth of **A** from **B**

† Values for middle latitude, i.e. for $\lambda + \frac{\Delta\lambda}{2}$

APPENDIX III

ALTERNATIVE FORMULÆ FOR DETERMINING LATITUDE, LONGITUDE, AND REVERSE AZIMUTH IN TRIANGULATION, ALSO THOSE FOR THE REVERSE PROCESS

Direct Process.—

$$\frac{\Delta A}{2} = \frac{\Delta L}{2}\left(\sin \lambda + \frac{\Delta\lambda}{2}\right) \tag{1}$$

$$\Delta\lambda = -\frac{c}{\rho}\cos\left(A + \frac{\Delta A}{2}\right)\text{cosec } 1'' \tag{2}$$

$$\Delta L = -\frac{c}{\nu}\,\frac{\sin\left(A + \frac{\Delta A}{2}\right)}{\cos\left(\lambda + \frac{\Delta\lambda}{2}\right)}\text{cosec } 1'' \tag{3}$$

where A, λ, L are the initial azimuth, latitude and longitude, and ΔA, Δλ, ΔL, are the increments to be added to the initial values to obtain those of the distant station at the end of the line of which c is the length in feet.

Now

$$\log \frac{\text{cosec } 1''}{\rho} = P \text{ (Table I.)}$$

and

$$\log \frac{\text{cosec } 1''}{\nu} = P + Q \text{ (Table I.)}$$

To obtain a value for $\frac{\Delta A}{2}$ the approximate position of the end of the line must be known by measurement from a plane-table or approximate computation, and the rough values for ΔL and Δλ thus obtained are substituted in (1). The rest is obvious.

The Reverse Process, i.e. the latitudes and longitudes of the two stations being known, to compute the azimuths and distance.—

Here we know ΔL, λ, and Δλ, and can therefore compute the value of ΔA from (1).

Dividing (3) by (2)

$$\frac{\Delta L}{\Delta\lambda} = \frac{\rho}{\nu}\tan\left(A + \frac{\Delta A}{2}\right)\sec\left(\lambda + \frac{\Delta\lambda}{2}\right) \tag{4}$$

where

$$\log \frac{\rho}{\nu} = Q \text{ (Table I.)}$$

From this $\left(A + \frac{\Delta A}{2}\right)$ can be computed, and then from (2) c can be found.

NOTE.—Both in the Direct Process and in the Reverse Process the values of P and Q to be used are those for the *mean latitude* $\left(\lambda + \frac{\Delta\lambda}{2}\right)$.

APPENDIX IV

TABLE IV.—COMPUTATION OF HEIGHTS

This table is to facilitate finding the subtended angle when only one angle has been observed.

$$\left.\begin{matrix} D_a \\ E_a \end{matrix}\right\} \quad \text{angle of} \left\{\begin{matrix} \text{Depression} \\ \text{Elevation} \end{matrix}\right\} \text{ at A}$$

$$\left.\begin{matrix} D_b \\ E_b \end{matrix}\right\} \quad \text{Do} \qquad \text{do} \qquad \text{at B}$$

c = distance in feet between **A** and **B**.

1st *Case*.—When reciprocal angles have been observed—

(*a*) If both angles are depressions take half the difference.
(*b*) If one angle only is an elevation take half the sum.

The result is an auxiliary angle called the subtended angle (S)—

$$i.e.\ S = \frac{D_b - D_a}{2} \quad \text{or} \quad \frac{E_a + D_b}{2}$$

2nd Case.—When one angle only (D_a or E_a) has been observed, we must first find the subtended angle.

Rule.—Subtract the quantity in Table IV from log c. The natural number corresponding to the resulting logarithm is an angle K in seconds of arc, then—

$$S = K - D, \text{ or } K + E$$

Having now found S in either case the difference in height in feet (h) between **A** and **B** is obtained from the formula—

$$h = c \tan S$$

EXAMPLE OF 2ND CASE.—

Given log c = 4·8907
and D = 0° 14′ 25″,

to find S in lat. 20° with coefficient of refraction ·07—

log c	= 4·8907
Number in Table IV under ·07 for lat. 20°	= 2·3716
log K	= 2·5191

		°	′	″
K = 330″	=	0	5	30
D	=	0	14	35
Hence S = K − D	=	0	9	5

See Form II, p. 31.

APPENDIX III

ALTERNATIVE FORMULÆ FOR DETERMINING LATITUDE, LONGITUDE, AND REVERSE AZIMUTH IN TRIANGULATION, ALSO THOSE FOR THE REVERSE PROCESS

Direct Process.—

$$\frac{\Delta A}{2} = \frac{\Delta L}{2}\left(\sin \lambda + \frac{\Delta\lambda}{2}\right) \qquad (1),$$

$$\Delta\lambda = \frac{c}{\rho}\cos\left(A + \frac{\Delta A}{2}\right)\operatorname{cosec} 1'' \qquad (2),$$

$$\Delta L = \frac{c}{\nu}\,\frac{\sin\left(A + \frac{\Delta A}{2}\right)}{\cos\left(\lambda + \frac{\Delta\lambda}{2}\right)}\operatorname{cosec} 1'' \qquad (3),$$

where A, λ, L are the initial azimuth, latitude, and longitude, and ΔA, Δλ, ΔL, are the increments to be added to the initial values to obtain those of the distant station at the end of the line, of which c is the length in feet.

Now

$$\log \frac{\operatorname{cosec} 1''}{\rho} = P \text{ (Table I.)}$$

and

$$\log \frac{\operatorname{cosec} 1''}{\nu} = P + Q \text{ (Table I.)}$$

To obtain a value for $\frac{\Delta A}{2}$ the approximate position of the end of the line must be known by measurement from a plane table or approximate computation, and the rough values for ΔL and Δλ thus obtained are substituted in (1). The rest is obvious.

The Reverse Process, i.e. the latitudes and longitudes of the two stations being known, to compute the azimuths and distance.—

Here we know ΔL, λ, and Δλ, and can therefore compute the value of ΔA from (1).

Dividing (3) by (2),

$$\frac{\Delta L}{\Delta\lambda} = \frac{\rho}{\nu}\tan\left(A + \frac{\Delta A}{2}\right)\sec\left(\lambda + \frac{\Delta\lambda}{2}\right) \qquad (4),$$

where

$$\log \frac{\rho}{\nu} = Q \text{ (Table I.)}$$

From this, $\left(A + \frac{\Delta A}{2}\right)$ can be computed, and then from (2) c can be found.

NOTE.—Both in the Direct Process and in the Reverse Process the values of P and Q to be used are those for the *mean latitude* $\left(\lambda + \frac{\Delta\lambda}{2}\right)$.

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TABLE IV.—COMPUTATION OF HEIGHTS

This table is to facilitate finding the subtended angle when only one angle has been observed—

D_a, E_a	angle of {Depression, Elevation}	at A
D_b, E_b	do. do.	at B
c	distance in feet between A and B	

1st case.—When reciprocal angles have been observed—

(a) If both angles are depressions take half the difference.
(b) If one angle only is an elevation take half the sum.

The result is an auxiliary angle called the subtended angle (S)—

$$i.e.\ S = \frac{D_b - D_a}{2} \text{ or } \frac{E_a + D_b}{2}$$

2nd case.—When one angle only (D_a or E_a) has been observed, we must first find the subtended angle.

Rule.—Subtract the quantity in Table IV from log c. The natural number corresponding to the resulting logarithm is an angle K in seconds of arc, then—

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Given log c = 4.8907
and D = 0° 14′ 25″,

to find S in lat. 20° with coefficient of refraction ·07—

log c = 4.8907
Number in Table IV under ·07 for lat. 20 } = 2.3716

log K = 2.5191

K = 330″ = 0° 5′ 30″
D = 0 14 35

Hence S = K − D = 0 9 5

See Form II, p. 31

Rope, 3 inch
Saws
Shovels

These should be supplemented by the tools in use in the country

CAMP EQUIPMENT

Baths, canvas
Beds, camp
Bedding and valises
Boxes for small stores
Buckets
Canteens and messing kit
Candle lamps
Chairs, camp
Curtains, mosquito
Hurricane lamps
Indiarubber sheets
Lamps, Lord's
Mess boxes and cooking utensils
Stoves, camp and cooking (for cold climates)
Tables, camp
Tents, Willesden canvas, double roof, ridge
Washing kit

APPENDIX VII

The account of sketching in dense forest has been extracted from the "Manual of Field Sketching." The method described is useful in exploratory expeditions, and may also prove to be of help in work for filling in minor tracks between points more accurately fixed.

SKETCHING IN DENSE FOREST*

In many countries, parts of Africa especially, it is impossible on account of dense bush, jungle or grass, to survey any features other than the line of the road or track. As the paths wind about very much the bearings have to be taken at short intervals, and such surveys being generally made while the sketcher is marching with a column or caravan it is inconvenient and would occasion too great a loss of time to plot the bearings and distances on the march. It is preferable therefore to enter them in a note book and to plot them on arrival in camp. The bearings are better taken with a prismatic compass if available, but if not with an ordinary pocket compass. Distances are best arrived at by noting the times at which the bearings are taken or objects passed and estimating the rate of marching during the interval. A convenient form of note book is formed by ruling four lines about half an inch apart down the centre of each page (see fig. 70). Supposing the day's traverse is being commenced from Yandahu village, start by obtaining a back bearing to the last village. This is entered as B B (back bearing) to village 208°, and a forward bearing to the next village in front entered as F B village (forward bearing) Yolu 71°. In the thickest bush country a native has generally a very good idea of the relative positions of the villages round his own, and these bearings from village to village are very useful as checks on the direction of the traverse.

If a distant view is obtainable it is advisable to take back readings to any marked *physical feature as far as possible and near which the track has passed, also to any* points in advance near which it appears probable it may pass. These long bearings of points say 30 or 40 miles apart, are more useful than those from village to village in checking and correcting the direction of the traverse.

In the example (fig 70) bearings were obtained to two hills about 30 miles ahead. The bearing of the track immediately ahead is then taken and entered in the centre column, and the time in the first or left hand column.

It is often impossible to see the general direction of the track, which probably disappears in the bush a few yards ahead; in this case it is necessary to have a native to show the direction, but constant watching is necessary, as many natives are very slow in understanding what is required, and seem unable to strike a mean between the direction of the path just a few yards in front and its position two or three miles ahead. To get a general direction of about half a mile of road, so as to take a bearing every ten minutes, is a very convenient mean, and will give fair results. On arrival at the point where the direction changes and a new bearing has to be taken the estimated rate of marching is entered in the third column.

If it be impossible from want of guides or other cause, to get satisfactorily the forward direction of the track, the best way of proceeding is to take the bearing upon which one happens to be marching at fixed intervals, say every five minutes of time, which may be

* The method here described is also best for long continuous sketches from horse or camel in hot climates. If the bearings are long it is better to dismount from the horse when taking them (not necessarily from the camel). The plotting is done on arrival in camp.

Rope 3-inch
Saws
Shovels

These should be supplemented by the tools in use in the country

CAMP EQUIPMENT

Baths canvas
Beds camp
Bedding and valises
Boxes for small stores
Buckets
Canteens and messing kit
Candle lamps
Chairs camp
Curtains mosquito
Hurricane lamps
Indiarubber sheets
Lamps Lord's
Mess boxes and cooking utensils
Stoves camp and cooking (for cold climates)
Tables camp
Tents Willesden canvas double roof ridge
Washing kit

note it and make allowance for it in the plotting. In the example, 6.14 is the time of arrival at the spot and 6.34 that of starting off again. It is sufficient to bracket the two times together to show what is intended.

The distances are plotted as straight lines: it is therefore necessary to allow in the "rate of marching" for the windings of the path. The maximum rate entered is 2¾ miles an

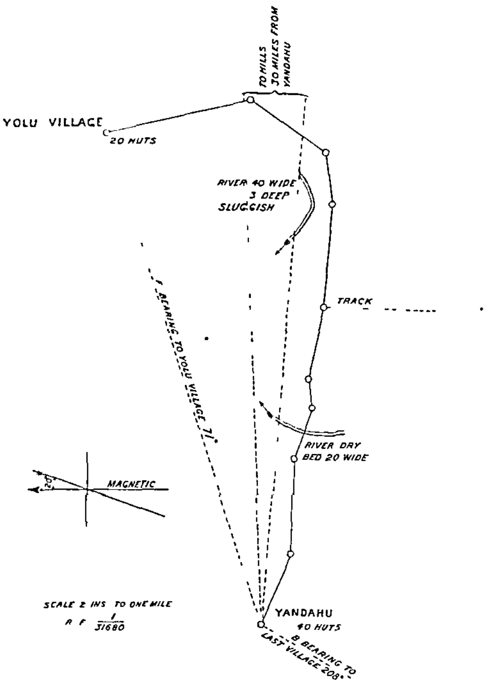


Fig. 71.—Plotting of the Compass Traverse. (Shown on fig. 70.)

hour, and this is as much as most caravans of porters will accomplish on an African bush track. If the track goes up and down hill, or is very stony or otherwise bad, it is necessary to reduce the rate entered accordingly.

After a little practice the error in length of traverses of this nature done with ordinary care should not exceed about eight miles in a hundred, and this is caused generally by a persistent over or under estimation of the rate of marching In direction the result is, as a rule very good it is wonderful how the number of small errors in bearings neutralise one another

The track included in fig 70 is plotted on fig 71

For plotting it is convenient to make one or two scales showing the distances to be plotted in minutes of marching at the different rates

Try to start and end up traverses of this nature on points of which the positions are known

When marching with a column or caravan it is a good thing to work, if possible, for a mile or so beyond the camping ground at the end of the march so as to be able in the morning to start off ahead and clear of the caravan

It is advisable to carry the watch by some means which enables the time to be readily noted A leather case, of the pattern used in South Africa, on the belt, or a band on the wrist are both convenient methods

It is a good thing to practise at first in a country of which there exists an accurate survey as this gives a means of checking the work and shows the topographer how much reliance he can place upon it

Excellent traverses have been executed in a similar way during canoe voyages down rivers in Africa and elsewhere

APPENDIX VIII

PHOTOGRAPHIC SURVEYING

A map of a tract of country may be constructed from photographs of it, and, under exceptional conditions, such a process may be a useful and economical one. But suitable conditions are rare, and the resulting map is necessarily less accurate than one made by the methods which have been described in this book.

Photographic surveying is practically equivalent to constructing a map by intersection only, excluding the use of interpolation, traversing, and local measurement. As a survey system it has two fundamental defects: the ground is not walked over, and the map is not made in the field but plotted in an office.

The conditions favourable to the use of phototopographic methods are:—

(i.) Inaccessibility of the tract of country to be surveyed, which may be, for instance, the upper portion of a high snow range.
(ii.) Well marked features. The method would, it is obvious, be useless on the rolling veldt, on plains, or in forests.
(iii.) A very short field season, such as may occur in sub-arctic regions.
(iv.) The want of skilled topographers.

The most successful instance of the use of photo-topography is to be found in the Canadian surveys of the Rocky Mountains.

Many treatises have been published on the subject; of those in English, the reader is referred to:—

'Photographic Surveying,' by E. Deville, Surveyor General of Dominion Lands, Ottawa. Government Printing Bureau, 1895.
'Photographic Methods and Instruments,' by J. A. Flemer, in the United States Coast and Geodetic Survey Report for 1897.
'Photographic Surveying,' by J. Bridges Lee, in the Royal Geographical Society's 'Hints to Travellers.' Eighth Edition, 1901.

The use of photo-topography is so rarely necessary that it is not considered desirable to increase the bulk of this book by the inclusion of a detailed description.

APPENDIX IX

USEFUL TRIGONOMETRICAL FORMULÆ

(i) **Plane Triangles—**

Given three sides, a, b, c,

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}},$$

where

$$s = \tfrac{1}{2}(a+b+c)$$

Given two sides and included angle a, b, C,

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

Given angles and a side, A, B, C, a,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Given two sides and angle opposite one of them, a, b, A,

$$\sin B = \frac{b}{a}\sin A$$

(ii) **Spherical Triangles—**

Given three sides, a, b, c,

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}},$$

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}},$$

where

$$s = \tfrac{1}{2}(a+b+c)$$

Given two sides and included angle, a, b, C,

$$\tan\frac{A+B}{2} = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{C}{2},$$

$$\tan\frac{A-B}{2} = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\frac{C}{2}$$

Given two sides and angle opposite one of them, a, b, A,

let

$$\tan\theta = \tan b\cos A,$$

$$\cos\theta' = \frac{\cos\theta\cos a}{\cos b},$$

then

$$c = \theta \pm \theta' \text{ (two solutions)}$$

Given three angles A, B, C:

$$\cos a = \frac{\sin (A + \theta)}{\sin B \sin C \sin \theta},$$

where

$$\cot \theta = \frac{\cos B \cos C}{\sin A}.$$

Given two angles and interjacent side A, B, c:

$$\cos C = \frac{\cos A \sin (B - \theta)}{\sin \theta},$$

where

$$\cot \theta = \cos c \tan A.$$

Or,

$$\tan \frac{a + b}{2} = \frac{\cos \frac{A - B}{2}}{\cos \frac{A + B}{2}} \tan \frac{c}{2},$$

$$\tan \frac{a - b}{2} = \frac{\sin \frac{A - B}{2}}{\sin \frac{A + B}{2}} \tan \frac{c}{2}.$$

Given two angles and side opposite one of them, A, B, a,

APPENDIX X

LENGTH OF THE METRE

In a book published by the Ordnance Survey entitled "Comparison of Standards of Length," by Captain A R Clarke, R E, F R S, 1866, will be found an elaborate account of the methods employed to determine the values of the principal national units of length in terms of each other The most important relation is the following —

THE METRE = 1 093623 yard
= 39 37043 inches

THE YARD = 0 914392 metre

REEVES'S TANGENT MICROMETER.

A micrometer which can be readily fitted to any theodolite by means of which angles can be read with much greater accuracy than by a Vernier, has been recently designed by Mr E. A Reeves of the Royal Geographical Society

The Tangent micrometer will be valuable when circumstances prevent the use of micrometer microscopes (and sometimes the extra bulk and large size of the boxes is a difficulty) A theodolite fitted with tangent micrometers is cheaper than one fitted with micrometer-microscopes.

By means of the addition of a micrometer drum combined with an indicator, a carefully constructed tangent screw serves also as a [illegible] consider- [illegible] to the [illegible] of this micrometer [illegible]

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THE METRE = 1.093623 yard
= 39.37043 inches

THE YARD = 0.914392 metre

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A [illegible] which can be readily fitted to any theodolite by means of which angles can be read with much greater accuracy than by a Vernier, has been recently designed by Mr E. A. Reeves of the Royal Geographical Society

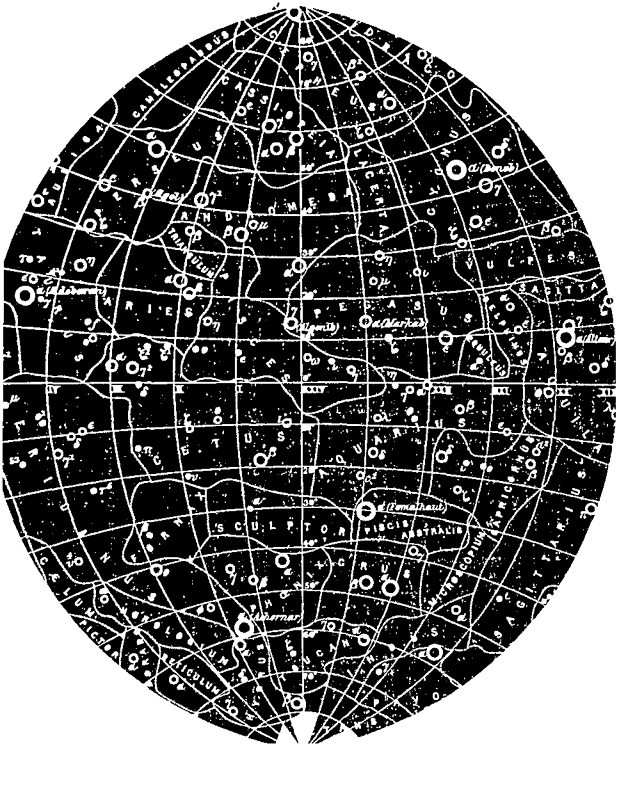
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STAR CHART

SHOWING ALL N.A. STARS (1904)

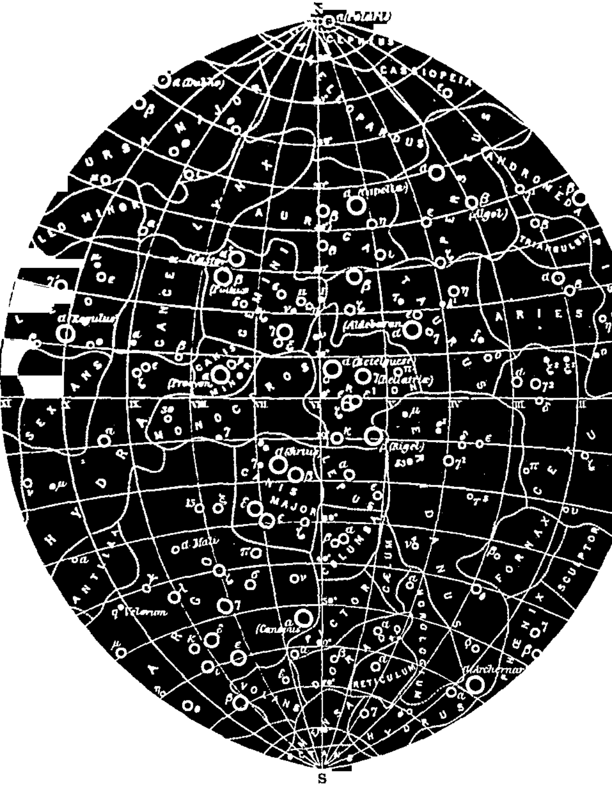
up to 4.5 mag in four plates



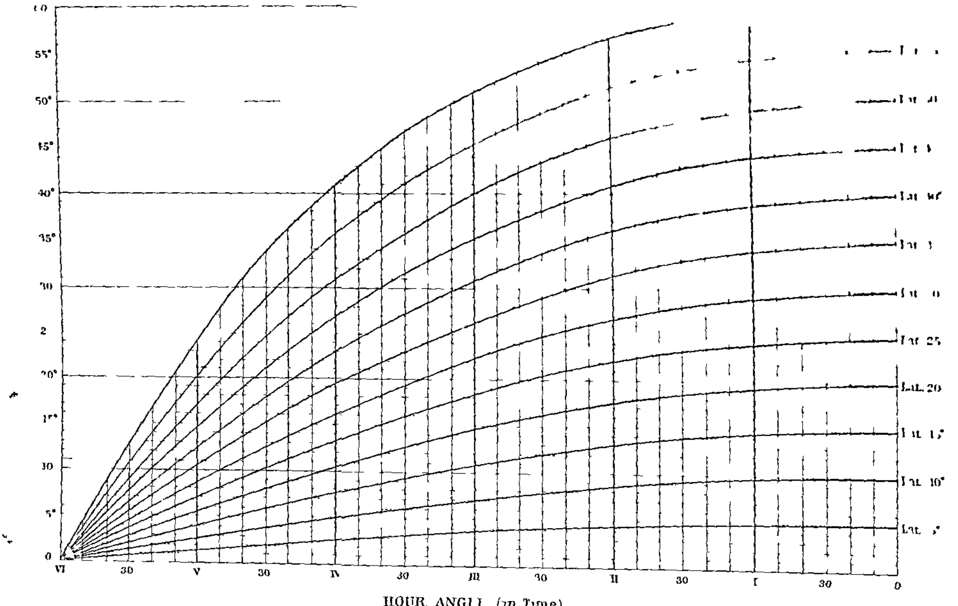
STAR CHART

SHOWING ALL N A STARS (1904)

up to 15 mag in four plates



MAGNITUDES


55°
50°
45°
40°
35°
20°
5°
0
VI
30
V
30
IV
30
III
30
II
30
I
30
0
Lat. 20
Lat. 10°
HOUR ANGLE (in Time)

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